MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 1

Candidate number

Thursday 6 May 2004 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Find

(a) $\dfrac{\mathrm{d}}{\mathrm{d}x}(3x^4 - 5x + 1)$;

(b) $\int (3x^4 - 5x + 1)\mathrm{d}x$.

Working:

Answers:

(a) ______________________

(b) ______________________

2. (a) Complete the row of Pascal's triangle which begins

1, 6, 15, ……………

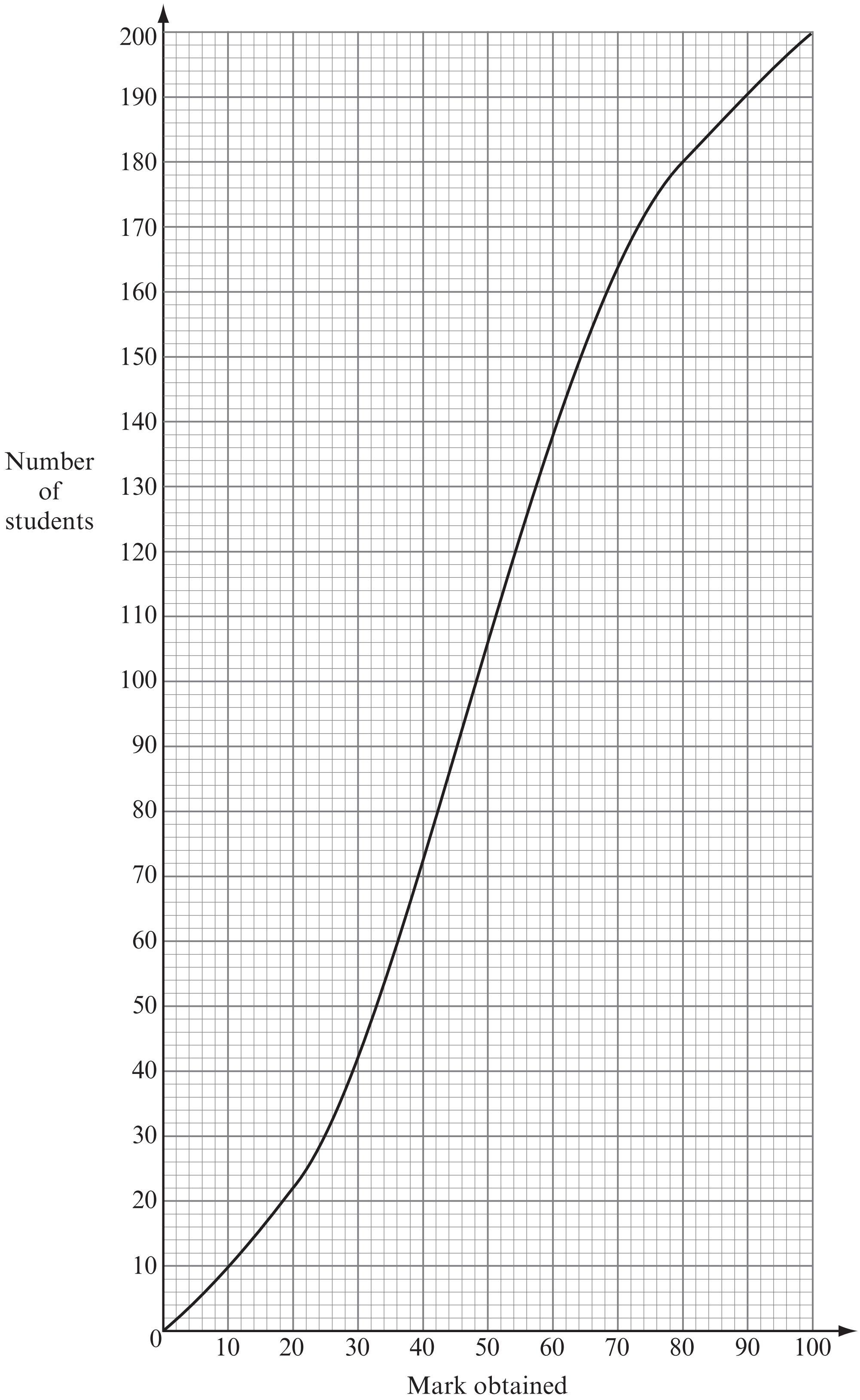
(b) Find the coefficient of the term in x^8 in the expansion of $(1+x^2)^6$.

