INTERNATIONAL BACCALAURÉAT BACHILLERATO IBO BACCALAUREATE INTERNATIONAL INTERNACIONAL

M04/521/S(2)M+

MARKSCHEME

May 2004

MATHEMATICAL METHODS

Standard Level

Paper 2

23 pages

QUESTION 1

(a) (i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ ***(A1)*** ***(N2)***

(ii) $|\overrightarrow{AB}| = \sqrt{25+1}$ ***(M1)***

$= \sqrt{26}\ (= 5.10 \text{ to } 3 \text{ s.f.})$ ***(A1)*** ***(N2)***

Note: An answer of 5.1 is subject to **AP**.

[4 marks]

(b) $\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix} d \\ 23 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} d-2 \\ 25 \end{pmatrix}$ ***(A1)(A1)***

[2 marks]

(c) (i) **EITHER**

$B\hat{A}D = 90^\circ \Rightarrow \overrightarrow{AB} \bullet \overrightarrow{AD} = 0$ or mention of scalar (dot) product. ***(M1)***

$\Rightarrow \begin{pmatrix} -5 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} d-2 \\ 25 \end{pmatrix} = 0$

$-5d + 10 + 25 = 0$ ***(A1)***

$d = 7$ ***(AG)***

OR

Gradient of AB $= -\dfrac{1}{5}$

Gradient of AD $= \dfrac{25}{d-2}$ ***(A1)***

$\left(\dfrac{25}{d-2}\right) \times \left(-\dfrac{1}{5}\right) = -1$ ***(A1)***

$\Rightarrow d = 7$ ***(AG)***

(ii) $\overrightarrow{OD} = \begin{pmatrix} 7 \\ 23 \end{pmatrix}$ (correct answer only) ***(A1)***

[3 marks]

continued…

Question 1 continued

(d) $\vec{AD} = \vec{BC}$ ***(M1)***

$\vec{BC} = \begin{pmatrix} 5 \\ 25 \end{pmatrix}$ ***(A1)***

$\vec{OC} = \vec{OB} + \vec{BC}$ ***(M1)***

$\vec{OC} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ 25 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 24 \end{pmatrix}$ ***(A1)*** ***(N3)***

Note: Many other methods, including scale drawing, are acceptable.

[4 marks]

(e) $|\vec{AD}| \left(\text{or } |\vec{BC}| \right) = \sqrt{5^2 + 25^2} = \sqrt{650}$ ***(A1)***

Area $= \sqrt{26} \times \sqrt{650}$ $(= 5.099 \times 25.5)$

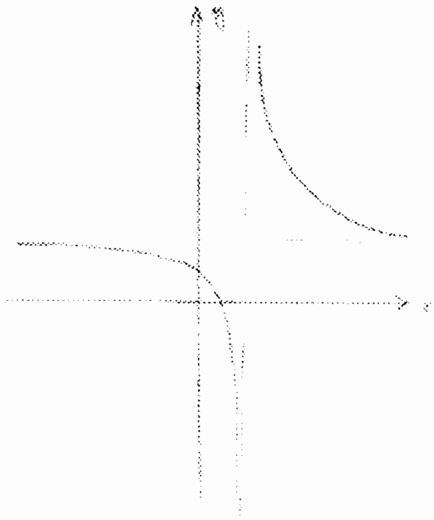
$= 130$ ***(A1)***

[2 marks]

Total [15 marks]

QUESTION 2

(a) *(A1)(A1)*



Note: Award *(A1)* for a second branch in approximately the correct position, and *(A1)* for the second branch having positive x and y intercepts. Asymptotes need not be drawn.

