INTERNATIONAL BACCALAUREATE
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MARKSCHEME

November 2003

MATHEMATICAL METHODS

Standard Level

Paper 2

23 pages

Paper 2 Markscheme

Instructions to Examiners

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*.
- write down each part mark total, indicated on the markscheme (for example, ***[3 marks]***) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise an error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii) award ***M*** marks for a correct method, and ***A*****(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	***M1***	Amount earned = \$ 600 × 1.02	√	***M1***
= \$ 612	***A1***	= \$602	×	***A0***
\$ (306 × 1.02) + (306 × 1.04)	***M1***	Amount = 301 × 1.02 + 301 × 1.04	√	***M1***
= \$ 630.36	***A1***	= \$ 620.06	√	***A1*(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc*. Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

Mean = 7906/134 ***(M1)***
= 59 ***(A1)***

OR

Mean = 59 ***(G2)***

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect ie, once the correct answer is seen, ignore further working.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan x.

5 Accuracy of Answers

There are two types of accuracy errors, incorrect level of accuracy, and rounding errors.

Unless the level of accuracy is specified in the question, candidates should be penalized **once only IN THE PAPER** for any accuracy error **(AP)**. This could be an incorrect level of accuracy **(only applies to fewer than three significant figures)**, or a rounding error. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy, or rounded incorrectly, maximum marks are **not** awarded, but on **all subsequent occasions** when accuracy errors occur, then maximum marks **are** awarded.

(a) **Level of accuracy**

(i) In the case when the accuracy of the answer is **specified in the question** (for example: “find the size of angle A to the nearest degree”) the maximum mark is awarded **only if** the correct answer is given to the accuracy required.

(ii) When the accuracy is **not** specified in the question, then the general rule applies:

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

However, if candidates give their answers to more than three significant figures, this is acceptable

(b) **Rounding errors**

Rounding errors should only be penalized at the **final answer** stage. This does **not** apply to intermediate answers, only those asked for as part of a question. Premature rounding which leads to incorrect answers should only be penalized at the answer stage.

Incorrect answers are wrong, and should not be considered under (a) or (b).

Examples

A question leads to the answer 4.6789….

- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy : 4.7 should be penalised the first time this type of error occurs, but 4.679 is **not** penalized, as it has more than three significant figures.
- 4.67 is incorrectly rounded – penalise on the first occurrence.
- 4.678 is incorrectly rounded, but has more than the required accuracy, do **not** penalize.

Note: All these “incorrect” answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalised as being incorrect answers, not as examples of accuracy errors.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

(i) Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10 % of their overall mark.. Note this on the front cover.

(ii) Calculator box not filled in.

Please apply a calculator penalty (***CP***) of 1 mark if this information is not provided. Note this on the front cover.

QUESTION 1

(a) $\vec{OB} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$ $\vec{OC} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$ ***(A1)(A1)***

[2 marks]

(b) $\vec{AD} = \vec{BC} = \vec{OC} - \vec{OB}$ ***(M1)***

$= \begin{pmatrix} 8 \\ 9 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$ ***(A1)***

$\vec{OD} = \vec{OA} + \vec{AD} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 9 \\ 2 \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix}$ $\left(\text{or} \begin{pmatrix} 8 \\ 9 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} \right)$

$\Rightarrow d = 11$ $\left(\text{accept} \begin{pmatrix} 11 \\ 4 \end{pmatrix} \right)$ ***(A1)***

[3 marks]

(c) $\vec{BD} = \begin{pmatrix} 11 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} 12 \\ -3 \end{pmatrix}$ ***(A1)***

[1 mark]

(d) (i) $l: \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix}$ $\left(\text{or} \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix} \right)$ ***(A2)***

(ii) At B, $t = 0$ by observation ***(A1)***

OR

$\begin{pmatrix} -1 \\ 7 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix} + t\begin{pmatrix} 12 \\ -3 \end{pmatrix}$

$\Rightarrow t = 0$ ***(A1)***

[3 marks]

continued...