Working:

Answer:

(b) ________________________________

3. The cumulative frequency curve below shows the marks obtained in an examination by a group of 200 students.



(This question continues on the following page)

(Question 3 continued)

(a) Use the cumulative frequency curve to complete the frequency table below.

Mark (x)	$0 \le x < 20$	$20 \le x < 40$	$40 \le x < 60$	$60 \le x < 80$	$80 \le x < 100$
Number of students	22				20

(b) Forty percent of the students fail. Find the pass mark.

Working:

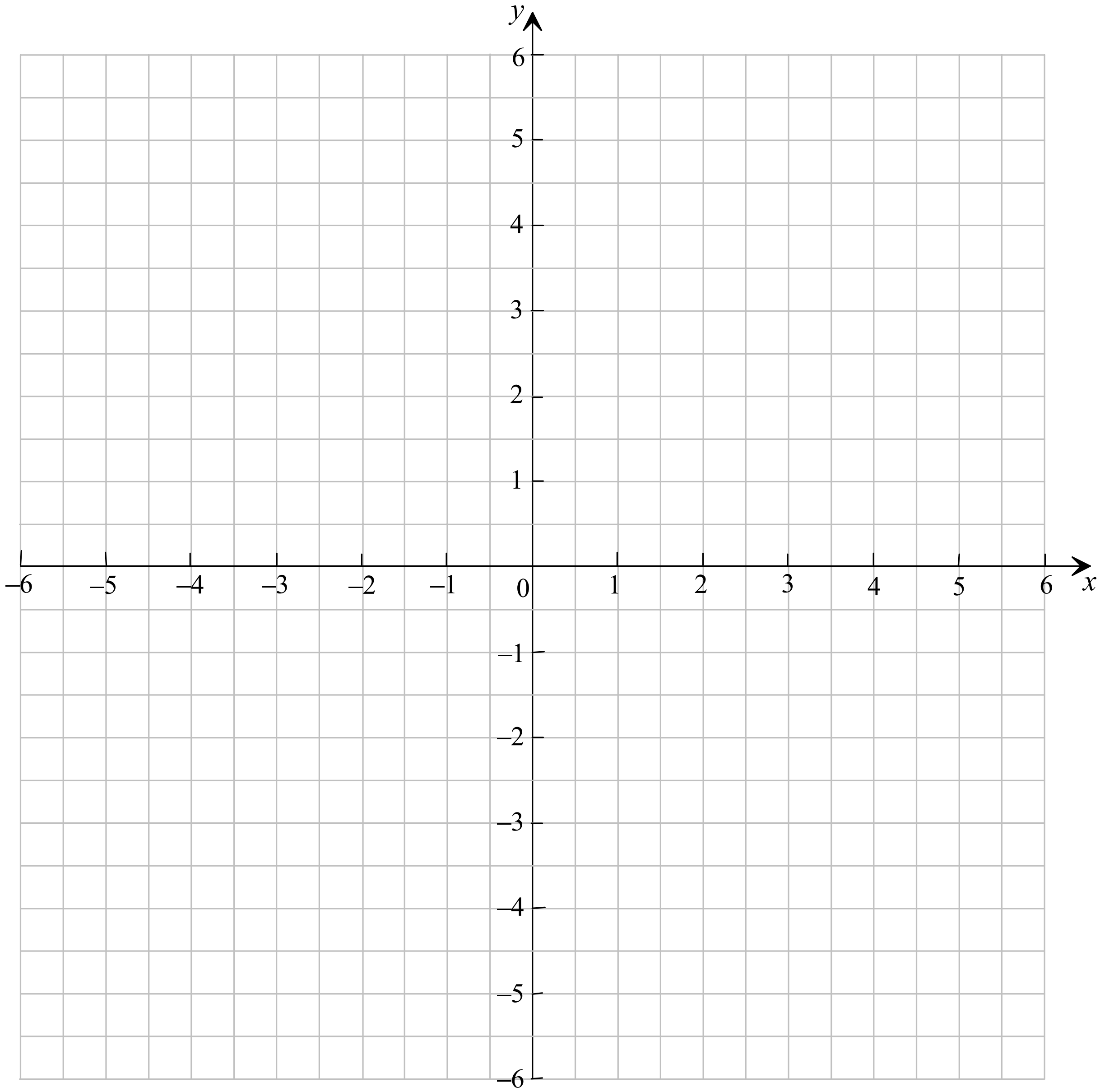
Answer:

(b) ____________________

4. Let $f(x) = 2x + 1$.

(a) On the grid below draw the graph of $f(x)$ for $0 \le x \le 2$.

