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M04/521/S(1)M+

MARKSCHEME

May 2004

MATHEMATICAL METHODS

Standard Level

Paper One

13 pages

QUESTION 1

(a) $f'(x) = 3x^2 - 4x - 0$ *(A1)(A1)(A1)*

$= 3x^2 - 4x$ *(C3)*

(b) Gradient $= f'(2)$ *(M1)*

$= 3 \times 4 - 4 \times 2$ *(A1)*

$= 4$ *(A1)* *(C3)*

QUESTION 2

Area of large sector $\frac{1}{2}r^2\theta = \frac{1}{2}16^2 \times 1.5$ *(M1)*

$= 192$ *(A1)*

Area of small sector $\frac{1}{2}r^2\theta = \frac{1}{2} \times 10^2 \times 1.5$ *(M1)*

$= 75$ *(A1)*

Shaded area = large area – small area = 192 – 75 *(M1)*

= 117 *(A1)* *(C6)*

QUESTION 3

(a)

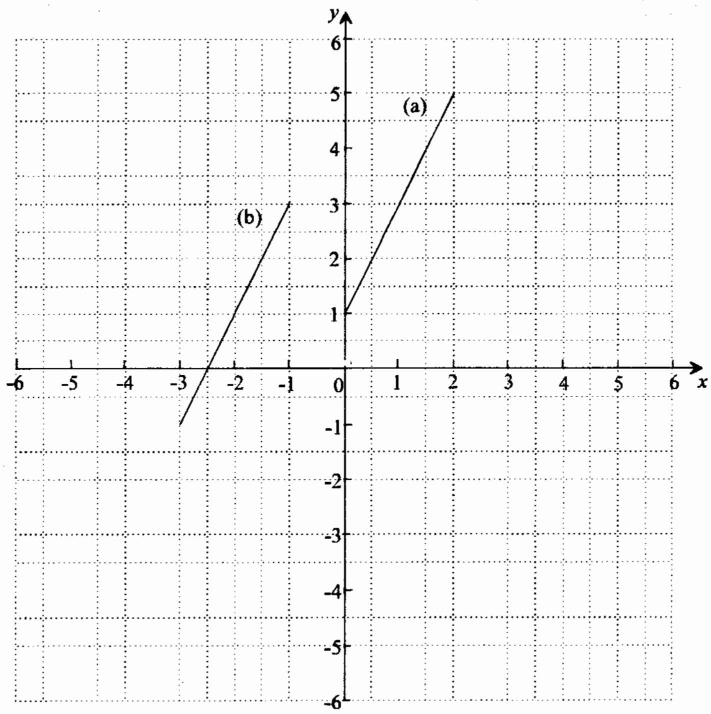
Mark (x)	$0 \le x < 20$	$20 \le x < 40$	$40 \le x < 60$	$60 \le x < 80$	$80 \le x < 100$
Number of Students	22	**50** (±1)	**66** (±1)	**42** (±1)	20

(A1)(A1)(A1) *(C3)*

(b) 40th Percentile ⇒ 80th student fails, (mark 42 %) *(M2)*

Pass mark 43 % (Accept mark > 42.) *(A1)* *(C3)*

QUESTION 4



(a) *(A1)(A1)* *(C2)*

(b) *(A1)(A3)* *(C4)*

(a) **Note:** Award *(A1)* for the correct line, *(A1)* for using the given domain.

(b) Correct domain *(A1)*

EITHER

The correct line drawn *(A3)*

OR

$g(x) = f(x+3) - 2$

$= (2(x+3)+1) - 2$ *(M1)*

$= 2x + 5$ *(A1)*

Candidate's line drawn *(A1)*

OR

$g(-3) = -1 \quad g(-1) = 3$ *(A1)(A1)*

Line joining $g(-3)$ and $g(-1)$ drawn *(A1)*

QUESTION 5

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ***(M1)***

$P(A \cap B) = \frac{1}{2} + \frac{3}{4} - \frac{7}{8}$

$= \frac{3}{8}$ ***(A1)*** ***(C2)***

(b) $P(A|B) = \frac{P(A \cap B)}{P(B)} \left(= \frac{\frac{3}{8}}{\frac{3}{4}} \right)$ ***(M1)***