Question 1 continued

(e) $\begin{pmatrix}7\\5\end{pmatrix}=\begin{pmatrix}-1\\7\end{pmatrix}+t\begin{pmatrix}12\\-3\end{pmatrix} \Rightarrow 7+1=12t=8$

$\Rightarrow t=\dfrac{2}{3}$ ***(A1)***

Note: The equation $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-1\\7\end{pmatrix}+t\begin{pmatrix}4\\-1\end{pmatrix}$ leads to $t=2$.

when $t=\dfrac{2}{3}$, $y=7+\left(\dfrac{2}{3}\right)(-3)$ ***(M1)***

$=7-2=5$ ***(A1)***

i.e. P on line ***(AG)***

OR

$5-7=-3t=-2$

$\Rightarrow t=\dfrac{2}{3}$ ***(A1)***

when $t=\dfrac{2}{3}$, $x=-1+\dfrac{2}{3}\times 12$ ***(M1)***

$=-1+8=7$ ***(A1)***

i.e. P on line ***(AG)***

[3 marks]

(f) $\vec{\text{CP}}=\begin{pmatrix}7\\5\end{pmatrix}-\begin{pmatrix}8\\9\end{pmatrix}=\begin{pmatrix}-1\\-4\end{pmatrix}$ ***(A1)***

$\begin{pmatrix}-1\\-4\end{pmatrix}\bullet\begin{pmatrix}12\\-3\end{pmatrix}=-12+12=0$ ***(M1)(A1)***

Scalar product of non-zero vectors $=0 \Rightarrow$ vectors are perpendicular ***(R1)(AG)***

OR

Geometric approach

$\text{CP}: m=4$ ***(A1)***

$\text{BD}: m_1=\dfrac{-1}{4}$ ***(A1)***

$mm_1=4\times\left(\dfrac{-1}{4}\right)=-1$ ***(A1)***

Product of gradients is $-1 \Rightarrow$ lines (vectors) are perpendicular ***(R1)(AG)***

[4 marks]

Total [16 marks]

QUESTION 2

Note: Do not penalize missing units in this question.

(a) $AB^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 75^\circ$ ***(A1)***

$= 12^2 (2 - 2\cos 75^\circ)$ ***(A1)***

$= 12^2 \times 2(1 - \cos 75^\circ)$

$AB = 12\sqrt{2(1 - \cos 75^\circ)}$ ***(AG)***

Note: The second ***(A1)*** is for transforming the initial expression to any simplified expression from which the given result can be clearly seen.

[2 marks]

(b) $P\hat{O}B = 37.5^\circ$ ***(A1)***

$BP = 12\tan 37.5^\circ$ ***(M1)***

$= 9.21\,\text{cm}$ ***(A1)***

OR

$B\hat{P}A = 105^\circ \quad B\hat{A}P = 37.5^\circ$ ***(A1)***

$$\frac{AB}{\sin 105^\circ} = \frac{BP}{\sin 37.5^\circ}$$ ***(M1)***

$$BP = \frac{AB\sin 37.5^\circ}{\sin 105^\circ} = 9.21\,(\text{cm})$$ ***(A1)***

[3 marks]

(c) (i) Area $\Delta OBP = \frac{1}{2} \times 12 \times 9.21 \quad \left(\text{or } \frac{1}{2} \times 12 \times 12\tan 37.5^\circ\right)$ ***(M1)***

$= 55.3\,(\text{cm}^2) \quad (\text{accept } 55.2\,\text{cm}^2)$ ***(A1)***

(ii) Area $\Delta ABP = \frac{1}{2}(9.21)^2 \sin 105^\circ$ ***(M1)***

$= 41.0\,(\text{cm}^2) \quad (\text{accept } 40.9\,\text{cm}^2)$ ***(A1)***

[4 marks]

(d) Area of sector $= \frac{1}{2} \times 12^2 \times 75 \times \frac{\pi}{180} \quad \left(\text{or } \frac{75}{360} \times \pi \times 12^2\right)$ ***(M1)***