(b) Let $g(x) = f(x+3) - 2$. On the grid below draw the graph of $g(x)$ for $-3 \le x \le -1$.

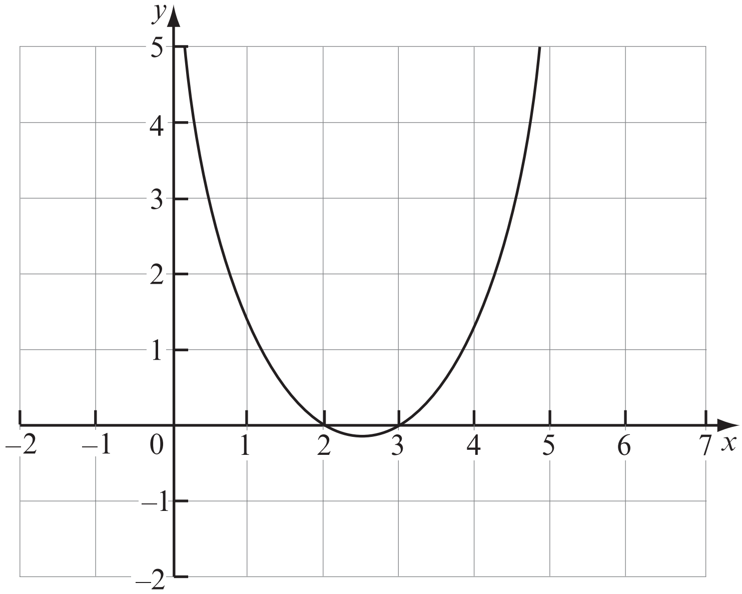


(This question continues on the following page)

(Question 4 continued)

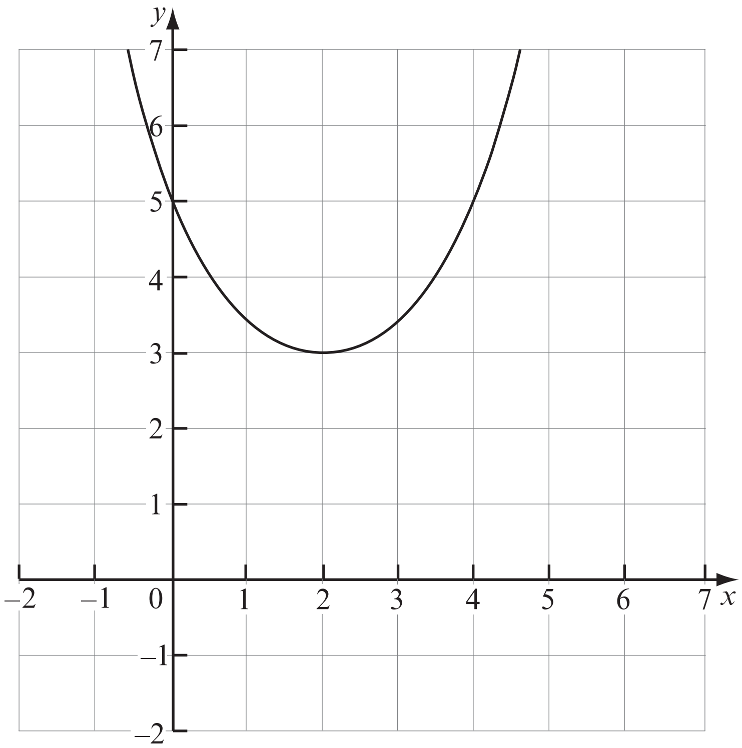
Working:

5. (a) The diagram shows part of the graph of a quadratic function $f(x) = x^2 + bx + c$, which intersects the x-axis at $x = 2$ and at $x = 3$.



Find the value of b and of c.

(b) The diagram shows part of the graph of another quadratic function g. It can be written in the form $g(x) = a(x-h)^2 + 3$. Its vertex is at (2, 3) and its y-intercept is 5.



(i) Write down the value of h.

(ii) Find the value of a.

(This question continues on the following page)

(Question 5 continued)

Working:

Answers:

(a) $b =$ ______

$c =$ ______

(b)(i) $h =$ ______

(ii) $a =$ ______

6. Let $f(x) = \frac{8}{x}$ and $g(x) = x^2$.

(a) Find $f^{-1}(x)$.