$= \frac{1}{2}$ ***(A1)*** ***(C2)***

(c) Yes, the events are independent ***(A1)*** ***(C1)***

EITHER

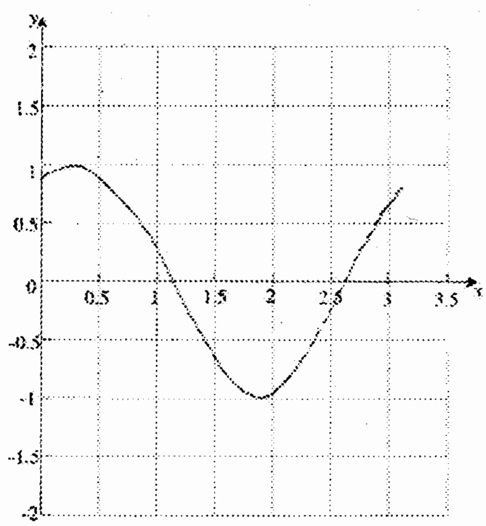
$P(A|B) = P(A)$ ***(R1)*** ***(C1)***

OR

$P(A \cap B) = P(A)P(B)$ ***(R1)*** ***(C1)***

QUESTION 6

(a) ***(A1)(A1)*** ***(C2)***



Note:	Award ***(A1)*** for the graph crossing the y-axis between 0.5 and 1, and ***(A1)*** for an approximate sine curve crossing the x-axis twice. Do **not** penalize for $x > 3.14$.

(b) (Maximum) $x = 0.285... \left(\frac{\pi}{4} - \frac{1}{2} \right)$ ***(A1)***

$x = 0.3$ (1 d.p.) ***(A1)*** ***(C2)***

(Minimum) $x = 1.856... \left(\frac{3\pi}{4} - \frac{1}{2} \right)$ ***(A1)***

$x = 1.9$ (1 d.p.) ***(A1)*** ***(C2)***

QUESTION 7

METHOD 1

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - \log_{10} y^2 - \log_{10}\sqrt{z}$ *(A1)(A1)(A1)*

$\log_{10} y^2 = 2\log_{10} y$ *(A1)*

$\log_{10}\sqrt{z} = \frac{1}{2}\log z$ *(A1)*

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - 2\log_{10} y - \frac{1}{2}\log z$

$= p - 2q - \frac{1}{2}r$ *(A1)* *(C2)(C2)(C2)*

METHOD 2

$x = 10^p,\ y^2 = 10^{2q},\ \sqrt{z} = 10^{\frac{r}{2}}$ *(A1)(A1)(A1)*

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10}\left(\frac{10^p}{10^{2q}10^{\frac{r}{2}}}\right)$ *(A1)*

$= \log_{10}\left(10^{p-2q-\frac{r}{2}}\right)\ \left(= p - 2q - \frac{r}{2}\right)$ *(A2)* *(C2)(C2)(C2)*

QUESTION 8

(a) Area of a triangle $= \frac{1}{2}\times 3\times 4\sin A$ *(A1)*

$\frac{1}{2}\times 3\times 4\sin A = 4.5$ *(A1)*

$\sin A = 0.75$ *(A1)*

$A = 48.6^\circ$ and $A = 131^\circ$ (or 0.848, 2.29 radians) *(A1)(A2)* *(C6)*

Note: Award *(C4)* for 48.6° only, *(C5)* for 131° only.

QUESTION 9

METHOD 1

$2\cos^2 x = 2\sin x\cos x$ *(M1)*

$2\cos^2 x - 2\sin x\cos x = 0$

$2\cos x(\cos x - \sin x) = 0$ *(M1)*

$\cos x = 0, (\cos x - \sin x) = 0$ *(A1)(A1)*

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ *(A1)(A1)* *(C6)*

METHOD 2

Graphical solutions

EITHER

for both graphs $y = 2\cos^2 x, y = \sin 2x$, *(M2)*

OR

for the graph of $y = 2\cos^2 x - \sin 2x$. *(M2)*

THEN

Points representing the solutions clearly indicated *(A1)*

1.57, 0.785 *(A1)*

$x = \frac{\pi}{2}, x = \frac{\pi}{4}$ *(A1)(A1)* *(C6)*

Notes: If no working shown, award ***(C4)*** for one correct answer.
Award ***(C2)(C2)*** for each correct decimal answer 1.57, 0.785.
Award ***(C2)(C2)*** for each correct degree answer $90^\circ, 45^\circ$.
Penalize a total of ***[1 mark]*** for any additional answers.

