INTERNATIONAL BACCALAURÉAT BACHILLERATO IBO BACCALAUREATE INTERNATIONAL INTERNACIONAL

N03/520/S(1)M+

MARKSCHEME

November 2003

MATHEMATICAL METHODS

Standard Level

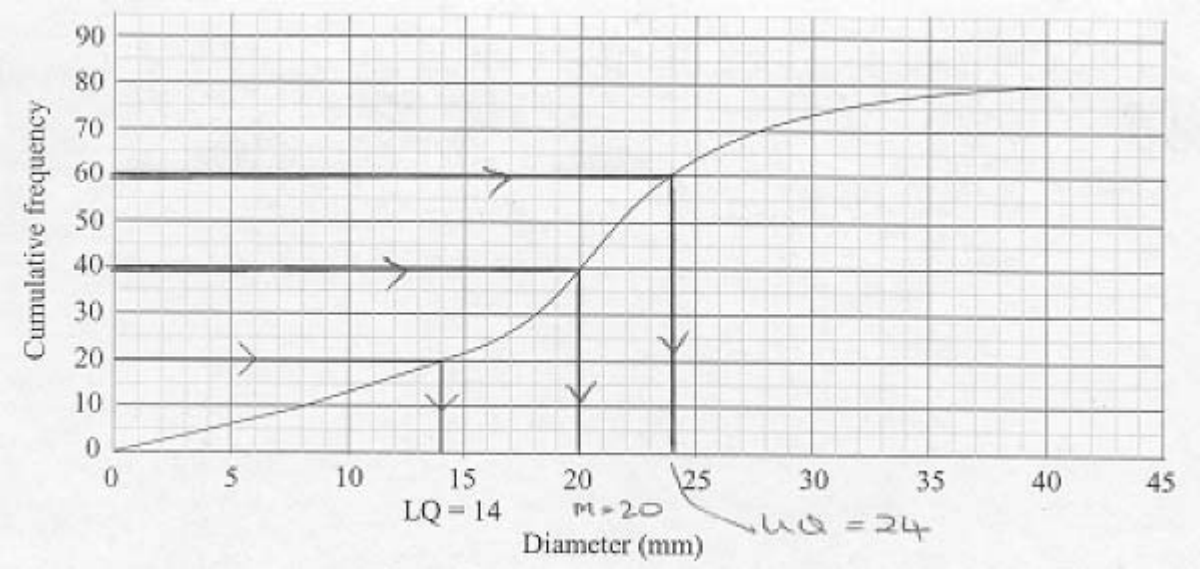
Paper 1

9 pages

IMPORTANT NOTE TO EXAMINERS:

This markscheme has been written in a slightly different style, and the instructions have been changed. The new instructions will be sent to you by your team leader, and you should make sure that you understand the changes before you start marking. Please ensure that you follow the instructions, including those indicating how to record the marks. If you have any queries, you should contact your team leader immediately.

QUESTION 1



(a) (i) Correct lines drawn on graph, ***A1*** ***C1***
median = 20 ***A1*** ***C1***

(ii) Correct lines drawn on graph, ***A1*** ***C1***
$\text{UQ} = Q_3 = 24$ ***A1*** ***C1***

(b) $\text{IQR} = Q_3 - Q_1 \text{ (or } \text{UQ} - \text{LQ})$ ***(M1)***
$= 10$ (accept 14 to 24) ***A1*** ***C2***

Note: Accept 14 to 24, 24 to 14, $14 - 24$ or $24 - 14$.

QUESTION 2

(a) (i) A is $\left(\frac{4}{3}, 0\right)$ ***A1 A1*** ***C2***

(ii) B is $(0, -4)$ ***A1 A1*** ***C2***

Notes: In each of parts (i) and (ii), award ***C1*** if A and B are interchanged, ***C1*** if intercepts given instead of coordinates.

(b) $\text{Area} = \frac{1}{2} \times 4 \times \frac{4}{3}$ ***M1***

$= \frac{8}{3} (= 2.67)$ ***A1*** ***C2***

QUESTION 3

One solution $\Rightarrow$ discriminant $= 0$ ***(M2)***

$3^2 - 4k = 0$ ***A2***

$9 = 4k$

$k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$ ***A2*** ***C6***

Note:	If candidates correctly solve an incorrect equation, award ***M2 A0 A2*(ft)**, if they have the first line or equivalent, otherwise award no marks.

QUESTION 4

$\mathrm{P(RR)} = \frac{7}{12} \times \frac{6}{11} \left(= \frac{7}{22}\right)$ ***M1 A1***

$\mathrm{P(YY)} = \frac{5}{12} \times \frac{4}{11} \left(= \frac{5}{33}\right)$ ***M1 A1***

$\mathrm{P(same\ colour)} = \mathrm{P(RR)} + \mathrm{P(YY)}$ ***(M1)***

$= \frac{31}{66} (= 0.470\ 3\ \text{s.f.})$ ***A1*** ***C6***

Note:	Award ***C2*** for $\left(\frac{7}{12}\right)^2 + \left(\frac{5}{12}\right)^2 = \frac{74}{144}$.