$= 94.2\,(\text{cm}^2) \quad \left(\text{accept } 30\pi \text{ or } 94.3\,(\text{cm}^2)\right)$ ***(A1)***

[2 marks]

(e) Shaded area $= 2 \times \text{area } \Delta OPB - \text{area sector}$ ***(M1)***

$= 16.4\,(\text{cm}^2) \quad (\text{accept } 16.2\,\text{cm}^2, 16.3\,\text{cm}^2)$ ***(A1)***

[2 marks]

Total [13 marks]

QUESTION 3

(a) (i) Area B $= \frac{1}{16}$, area C $= \frac{1}{64}$ ***(A1)(A1)***

(ii) $\dfrac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ $\dfrac{\frac{1}{64}}{\frac{1}{16}} = \frac{1}{4}$ (Ratio is the same.) ***(M1)(R1)***

(iii) Common ratio $= \frac{1}{4}$ ***(A1)***

[5 marks]

(b) (i) Total area $(S_2) = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ $(= 0.3125)$ (0.313, 3 s.f.) ***(A1)***

(ii) Required area $= S_8 = \dfrac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^8\right)}{1-\frac{1}{4}}$ ***(M1)***

$= 0.3333282(471...)$ ***(A1)***

$= 0.333328$ (6 s.f.) ***(A1)***

Note: Accept result of adding together eight areas correctly.

[4 marks]

(c) Sum to infinity $= \dfrac{\frac{1}{4}}{1-\frac{1}{4}}$ ***(A1)***

$= \frac{1}{3}$ ***(A1)***

[2 marks]

Total [11 marks]

QUESTION 4

(a) (i) $f'(x) = -2e^{-2x}$ ***(A1)***

(ii) $f'(x)$ is always negative ***(R1)***

[2 marks]

(b) (i) $y = 1 + e^{-2\times-\frac{1}{2}}$ $(= 1 + e)$ ***(A1)***

(ii) $f'\left(-\frac{1}{2}\right) = -2e^{-2\times-\frac{1}{2}}$ $(= -2e)$ ***(A1)***

Note: In part (b) the answers do not need to be simplified.

[2 marks]

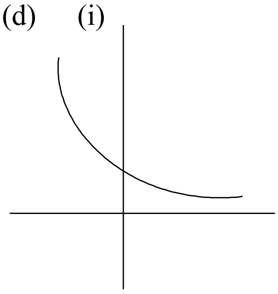
(c) $y - (1 + e) = -2e\left(x + \frac{1}{2}\right)$ ***(M1)***

$y = -2ex + 1$ $(y = -5.44x + 1)$ ***(A1)(A1)***

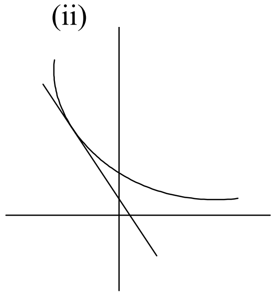
[3 marks]

continued...

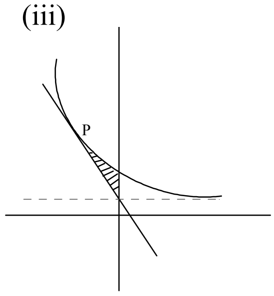
(d) (i)



(ii)