QUESTION 10

(a) $d = \int_0^4 (4t + 5 - 5e^{-t})dt$ *(M1)(A1)(A1)* *(C3)*

Note: Award ***(M1)*** for $\int$, ***(A1)*** for **both** limits, ***(A1)*** for $4t + 5 - 5e^{-t}$

(b) $d = \left[2t^2 + 5t + 5e^{-t}\right]_0^4$ *(A1)(A1)*

Note: Award ***(A1)*** for $2t^2 + 5t$, ***(A1)*** for $5e^{-t}$.

$= (32 + 20 + 5e^{-4}) - (5)$

$= 47 + 5e^{-4}$ (47.1, 3sf) *(A1)* *(C3)*

QUESTION 11

(a) (i) Neither

(ii) Geometric series

(iii) Arithmetic series

(iv) Neither *(C3)*

Note:	Award *(A1)* for geometric correct, *(A1)* for arithmetic correct and *(A1)* for **both** "neither". These may be implied by blanks **only** if GP **and** AP correct.

(b) (Series (ii) is a GP with a sum to infinity)

Common ratio $\frac{3}{4}$ *(A1)*

$S_\infty = \frac{a}{1-r}\left(= \frac{1}{1-\frac{3}{4}}\right)$ *(M1)*

$= 4$ *(A1)* *(C3)*

Note:	Do **not** allow **ft** from an incorrect series.

QUESTION 12

List of frequencies with p in the middle

e.g. $5+10, p, 6+2 \Rightarrow 15, 8, \text{or } 15 < \frac{23+p}{2}$, or $p > 7$. *(M1)*

Consideration that $p < 10$ because 2 is the mode or discretionary for further processing. *(M1)*

Possible values of p are 8 and 9 *(A2)(A2)* *(C6)*

QUESTION 13

Direction vectors are $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} - \boldsymbol{j}$. *(A2)*

$\boldsymbol{a} \cdot \boldsymbol{b} = (1+3)$ *(A1)*

$|\boldsymbol{a}| = \sqrt{10}, |\boldsymbol{b}| = \sqrt{2}$ *(A1)*

$\cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\left(= \frac{4}{\sqrt{10}\sqrt{2}}\right)$ *(M1)*

$\cos\theta = \frac{4}{\sqrt{20}}$ *(A1)* *(C6)*

QUESTION 14

Discriminant $\Delta = b^2 - 4ac\left(= (-2k)^2 - 4\right)$ *(A1)*

$\Delta > 0$ *(M2)*

Note: Award *(M1)(M0)* for $\Delta \geq 0$.

$(2k)^2 - 4 > 0 \quad \Rightarrow 4k^2 - 4 > 0$

EITHER

$4k^2 > 4 \ (k^2 > 1)$ *(A1)*

OR

$4(k-1)(k+1) > 0$ *(A1)*

OR

$(2k-2)(2k+2) > 0$ *(A1)*

THEN

$k < -1$ or $k > 1$ *(A1)(A1)* *(C6)*

Note: Award *(A1)* for $-1 < k < 1$.

QUESTION 15

$\binom{10}{3} 2^7 (ax)^3 \left(\text{accept } \binom{10}{7}\right)$ *(A1)(A1)(A1)*

$\binom{10}{3} = 120$ *(A1)*

$120 \times 2^7 a^3 = 414\,720$ *(M1)*

$a^3 = 27$

$a = 3$ *(A1)* *(C6)*

Note: Award *(A1)(A1)(A0)* for $\binom{10}{3} 2^7 ax^3$. If this leads to the answer $a = 27$, do not award the final *(A1)*.