[2 marks]

(b) (i) x-intercept $= \frac{1}{2}$ $\left(\text{Accept } \left(\frac{1}{2}, 0\right),\ x = \frac{1}{2}\right)$ *(A1)*

y-intercept $= 1$ (Accept $(0, 1)$, $y = 1$) *(A1)*

(ii) horizontal asymptote $y = 2$ *(A1)*

vertical asymptote $x = 1$ *(A1)*

[4 marks]

(c) (i) $f'(x) = 0 - (x-1)^{-2}$ $\left(= \frac{-1}{(x-1)^2}\right)$ *(A2)*

(ii) no maximum / minimum points.

since $\frac{-1}{(x-1)^2} \neq 0$. *(R1)*

[3 marks]

(d) (i) $2x + \ln(x-1) + c$ (accept $\ln|x-1|$) *(A1)(A1)(A1)*

(ii) $A = \int_2^4 f(x)\,dx$ $\left(\text{Accept } \int_2^4 \left(2 + \frac{1}{x-1}\right)dx,\ \left[2x + \ln(x-1)\right]_2^4\right)$ *(M1)(A1)*

Notes: Award *(A1)* for **both** correct limits. Award *(M0)(A0)* for an incorrect function.

(iii) $A = \left[2x + \ln(x-1)\right]_2^4$

$= (8 + \ln 3) - (4 + \ln 1)$ *(M1)*

$= 4 + \ln 3 (= 5.10, \text{ to } 3 \text{ s.f.})$ *(A1)* *(N2)*

[7 marks]

Total [16 marks]

QUESTION 3

(a) (i) $10+4\sin 1=13.4$ ***(A1)***

(ii) At 2100, $t=21$ ***(A1)***

$10+4\sin 10.5=6.48$ ***(A1)*** *(N2)*

Note:	Award ***(A0)(A1)*** if candidates use $t=2100$ leading to $y=12.6$. No other **ft** allowed.

[3 marks]

(b) (i) 14 metres ***(A1)***

(ii) $14=10+4\sin\left(\frac{t}{2}\right) \Rightarrow \sin\left(\frac{t}{2}\right)=1$ ***(M1)***

$\Rightarrow t=\pi$ (3.14) (correct answer only) ***(A1)*** *(N2)*

[3 marks]

(c) (i) 4 ***(A1)***

(ii) $10+4\sin\left(\frac{t}{2}\right)=7$ ***(M1)***

$\Rightarrow \sin\left(\frac{t}{2}\right)=-0.75$ ***(A1)***

$\Rightarrow t=7.98$ ***(A1)*** *(N3)*

(iii) depth <7 from $8-11=3$ hours ***(M1)***

from 2030 – 2330 = 3 hours ***(M1)***

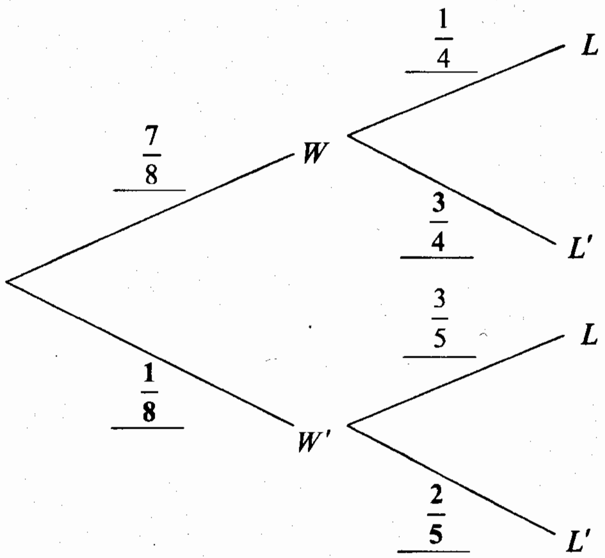
therefore, total = 6 hours ***(A1)*** *(N3)*

[7 marks]

Total [13 marks]

QUESTION 4

(a)



(A1)(A1)(A1)(A1)

Note: Award *(A1)* for the given probabilites $\left(\frac{7}{8}, \frac{1}{4}, \frac{3}{5}\right)$ in the correct positions, and *(A1)* for each **bold** value.