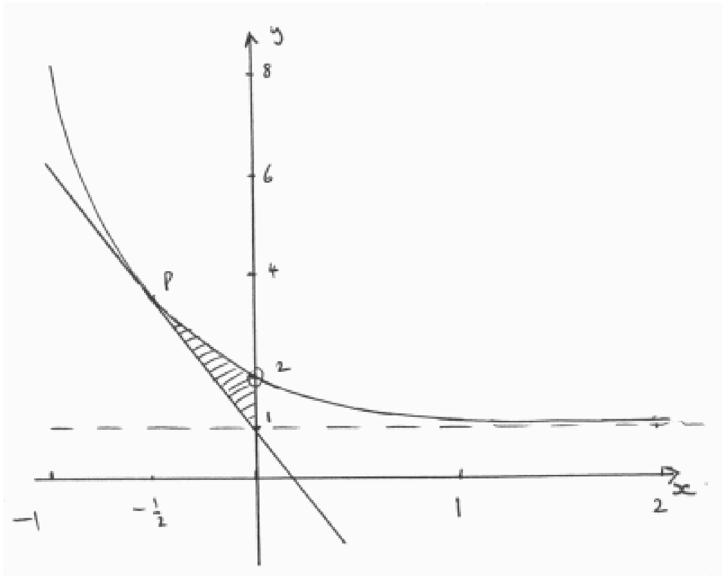
(iii)



(A1)(A1)(A1)

Note:	Award ***(A1)*** for each correct answer. Do **not** allow **(ft)** on an incorrect answer to part (i). The correct final diagram is shown below. Do not penalize if the horizontal asymptote is missing. Axes do not need to be labelled.

(i) (ii) (iii)



(iv) Area $= \int_{-\frac{1}{2}}^{0} \left[\left(1 + e^{-2x}\right) - \left(-2ex + 1\right) \right] dx$ (or equivalent) ***(M1)(M1)***

Notes:	Award ***(M1)*** for the limits, ***(M1)*** for the function. Accept difference of integrals as well as integral of difference. Area below line may be calculated geometrically.

$$\text{Area} = \int_{-\frac{1}{2}}^{0} \left(e^{-2x} + 2ex\right) dx$$

$$= \left[-\frac{1}{2} e^{-2x} + ex^2 \right]_{-\frac{1}{2}}^{0}$$ ***(A1)***

$= 0.1795\ldots = 0.180$ (3 s.f.) ***(A1)***

OR

Area $= 0.180$ ***(G2)***

[7 marks]

Total [14 marks]

QUESTION 5

Note: Do not penalize missing units in this question.

(a) (i) At release(P), $t = 0$ *(M1)*

$s = 48 + 10\cos 0$

$= 58$ cm below ceiling *(A1)*

(ii) $58 = 48 + 10\cos 2\pi t$ *(M1)*

$\cos 2\pi t = 1$ *(A1)*

$t = 1$ sec *(A1)*

OR

$t = 1$ sec *(G3)*

[5 marks]

(b) (i) $\frac{ds}{dt} = -20\pi \sin 2\pi t$ *(A1)(A1)*

Note: Award ***(A1)*** for -20π, and ***(A1)*** for $\sin 2\pi t$.

(ii) $v = \frac{ds}{dt} = -20\pi \sin 2\pi t = 0$ *(M1)*

$\sin 2\pi t = 0$

$t = 0, \frac{1}{2} \ldots$ (at least 2 values) *(A1)*

$s = 48 + 10\cos 0$ **or** $s = 48 + 10\cos \pi$ *(M1)*

$= 58$ cm (at P) $= 38$ cm (20 cm above P) *(A1)(A1)*

Note: Accept these answers without working for full marks. May be deduced from recognizing that amplitude is 10.

[7 marks]

(c) $48 + 10\cos 2\pi t = 60 + 15\cos 4\pi t$ *(M1)*

$t = 0.162$ secs *(A1)*

OR

$t = 0.162$ secs *(G2)*

[2 marks]

(d) 12 times *(G2)*

Notes: If either of the correct answers to parts (c) and (d) are missing and suitable graphs have been sketched, award ***(G2)*** for sketch of suitable graph(s); ***(A1)*** for $t = 0.162$; ***(A1)*** for 12.