QUESTION 5

$\ldots + 6 \times 2^2 (ax)^2 + 4 \times 2 (ax)^3 + (ax)^4$ ***(M1)(M1)(M1)***

$= \ldots + 24a^2x^2 + 8a^3x^3 + a^4x^4$ ***A1 A1 A1*** ***C6***

Notes:	Award ***C3*** if brackets omitted, leading to $24ax^2 + 8ax^3 + ax^4$. Award ***C4*** if correct expression with brackets as in first line of markscheme is given as final answer.

QUESTION 6

Arithmetic sequence ***(M1)***

$a = 200 \qquad d = 30$ ***(A1)***

(a) Distance in final week $= 200 + 51 \times 30$ ***M1***

$= 1730$ m ***A1*** ***C3***

(b) Total distance $= \frac{52}{2}[2.200 + 51.30]$ ***M1***

$= 50180$ m ***A1*** ***C3***

Note:	Penalize once for absence of units *i.e.* award ***A0*** the first time units are omitted, ***A1*** the next time .

QUESTION 7

B, or $\boldsymbol{r} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ ***C3***

D, or $\boldsymbol{r} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ***C3***

Note: Award ***C4*** for B, D and one incorrect, ***C3*** for one correct and nothing else, ***C1*** for one correct and one incorrect, ***C0*** for anything else.

QUESTION 8

(a) (i) $p = 2$ ***A2*** ***C2***

(ii) $10 = \dfrac{q}{3-2}$ (or equivalent) ***(M1)***

$q = 10$ ***A1*** ***C2***

(b) Reflection, in x-axis ***A1 A1*** ***C2***

QUESTION 9

(a) $\begin{pmatrix} 60 \\ 25 \end{pmatrix} \bullet \begin{pmatrix} -30 \\ 40 \end{pmatrix} = 60\times(-30) + 25\times 40$ ***M1***

$= -800$ ***A1*** ***C2***

(b) $\cos\theta = \dfrac{-800}{\sqrt{60^2 + 25^2}\sqrt{(-30)^2 + 40^2}}$ ***M1 A1***

Note: Trig solutions:
Award ***M1*** for attempt to use a correct strategy, ***A1*** for correct values.

$\cos\theta = -0.246...$ ***A1***

. . . . . .

$\theta = 104.25...^\circ$ (or $255.75...^\circ$) ***A1***

She turns through 104° (or 256°) ***C4***

Note: Accept answers in radians *i.e.* 1.82 or 4.46.

QUESTION 10

(a) Initial mass $\Rightarrow t = 0$ ***(A1)***

mass $= 4$ ***A1*** ***C2***

(b) $1.5 = 4e^{-0.2t}$ (or $0.375 = e^{-0.2t}$) ***M2***

$\ln 0.375 = -0.2t$ ***M1***

$t = 4.90$ hours ***A1*** ***C4***

QUESTION 11

$y = \int \frac{dy}{dx} dx$ ***(M1)***

$= \frac{x^4}{4} + \frac{2x^2}{2} - x + c$ ***A1 A1***

Note: Award ***(A1)*** for first 3 terms, ***(A1)*** for "+ c ".

$13 = \frac{16}{4} + 4 - 2 + c$ ***(M1)***

$c = 7$ ***A1***

. . . . . .

$y = \frac{x^4}{4} + x^2 - x + 7$ ***A1*** ***C6***

QUESTION 12

(a) $a = 3, b = 4$ ***(A1)***

$f(x) = (x-3)^2 + 4$ ***A1*** ***C2***

(b) $y = (x-3)^2 + 4$

METHOD 1

$x = (y-3)^2 + 4$ ***(M1)***

$x - 4 = (y-3)^2$

$\sqrt{x-4} = y - 3$ ***(M1)***

$y = \sqrt{x-4} + 3$ ***A1*** ***C3***

METHOD 2

$y - 4 = (x-3)^2$ ***(M1)***

$\sqrt{y-4} = x - 3$ ***(M1)***

$\sqrt{y-4} + 3 = x$

$y = \sqrt{x-4} + 3$

$\Rightarrow f^{-1}(x) = \sqrt{x-4} + 3$ ***A1*** ***C3***

(c) $x \geq 4$ ***A1*** ***C1***

QUESTION 13

(a) $(3\sin x - 2)(\sin x - 3)$ ***A1 A1*** ***C2***

Note: Award ***A1*** if $3x^2 - 11x + 6$ correctly factorized to give $(3x-2)(x-3)$ (or equivalent with another letter).