[4 marks]

(b) Probability that Dumisani will be late is $\frac{7}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{3}{5}$ *(A1)(A1)*

$= \frac{47}{160} (0.294)$ *(A1)* *(N2)*

[3 marks]

(c) $P(W|L) = \frac{P(W \cap L)}{P(L)}$

$P(W \cap L) = \frac{7}{8} \times \frac{1}{4}$ *(A1)*

$P(L) = \frac{47}{160}$ *(A1)*

$P(W|L) = \frac{\frac{7}{32}}{\frac{47}{160}}$ *(M1)*

$= \frac{35}{47} (= 0.745)$ *(A1)* *(N3)*

[4 marks]

Total [11 marks]

QUESTION 5

(a) (i) 2420 *(A1)*

(ii) $1420 + 100n > 2000$ *(M1)*

$n > 5.8$

1999 (accept 6^{th} year or $n = 6$) *(A1)* *(N1)*

Note: Award *(A0)* for 2000, or after 6 years, or $n = 6$, 2000.

[3 marks]

(b) (i) $1\,200\,000(1.025)^{10} = 1\,536\,101$ (accept $1\,540\,000$ or 1.54 (million)) *(A1)*

(ii) $\dfrac{1\,536\,101 - 1\,200\,000}{1\,200\,000} \times 100$ *(M1)*

28.0 % (accept 28.3 % from $1\,540\,000$) *(A1)* *(N2)*

(iii) $1\,200\,000\,(1.025)^n > 2\,000\,000$ (accept an equation) *(M1)*

$n \log 1.025 > \log\left(\dfrac{2}{1.2}\right) \Rightarrow n > 20.69$ *(M1)(A1)*

2014 (accept 21^{st} year or $n = 21$) *(A1)* *(N3)*

Notes: Award *(A0)* for 2015, after 21 years, or $n = 21$, so 2015.

[7 marks]

(c) (i) $\dfrac{1\,200\,000}{1420} = 845$ *(A1)*

(ii) $\dfrac{1\,200\,000(1.025)^n}{1420 + 100n} < 600$ *(M1)(M1)*

$\Rightarrow n > 14.197$

15 years *(A2)* *(N2)*

[5 marks]

Total [15 marks]

QUESTION 6

(i) (a) (i) **EITHER**

$\text{P(men)} \times \text{P(no)} \times \text{Total}$ (may be implied) *(M1)*

$a = \left(\frac{40}{75} \times \frac{21}{75}\right) \times 75$ *(A1)(A1)*

$a = 11.2$ *(AG)*

OR

$\frac{\text{(row total)} \times \text{(column total)}}{\text{total}}$ (may be implied) *(M1)*

$a = \frac{40 \times 21}{75}$ *(A1)(A1)*

$a = 11.2$ *(AG)*

Note: Award ***(M0)(A0)*** for showing the matrix obtained from GDC.

(ii) $d = 13.1$ *(A1)*

(iii) $\chi^2_{\text{calc}} = 4.15$ (Accept $4.08 \le \chi^2 \le 4.15$) *(A2)*

[6 marks]

(b) **EITHER**

critical value of $\chi^2 = 4.605$ *(A1)*

since $\chi^2_{\text{calc}} (= 4.15) < 4.605$, (answers are independent of gender) *(R2)*

OR

$p = 0.126$ *(A1)*

since $p > 0.100$ (answers are independent of gender) *(R2)*

[3 marks]

continued…

Question 6 continued

(ii) (a) a two-tailed test *(A1)*

[1 mark]

Note: In parts (b) and (c), award no marks to candidates who omit $\sqrt{36}$.

(b) (i) $Z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}} \quad \left(= \dfrac{79 - 82}{\dfrac{8}{\sqrt{36}}} \right)$ *(M1)*

$= -2.25$ *(A1)* *(N2)*

(ii) $2 \times \mathrm{P}(Z > 2.25)$ *(M1)*

$= 0.0244$ *(A1)*

since $0.0244 < 0.05$ reject $\mathbf{H_0}$ *(R1)* *(N3)*

Note: Award ***(A0)*** for the answer "reject H_0" with no explanation.

[5 marks]

(c) $Z = \left(\dfrac{79 - 80}{\dfrac{8}{\sqrt{36}}} \right) = -0.75$ *(A1)*

$\mathrm{P}(Z < -0.75) = 0.227$ (3 s.f.) *(A1)* *(N2)*

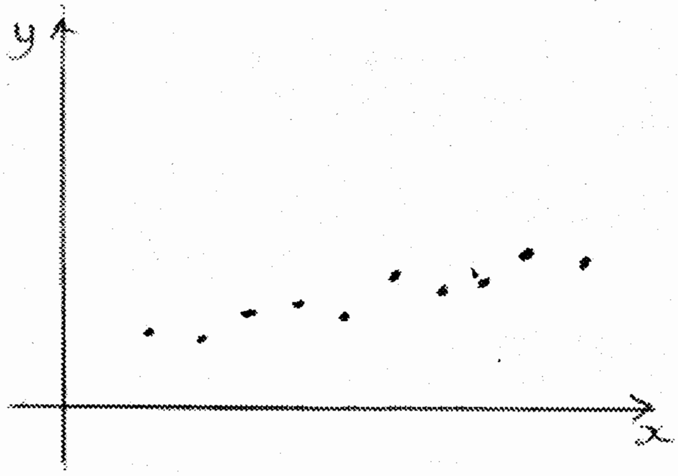
[2 marks]

continued…

Question 6 continued

(iii) (a) (i) minimum value $= -1$; maximum value $= 1$ ***(A1)(A1)***

(ii)



(A1)

(iii) linear, strong positive ***(A2)***

[5 marks]

(b) (i) regression line passes through $(\bar{x}, \bar{y})$ ***(M1)***

gradient of regression line $= \dfrac{49.2-46}{660-500} = 0.02$ ***(A1)***

equation of regression line: $\dfrac{y-46}{x-500} = 0.02$ $(\Rightarrow y = 0.02x + 36)$ ***(A1)*** ***(N3)***

(ii) $y = \$\,47$ ***(A1)***

[4 marks]

(c) $46 \pm 1.96\dfrac{8.5}{\sqrt{15}}$ ***(A1)(A1)(A1)***

confidence interval is (41.7, 50.3) ***(A1)*** ***(N4)***

[4 marks]

Total [30 marks]

QUESTION 7

(i) (a) $x^2 \sin(x^3 + \pi) = 0\left(\text{Accept } \sin(x^3 + \pi) = 0 \text{ or } x^3 + \pi = n\pi\right)$ ***(M1)***

$x = (2\pi)^{\frac{1}{3}}$ $(= 1.85)$ (Accept $(1.85, 0)$) ***(A1)*** ***(N2)***

[2 marks]

(b) $(u = x^3 + \pi) \Rightarrow \mathrm{d}u = 3x^2\,\mathrm{d}x$ ***(M1)***

$\int f(x)\,\mathrm{d}x = \frac{1}{3}\int \sin u\,\mathrm{d}u$ ***(A1)***

$= -\frac{1}{3}\cos(x^3 + \pi) + c$ ***(A1)(A1)*** ***(N4)***

Note: Award the final ***(A1)*** for the constant of integration.