(b) (i) Write down $(f^{-1} \circ g)(x)$.

(ii) Solve the equation $(f^{-1} \circ g)(x) = x$.

Working:

Answers:

(a) ______________________

(b)(i) ______________________

(ii) ______________________

7. The displacement, s metres, of a car t seconds after it starts from a fixed point A is given by $s = 4t + 5 - 5e^{-t}$.

(a) Find an expression for its velocity (in ms^{-1}) after t seconds.

(b) Find the acceleration (in ms^{-2}) at A.

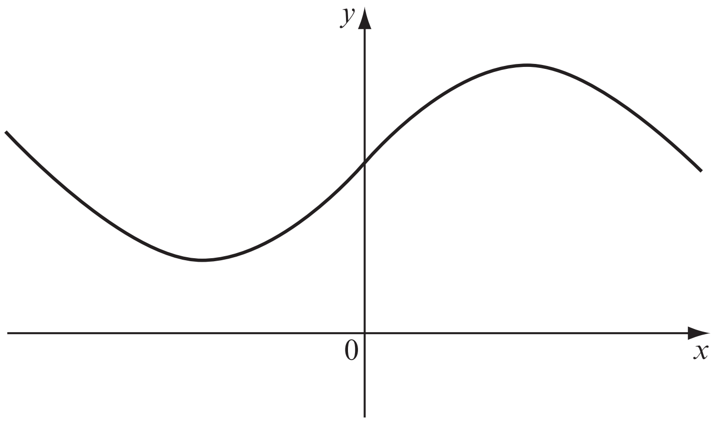
Working:

Answers:

(a) ______________________

(b) ______________________

8. The diagram shows part of the graph of a sine curve with equation $f(x) = p + q \sin kx$.
The period is 4π, the minimum value is 3 and the maximum value is 11.



Find the value of

(a) (i) p;

(ii) q.

(b) k.

Working:

Answers:

(a)(i) ____________________

(ii) ____________________

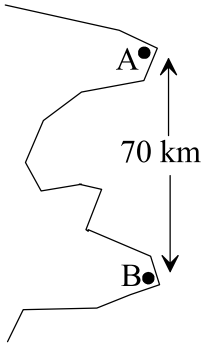
(b) ____________________

9. Solve the equation $2\cos^2 x = \sin 2x$ for $0 \le x \le \pi$, giving your answers in terms of π.

Working:

Answers:

10. The diagram shows two ports, A and B, which are 70km apart. A boat sets out from A travelling in a straight line at 25 kmh^{-1}. At the same time a second boat sets out from B travelling in a straight line at 50 kmh^{-1}. They meet two hours later at point C.



(a) Complete the diagram to represent this information.

(b) Calculate the size of the acute angle between their paths.

Working:

Answer:

(b) ______________________

11. The following table shows four series of numbers. One of these series is geometric, one of the series is arithmetic and the other two are neither geometric nor arithmetic.

(a) Complete the table by stating the type of series that is shown.

Series		Type of series
(i)	$1+11+111+1111+11111+\dots$	
(ii)	$1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\dots$	
(iii)	$0.9+0.875+0.85+0.825+0.8+\dots$	
(iv)	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\dots$	

(b) The geometric series can be summed to infinity. Find this sum.

Working:

Answer:

(b) ______________________

12. Write each of the following in its simplest form

(a) $e^{\ln x}$;

(b) $e^{(\ln x + \ln y)}$;

(c) $\ln(e^{x+y})^2$.

Working:

Answers:

(a) ______________________

(b) ______________________

(c) ______________________

13. Two lines L_1 and L_2 have these vector equations.

$$L_1 : \boldsymbol{r} = 2\boldsymbol{i} + 3\boldsymbol{j} + t(\boldsymbol{i} - 3\boldsymbol{j})$$
$$L_2 : \boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + s(\boldsymbol{i} - \boldsymbol{j})$$

The angle between L_1 and L_2 is θ. Find the cosine of the angle θ.

Working:

Answer:

14. The equation $x^2 + kx + 9 = 0$ has two distinct real roots. Find the set of all possible values of k.

Working:

Answer:

15. A collection of five whole numbers has a mode of 3, a median of 4 and a mean of 5. List all the possible collections of five numbers.

Working:

Answer: