MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 1

Candidate number

Thursday 6 May 2004 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$.

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$.

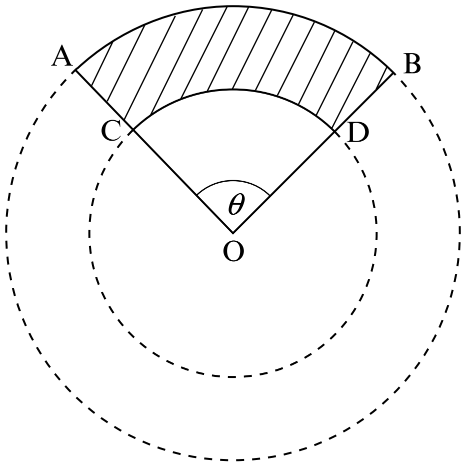
Working:

Answers:

(a) ______________________

(b) ______________________

2. The diagram below shows two circles which have the same centre O and radii 16 cm and 10 cm respectively. The two arcs AB and CD have the same sector angle $\theta = 1.5$ radians.

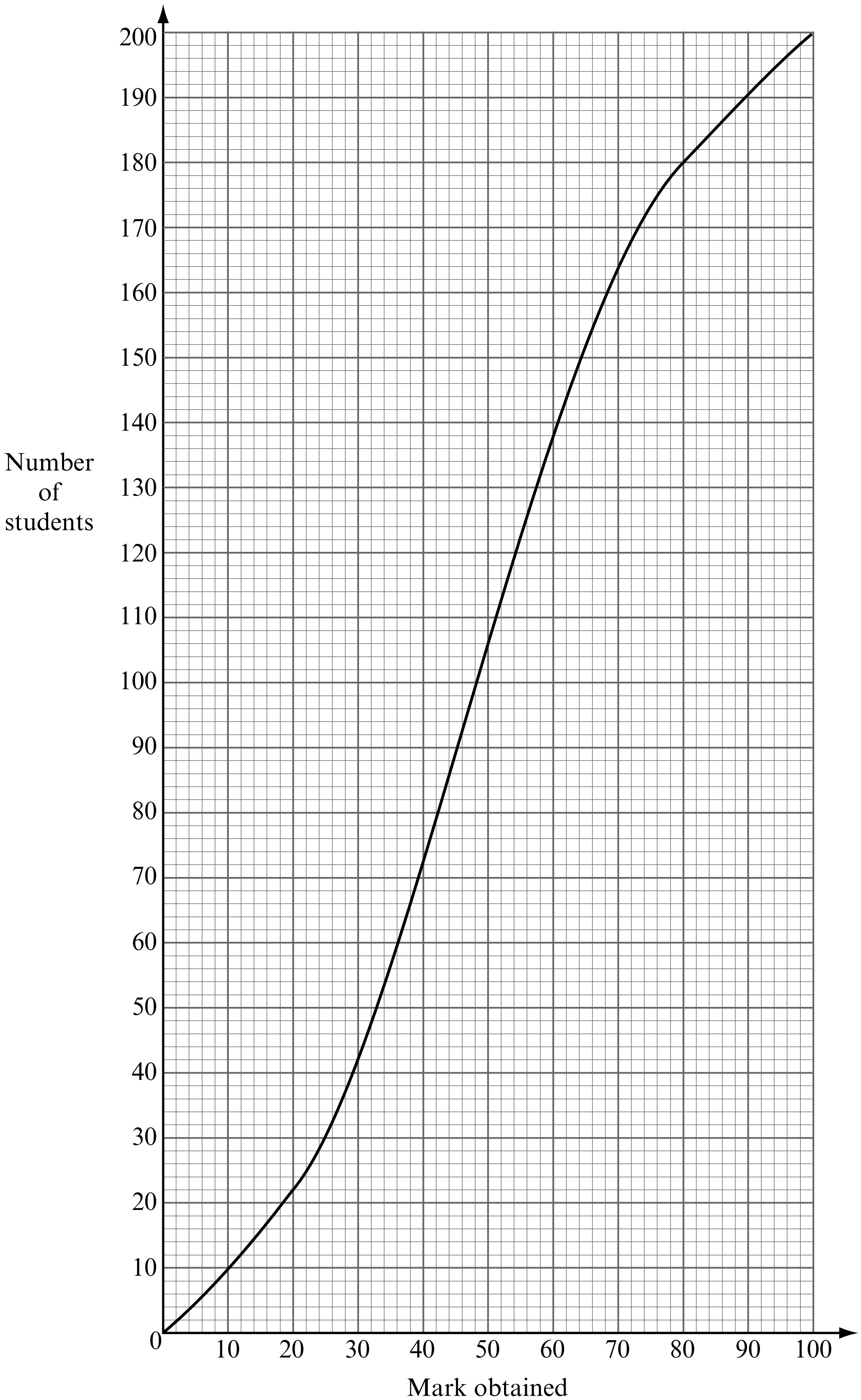


Find the area of the shaded region.

Working:

Answer:

3. The cumulative frequency curve below shows the marks obtained in an examination by a group of 200 students.



(This question continues on the following page)

(Question 3 continued)

(a) Use the cumulative frequency curve to complete the frequency table below.

Mark (x)	$0 \le x < 20$	$20 \le x < 40$	$40 \le x < 60$	$60 \le x < 80$	$80 \le x < 100$
Number of students	22				20

(b) Forty percent of the students fail. Find the pass mark.

Working:

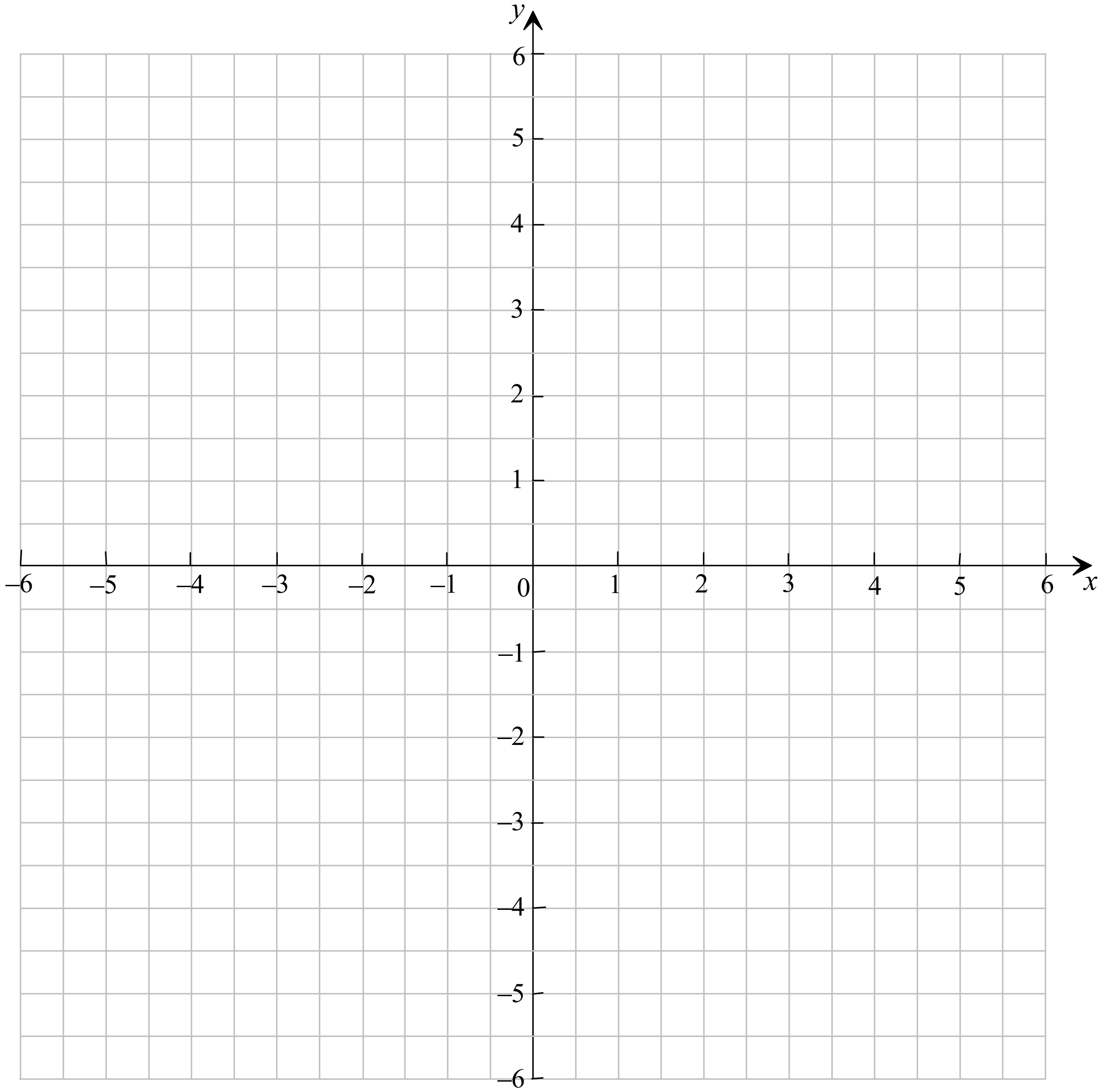
Answer:

(b) __________________________________

4. Let $f(x) = 2x + 1$.

(a) On the grid below draw the graph of $f(x)$ for $0 \le x \le 2$.

(b) Let $g(x) = f(x+3) - 2$. On the grid below draw the graph of $g(x)$ for $-3 \le x \le -1$.



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(Question 4 continued)

Working:

5. Let A and B be events such that $\mathrm{P}(A) = \frac{1}{2}$, $\mathrm{P}(B) = \frac{3}{4}$ and $\mathrm{P}(A \cup B) = \frac{7}{8}$.

(a) Calculate $\mathrm{P}(A \cap B)$.