[4 marks]

(c) $A = \int_{\pi^{\frac{1}{3}}}^{(2\pi)^{\frac{1}{3}}} f(x)\,\mathrm{d}x$

$= \left[-\frac{1}{3}\cos(x^3 + \pi)\right]_{\pi^{\frac{1}{3}}}^{(2\pi)^{\frac{1}{3}}}$ ***(A1)***

$= -\frac{1}{3}\left\{\cos(2\pi + \pi) - \cos(\pi + \pi)\right\}$ ***(A1)***

$= \frac{2}{3} (= 0.667)$ ***(A1)*** ***(N1)***

[3 marks]

continued…

Question 7 continued

(ii) (a) (i) perimeter of $R = 2(1+x)$ ***(A1)***

(ii) perimeter of $Q = 4\sqrt{1+x^2}$ ***(A1)***

[2 marks]

(b) $$g'(x) = \frac{0.5\sqrt{1+x^2} - (0.5)(x+1)\frac{2x}{2\sqrt{1+x^2}}}{(\sqrt{1+x^2})^2}$$ ***(A1)(A1)***

Note: Award ***(A1)*** for correctly using the quotient rule, ***(A1)*** for using the chain rule correctly to find $\frac{\mathrm{d}}{\mathrm{d}x}(\sqrt{1+x^2})$.

$$g'(x) = \frac{0.5(1+x^2) - (0.5)(x+1)x}{(1+x^2)^{\frac{3}{2}}}$$ (any evidence of correct simplification) ***(A1)***

$$= \frac{0.5(1-x)}{(1+x^2)^{\frac{3}{2}}}$$ ***(AG)***

[3 marks]

(c) For seeing $g'(x) = 0$ in some form ***(M1)***

$0.5(1-x) = 0$ ***(A1)***

$x = 1$ ***(A1)***

maximum value is $\frac{1}{\sqrt{2}} (= 0.707)$ ***(A2)*** ***(N5)***

[5 marks]

(iii) (a) $f'(x) = 5x^4$ ***(A1)***

$$x_{n+1} = x_n - \frac{x_n^5 - 5}{5x_n^4}$$ ***(A1)***

$$= \frac{4x_n^5 + 5}{5x_n^4}$$ ***(A1)***

$$= 0.8x_n + \frac{1}{x_n^4}$$ ***(AG)***

[3 marks]

continued…

Question 7 (iii) continued

(b) (i) $(x_1 = 1)$

$(x_2 = 1.8)$

$x_3 = 1.53526$ ***(A1)***

$x_4 = 1.40821$

$x_5 = 1.38086$

$x_6 = 1.379731505 = 1.3797$ (5 s.f.)

$x_7 = 1.379729661 = 1.3797$ (5 s.f.) ***(M1)***

Note:	Award ***(A1)*** for evidence of N-R method, *e.g.* x_3 or x_4 or x_5 or x_6. Award ***(M1)*** for showing the error is < 0.0001 by showing either there is no change in the 5th digit or by comparison with $\sqrt[5]{5}$.

(ii) root $= 1.3797$ ***(A1)***

Note:	Do not award this ***(A1)*** if the ***(M1)*** is not earned in part (b) above.

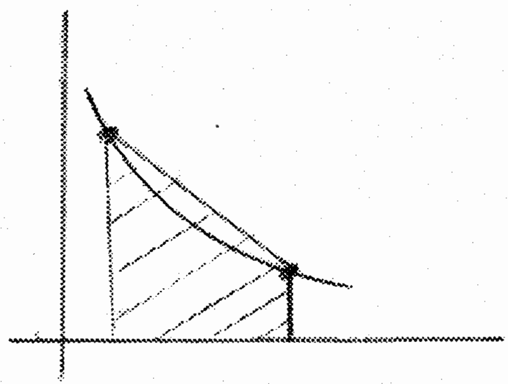
[3 marks]

(iv) (a) area of shaded region $S = \int_1^2 \frac{1}{x}\mathrm{d}x = 0.693(147\ldots)$ ***(A1)***

$= 0.69315$ (5 s.f.) (correct answer only) ***(A1)***

[2 marks]

(b) trapezium rule will **overestimate** the area ***(R1)***
because the graph is concave up or shown in a diagram ***(R1)***



Carl used trapezium rule ***(A1)***

Note:	Award ***(R0)(R0)(A0)*** for "Carl used trapezium rule" without a (correct) reason.