(b) (i) $(3\sin x - 2)(\sin x - 3) = 0$

$\sin x = \frac{2}{3}$ $\sin x = 3$ ***A1 A1*** ***C2***

(ii) $x = 41.8^\circ$, 138° ***A1 A1*** ***C2***

Notes: Penalize ***[1 mark]*** for any extra answers and ***[1 mark]*** for answers in radians.
i.e. Award ***A1 A0*** for 41.8°, 138° and any extra answers.
Award ***A1 A0*** for 0.730, 2.41.
Award ***A0 A0*** for 0.730, 2.41 and any extra answers.

QUESTION 14

(a) $\int \left(1 + 3\sin\left(x+2\right)\right) dx = x - 3\cos(x+2) + c$ ***A1 A1 A1*** ***C3***

Notes: Award ***A1*** for x, ***A1*** for $-\cos(x+2)$, ***A1*** for coefficient 3,
i.e. ***A1 A1*** for the second term, which may be written as $+3\left(-\cos\ (x\ +2)\right)$
Do **not** penalize the omission of c.

(b) $1 + 3\sin(x+2) = 0$ ***(M1)***

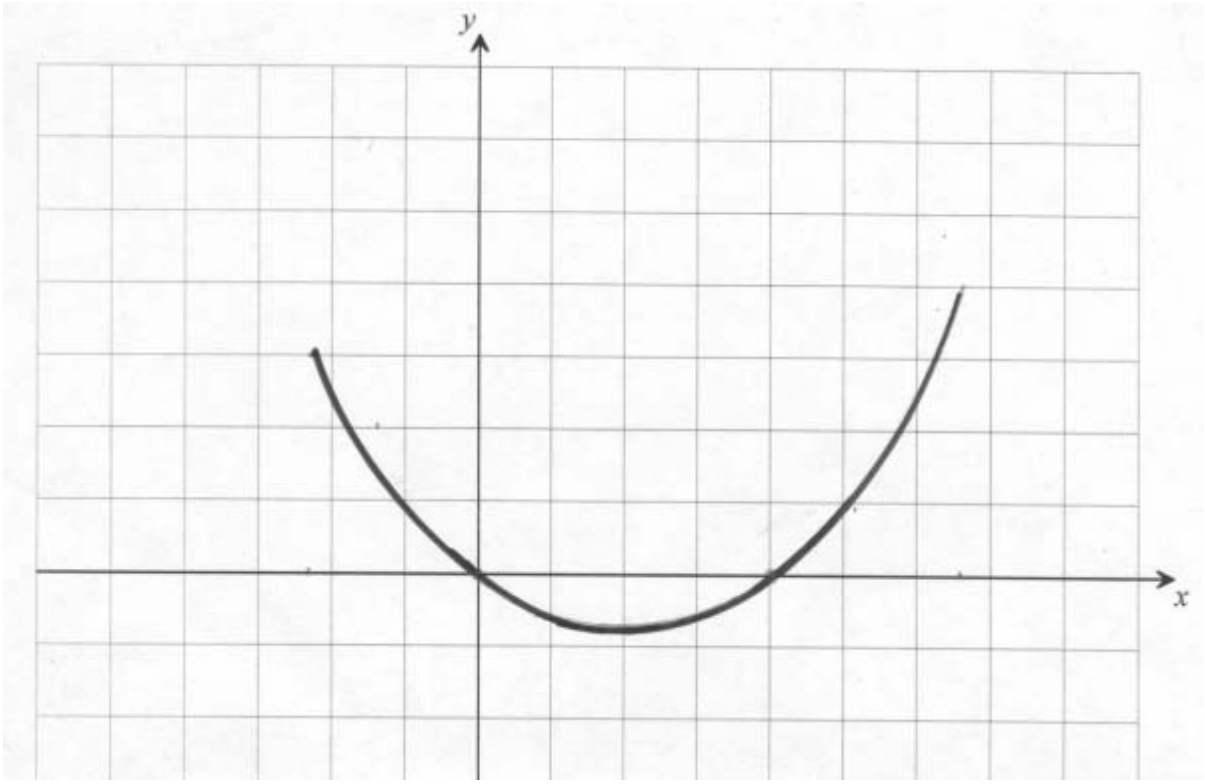
$\sin(x+2) = -\frac{1}{3}$

$x + 2 = -0.3398, \pi + 0.3398, ...$ ***A1***

$x = -2.3398, 1.4814, ...$

Required value of $x = 1.48$ ***A1*** ***C3***

QUESTION 15



A2 A1 A1 A2 ***C6***

Note:	Award ***A2*** for correct shape (approximately parabolic), ***A1 A1*** for intercepts at 0 and 4, ***A2*** for minimum between $x = 1.5$ and $x = 2.5$.