(b) Calculate $\mathrm{P}(A|B)$.

(c) Are the events A and B independent? Give a reason for your answer.

Working:

Answers:

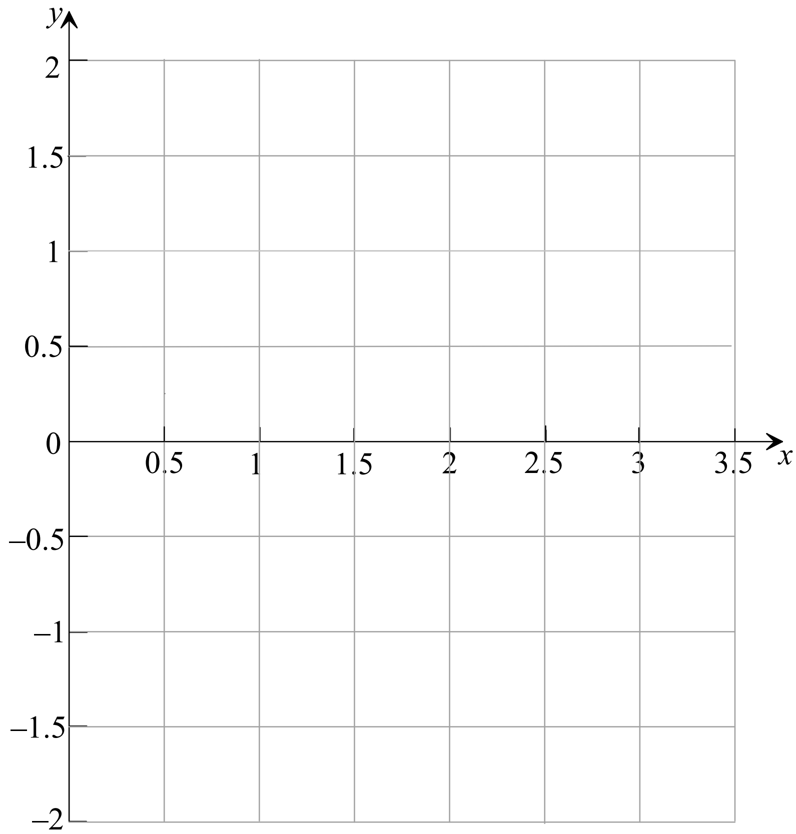
(a) ______________________________

(b) ______________________________

(c) ______________________________

6. Let $f(x) = \sin(2x+1)$, $0 \le x \le \pi$.

(a) Sketch the curve of $y = f(x)$ on the grid below.



(b) Find the x-coordinates of the maximum and minimum points of $f(x)$, giving your answers correct to one decimal place.

Working:

Answer:

(b) ______________________

7. Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10}\left(\dfrac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

Working:

Answer:

8. In a triangle ABC, $\text{AB} = 4\text{ cm}$, $\text{AC} = 3\text{ cm}$ and the area of the triangle is 4.5 cm^2.

Find the **two** possible values of the angle BAC.

Working:

Answers:

9. Solve the equation $2\cos^2 x = \sin 2x$ for $0 \le x \le \pi$, giving your answers in terms of π.

Working:

Answers:

10. A car starts by moving from a fixed point A. Its velocity, $v\,\mathrm{m\,s^{-1}}$ after t seconds is given by $v = 4t + 5 - 5\mathrm{e}^{-t}$. Let d be the displacement from A when $t = 4$.

(a) Write down an integral which represents d.

(b) Calculate the value of d.

Working:

Answers:

(a) ______________________________

(b) ______________________________

11. The following table shows four series of numbers. One of these series is geometric, one of the series is arithmetic and the other two are neither geometric nor arithmetic.

(a) Complete the table by stating the type of series that is shown.

Series		Type of series
(i)	$1+11+111+1111+11111+\dots$	
(ii)	$1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\dots$	
(iii)	$0.9+0.875+0.85+0.825+0.8+\dots$	
(iv)	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\dots$	

(b) The geometric series can be summed to infinity. Find this sum.

Working:

Answer:

(b) ______________________________

12. The table below shows the marks gained in a test by a group of students.

Mark	1	2	3	4	5
Number of students	5	10	p	6	2

The median is 3 and the mode is 2. Find the **two** possible values of p.

Working:

Answers:

13. Two lines L_1 and L_2 have these vector equations.

$$L_1 : \boldsymbol{r} = 2\boldsymbol{i} + 3\boldsymbol{j} + t(\boldsymbol{i} - 3\boldsymbol{j})$$
$$L_2 : \boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} + s(\boldsymbol{i} - \boldsymbol{j})$$

The angle between L_1 and L_2 is θ. Find the cosine of the angle θ.

Working:

Answer:

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14. The equation $x^2 - 2kx + 1 = 0$ has two distinct real roots. Find the set of all possible values of k.

Working:

Answer:

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15. When the expression $(2+ax)^{10}$ is expanded, the coefficient of the term in x^3 is 414 720. Find the value of a.

Working:

Answer:

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