[3 marks]

Total [30 marks]

QUESTION 8

(i) (a) (i) $\boldsymbol{H} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ *(A1)*

(ii) $\boldsymbol{S} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ *(A1)*

(iii) $\boldsymbol{R} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ *(A1)*

[3 marks]

(b) $\boldsymbol{H}^{-1}$ = shear of scale factor -2 in the direction of the x-axis *(A1)*

$\boldsymbol{S}^{-1}$ = stretch of scale factor $\frac{1}{2}$ in the direction of the y-axis *(A1)*

$\boldsymbol{R}^{-1}$ = reflection in the x-axis *(A1)*

Notes:	All components of the description are needed to receive marks. Award no marks if the inverse matrix is given.

[3 marks]

(c) (i) $\boldsymbol{M} = \boldsymbol{HSR}$ *(A1)*

(ii) $\begin{pmatrix} 1 & -4 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ *(M1)*

$$\left.\begin{aligned} x - 4y &= x \\ -2y &= y \end{aligned}\right\}$$ *(A1)*

all points $(x, 0)$ are invariant under $\boldsymbol{M}$. *(A2)* *(N3)*

(iii) **EITHER**

$\begin{pmatrix} 1 & -4 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} t \\ -t+2 \end{pmatrix} = \begin{pmatrix} 5t-8 \\ 2t-4 \end{pmatrix}$ *(A1)*

OR

$x = 5t - 8$
$y = 2t - 4$ *(A1)*

THEN

Image $y = \frac{2}{5}x - \frac{4}{5}$ *(A1)(A1)* *(N2)*

Notes:	One alternative method is to find two points on $y = -x + 2$, find their images and then find the line between them. Another valid alternative method is to express x and y in terms of x' and y' using $\boldsymbol{M}^{-1}$.

[8 marks]

continued...

Question 8 (i) continued

(d) (i) $\boldsymbol{Mu} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, $\boldsymbol{Mv} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ ***(A1)***

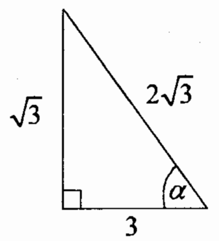
(ii) $\boldsymbol{Mw} = 3\boldsymbol{Mu} - 2\boldsymbol{Mv}$ $\left(\text{or } = 3\begin{pmatrix} -3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 4 \\ -1 \end{pmatrix}\right)$ ***(M1)***

$a = -17$; $b = 8$ ***(A1)(A1)*** ***(N2)***

Note: $\boldsymbol{u}$ and $\boldsymbol{v}$ may be found using $\boldsymbol{M}^{-1}$.

[4 marks]

(ii) (a) (i) **EITHER**



(M1)

OR

$\tan\alpha = \frac{\sqrt{3}}{3}$ ***(M1)***

THEN

$\sin\alpha = \frac{1}{2}$, $\cos\alpha = \frac{\sqrt{3}}{2}$ ***(A1)***

$\boldsymbol{F} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ ***(A1)(A1)*** ***(N3)***

(ii) $(0, 2)$ (or any point on L) is invariant under $\boldsymbol{T}$. ***(M1)***

$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ ***(A1)***

$h = -\sqrt{3}, k = 3$ ***(A1)(A1)***

$\left(\text{vector} \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}\right)$ ***(N3)***

[8 marks]

continued…

Question 8 (ii) continued

(b) (i) **EITHER**

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}$$ ***(R1)***

OR

$$\boldsymbol{T}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} -\sqrt{3} \\ 3 \end{pmatrix}$$ ***(R1)***

(ii) $d = \frac{1}{2}$ distance from $(0, 0)$ to $\boldsymbol{T}\ (0, 0)$. ***(M1)***

$d = \frac{1}{2}\sqrt{(-\sqrt{3})^2 + 3^2}$ ***(A1)***

$d = \sqrt{3}$ ***(A1)*** ***(N2)***

[4 marks]

Total [30 marks]