[2 marks]

Total [16 marks]

QUESTION 6

(i) (a) $\mathrm{P}(M \geq 350) = 1 - \mathrm{P}(M < 350) = 1 - \mathrm{P}\left(Z < \frac{350 - 310}{30}\right)$ ***(M1)***

$= 1 - \mathrm{P}(Z < 1.333) = 1 - 0.9088$

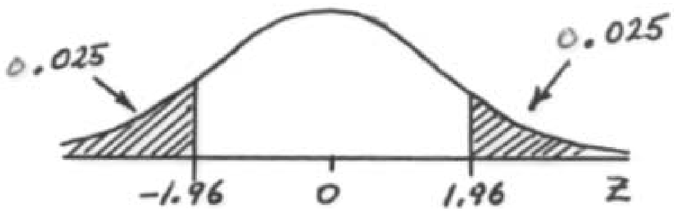
$= 0.0912$ (accept 0.0910 to 0.0920) ***(A1)***

OR

$\mathrm{P}(M \geq 350) = 0.0912$ ***(G2)***

[2 marks]

(b)



$\mathrm{P}(Z < 1.96) = 1 - 0.025 = 0.975$ ***(A1)***

$1.96(30) = 58.8$ ***(M1)***

$310 - 58.8 < M < 310 + 58.8 \Rightarrow \quad a = 251, b = 369$ ***(A1)***

OR

$251 < M < 369$ ***(G3)***

Note: Award ***(G1)*** if only one of the end points is correct.

[3 marks]

(c) $\frac{30}{\sqrt{36}} = 5\text{ kg} =$ standard error of mean ***(M1)***

$\mathrm{P}(\bar{M} \leq 300) = \mathrm{P}\left(Z < \frac{300 - 310}{5}\right) = \mathrm{P}(Z < -2)$ ***(M1)***

$= 0.0228$ (accept 0.0227) ***(A1)***

OR

$\mathrm{P}(\bar{M} \leq 300) = 0.0228$ ***(G3)***

[3 marks]

(d) The probability of obtaining a sample mean of 300 or less if the population mean is 310 is 2.28 % (or very small, or less than 5 %). ***(R1)***

Since a probability of less than 5 % means the event is very unlikely, the conclusion that the population mean is in fact less than 310 kg is justified. ***(R1)***

[2 marks]

continued...

Question 6 continued

(ii) (a)

	Pass	Fail	
Girls	90	30	***(A1)(A1)***
Boys	60	20	***(A1)(A1)***

[4 marks]

(b) $\chi^2 = \frac{(96-90)^2}{90} + \frac{(24-30)^2}{30} + \frac{(54-60)^2}{60} + \frac{(26-20)^2}{20}$ ***(M1)***

$= 4.00$ (or 4) ***(A1)***

OR

$\chi^2 = 4$ ***(G2)***

[2 marks]

(c) Girls are better than boys on this geography test. ***(A1)***

[1 mark]

(iii) (a) (i) $c = |8 - [1.3(4) + 0.5]|$ ***(M1)(A1)***

$= 2.3$ ***(AG)***

Note: Do not require absolute value for ***(M1)***.

$d = |7 - [1.3(6) + 0.5]| = 1.3$ ***(A1)***

Note: Also accept –1.3.

(ii) $V = 1.1^2 + 2.3^2 + 1.3^2 + 0.1^2 = 8.2$ ***(A1)***

(iii) Sum > V ***(A1)***

[5 marks]

(b) Statement (iii) is true. ***(A2)***

[2 marks]

continued...

Question 6 continued

(c) $(y-5)=\frac{s_{xy}}{s_x^2}(x-7)$ ***(M1)***

$\Rightarrow y-5=\frac{6.5}{10.5}(x-7)=\frac{13}{21}(x-7)$ ***(A1)***

$\Rightarrow y=\frac{13}{21}x+\frac{2}{3}=0.619x+0.667$ ***(A1)***

OR

$y=0.619x+0.667$ ***(G3)***

Note: Award ***(G1)*** for 0.619, ***(G1)*** for 0.667 and ***(G1)*** for the correct equation.

[3 marks]

(d) The least squares regression line passes through $(\bar{x}, \bar{y})$. ***(R2)***
$\bar{x}$ is the average value of the x coordinates and $\bar{y}$ is the average value of the y coordinates. ***(R1)***

[3 marks]

Total [30 marks]

QUESTION 7

(i) (a) $x = 1$ ***(A1)***

[1 mark]

(b) (i) $f(-1000) = 2.01$ ***(A1)***

(ii) $y = 2$ ***(A1)***

[2 marks]

(c) $f'(x) = \dfrac{(x-1)^2(4x-13) - 2(x-1)(2x^2 - 13x + 20)}{(x-1)^4}$ ***(A1)(A1)***

$= \dfrac{(4x^2 - 17x + 13) - (4x^2 - 26x + 40)}{(x-1)^3}$ ***(A1)***

$= \dfrac{9x - 27}{(x-1)^3}$ ***(AG)***

Notes:	Award ***(M1)*** for the **correct** use of the quotient rule, the first ***(A1)*** for the placement of the correct expressions into the quotient rule. Award the second ***(A1)*** for doing sufficient simplification to make the given answer reasonably obvious.

[3 marks]

(d) $f'(3) = 0 \quad \Rightarrow$ stationary (or turning) point ***(R1)***

$f''(3) = \dfrac{18}{16} > 0 \quad \Rightarrow$ minimum ***(R1)***

[2 marks]

(e) Point of inflexion $\Rightarrow \quad f''(x) = 0 \quad \Rightarrow \quad x = 4$ ***(A1)***

$x = 4 \quad \Rightarrow \quad y = 0 \quad \Rightarrow$ Point of inflexion $= (4, 0)$ ***(A1)***

OR

Point of inflexion $= (4, 0)$ ***(G2)***

[2 marks]

continued...

Question 7 continued

(ii)

Note:	In this part, do not penalize correct answers given to more than the required accuracy.

(a) $\dfrac{10x}{(x^2+1)^2} = 2 \Rightarrow 10x = 2(x^2+1)^2 \Rightarrow x = \dfrac{(x^2+1)^2}{5}$ ***(M1)***

[1 mark]

(b) (i) 0.21632 ***(G1)***

(ii) $0.2197897344 = 0.21979$ (5 s.f.) ***(G2)***

[3 marks]

(c) (i) $g'(x) = \dfrac{2(x^2+1)}{5}(2x)$ ***(M1)(A1)***

$= \dfrac{4x(x^2+1)}{5}$

OR

$g'(x) = \dfrac{4x(x^2+1)}{5}$ ***(A2)***

(ii) (a) $g'(0.2) = 0.166$ ***(A1)***

(b) $g'(1.2) = 2.34$ ***(A1)***

(iii) For fixed-point iteration to converge $|g'(x)|$ must be less than 1 in the vicinity of the root. ***(R1)***
This is not the case for the root near 1.2 . ***(R1)***

[6 marks]

(d) $1.2068386268 = 1.20684$ (6 s.f.) ***(G2)***

[2 marks]

Question 7 continued

(e) (i) $u = x^2 + 1 \Rightarrow \mathrm{d}u = 2x\mathrm{d}x \ \left(\text{or } \frac{\mathrm{d}u}{\mathrm{d}x} = 2x\right)$ ***(M1)***

$\int \frac{10x}{(x^2+1)^2}\mathrm{d}x = \int \frac{5}{u^2}\mathrm{d}u$ ***(A1)***

$= -\frac{5}{u} + C$ ***(A1)***

$= -\frac{5}{x^2+1} + C$ ***(A1)***

OR

By inspection $-\frac{5}{x^2+1} + C$ ***(A4)***

Notes:	With either approach, do not penalize missing *C*. If integration is done by inspection, award ***(A3)*** if a constant other than 5 or a missing negative sign is only error: with wrong constant and missing negative sign, award ***(A2)*** if denominator correct.

(ii) $\left[-\frac{5}{x^2+1}\right]_0^a = -\frac{5}{a^2+1} - \left(-\frac{5}{1}\right)$ ***(M1)***

$\Rightarrow 5 - \frac{5}{a^2+1} = 4$ ***(A1)***

$\Rightarrow \frac{5}{a^2+1} = 1 \Rightarrow a^2 = 4$ ***(A1)***

$\Rightarrow a = 2$ (Do not accept ± 2, or -2). ***(A1)***

OR

$a = 2$ ***(G4)***

[8 marks]

Total [30 marks]

QUESTION 8

(i) (a) $\det \boldsymbol{A} = 5(1) - 7(-2) = 19$

$$\boldsymbol{A}^{-1} = \frac{1}{19}\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$$ ***(A2)***

Note: Award ***(A1)*** for $\begin{pmatrix} 1 & 2 \\ -7 & 5 \end{pmatrix}$, ***(A1)*** for dividing by 19.

OR

$$\boldsymbol{A}^{-1} = \begin{pmatrix} 0.0526 & 0.105 \\ -0.368 & 0.263 \end{pmatrix}$$ ***(G2)***

[2 marks]

(b) (i) $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C} \Rightarrow \boldsymbol{XA} = \boldsymbol{C} - \boldsymbol{B}$ ***(M1)***

$\Rightarrow \boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$ ***(A1)***

OR

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1}$ ***(A2)***

(ii) $$(\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1} = \begin{pmatrix} -11 & -7 \\ -13 & 9 \end{pmatrix}\begin{pmatrix} \frac{1}{19} & \frac{2}{19} \\ \frac{-7}{19} & \frac{5}{19} \end{pmatrix}$$ ***(A1)***

$$\Rightarrow \boldsymbol{X} = \begin{pmatrix} \frac{38}{19} & \frac{-57}{19} \\ \frac{-76}{19} & \frac{19}{19} \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$ ***(A1)***

OR

$$\boldsymbol{X} = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$ ***(G2)***

Note: If premultiplication by $\boldsymbol{A}^{-1}$ is used, award ***(M1)(M0)*** in part (i) but award ***(A2)*** for $\begin{pmatrix} \frac{-37}{19} & \frac{11}{19} \\ \frac{12}{19} & \frac{94}{19} \end{pmatrix}$ in part (ii).

[4 marks]

continued...

(Question 8 continued)

(ii) (a) Area scale factor $= \det \boldsymbol{M}$ ***(M1)***

$= 0.6(1.4) - 0.2(-0.8) = 1$ ***(A1)***

[2 marks]

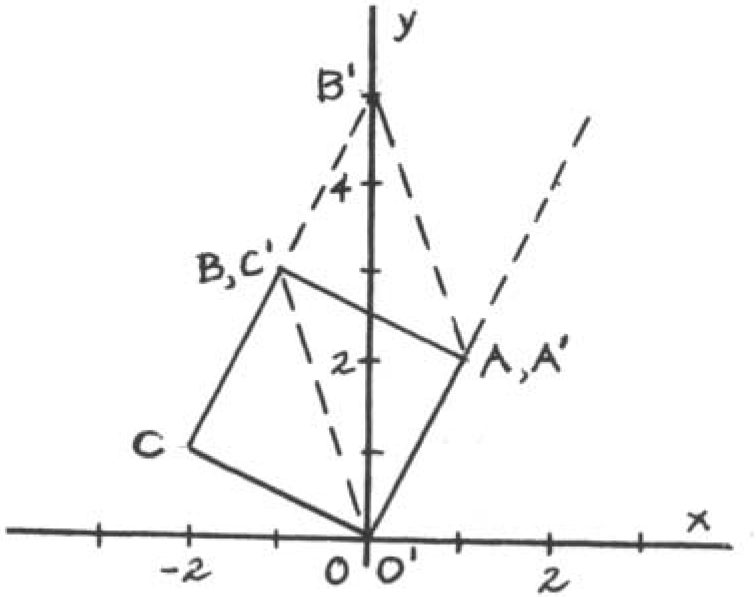
(b) (i) (a) $\begin{pmatrix} 0.6 & 0.2 \\ -0.8 & 1.4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ (may be implied) ***(M1)***

$\Rightarrow \boldsymbol{A}' = (1, 2)$ ***(A1)***

(b) $\begin{pmatrix} 0.6 & 0.2 \\ -0.8 & 1.4 \end{pmatrix}\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\Rightarrow \boldsymbol{C}' = (-1, 3)$ ***(A1)***

(ii)



Notes: Award ***(A1)*** for correct placement of OABC. Award ***(A1)*** for correct placement of O′A′B′C′.

(iii) Transformation is a **shear** ***(A1)***

with invariant line with equation $y = 2x$ ***(A1)***

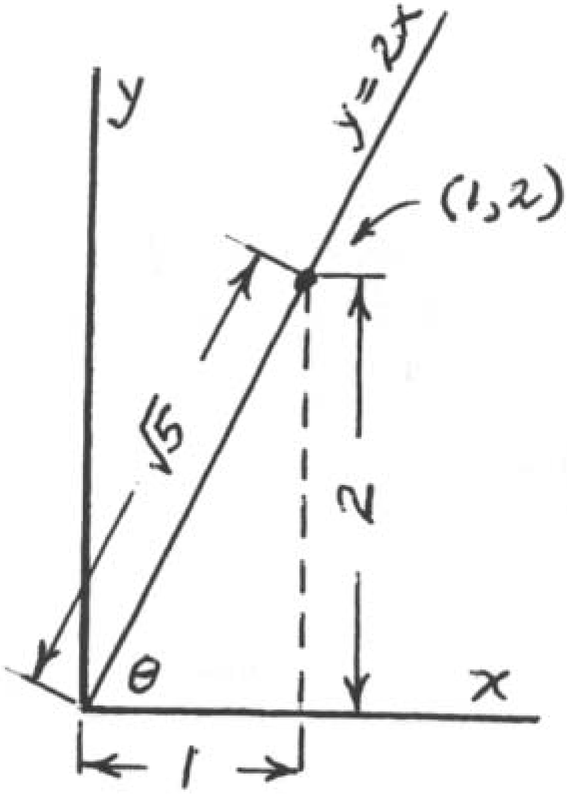
and scale factor 1. ***(A1)***

[8 marks]

continued...

(Question 8(ii) continued)

(c)



(M1)

$\cos\theta = \frac{1}{\sqrt{5}}$ ***(M1)(AG)***

$\sin\theta = \frac{2}{\sqrt{5}}$ ***(A1)***

Notes:	Award ***(M1)*** for a valid method. Award ***(M1)*** for applying method to give $\cos\theta$.

[3 marks]

continued...

(Question 8 continued)

(d) (i) (a) ***P*** – A rotation about $(0, 0)$ ***(A1)***

through 63.4° (or arctan 2 or θ) anti-clockwise . ***(A1)***

(b) ***Q*** – A shear with x-axis as invariant line ***(A1)***
and scale factor 1. ***(A1)***

(c) ***R*** – A rotation about $(0, 0)$ ***(A1)***

through 63.4° (or arctan 2 or θ) clockwise . ***(A1)***

Notes:	For parts (a) and (c), if centre of rotation not included, award ***(A0)*** in first line of (a) but ***(A1)*** in first line of (c). Positive rotation may be used instead of anti-clockwise, and negative instead of clockwise. In part (b), both shear and x-axis invariant line required for ***(A1)***.

(ii) The first transformation applied is ***R***. ***(A1)***

[7 marks]

(iii) For points (x, y) on the invariant line

$$\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -4.4 \\ 8.8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$ ***(M1)***

$\Rightarrow 0.6x + 0.8y - 4.4 = x \Rightarrow 0.4x - 0.8y + 4.4 = 0$ ***(A1)***

$\Rightarrow 0.8x - 0.6y + 8.8 = y \Rightarrow 0.8x - 1.6y + 8.8 = 0$ ***(A1)***

Both equations are equivalent to $x - 2y + 11 = 0$ ***(A1)***

[4 marks]

Total [30 marks]