IBO

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Tuesday 6 May 2003 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

SECTION A

*Answer all **five** questions from this section.*

1. *[Maximum mark: 18]*

(i) The sets U, P, R and S are defined as follows:

$U = \{$all quadrilaterals$\}$

$P = \{$all parallelograms$\}$

$R = \{$all rectangles$\}$

$S = \{$all squares$\}$

(a) Draw a Venn Diagram illustrating the relationships of the above sets. *[4 marks]*

(b) Draw a separate Venn Diagram for each of the examples below. Indicate by shading each of the following:

(i) $(P \cup S)'$

(ii) $(R \cup S) \cap P$ *[4 marks]*

(ii) Consider each of the following statements:

p: Alex is from Uruguay
q: Alex is a scientist
r: Alex plays the flute

(a) Write each of the following arguments in symbols:

(i) If Alex is not a scientist then he is not from Uruguay.

(ii) If Alex is a scientist, then he is either from Uruguay or plays the flute. *[3 marks]*

(b) Write the following argument in words:

$\neg r \Rightarrow \neg(q \vee p)$ *[3 marks]*

(This question continues on the following page)

(Question 1 (ii) continued)

(c) Construct a truth table for the argument in part (b) using the values below for *p*, *q*, *r* and ¬*r*. Test whether or not the argument is logically valid.

| *p*	*q*	*r*	¬*r*
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

[4 marks]

2.   *[Maximum mark: 11]*

Raul, in house R, is directly across the lake from Sylvia, in house S.   The houses are two kilometres apart.   When both Raul and Sylvia are facing due north, they see a speedboat B in the lake between the two houses.   Raul in house R can see the boat at 35° east of where he is facing.   Sylvia in house S can see the same boat at 65° west of where she is facing.

(a)   Copy and complete the diagram below, indicating which is the 35° angle, and which is the 65° angle.

diagram not to scale
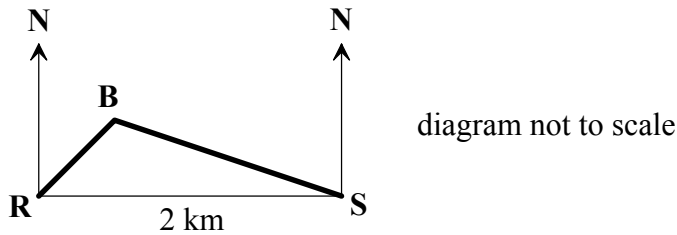
[2 marks]

(b)   (i)   Calculate the size of RB̂S.

(ii)   At this moment, how far is the boat (B) from Raul's house (R)? Please give your answer to the nearest 100 metres.   *[5 marks]*

(c)   Raul and Sylvia then see a sailboat on the lake at point Q, which is 2.6 km from Raul (R) and 3.5 km from Sylvia (S).   Calculate the size of RQ̂S at that moment, giving your answer to the nearest degree.   *[4 marks]*

3. *[Maximum mark: 13]*

Fifty students at Layton High School recorded how much money each student in their class spent on video rentals this month (to the nearest dollar). The results are shown in the frequency table below:

Class interval in $	Boundaries in $	Frequency
1 – 10	0.50 – 10.50	10
11 – 20	10.50 – 20.50	20
21 – 30	20.50 – 30.50	10
31 – 40	30.50 – 40.50	0
41 – 50	40.50 – 50.50	4
51 – 60	50.50 – 60.50	2
61 – 70	60.50 – 70.50	4

(a) On graph paper using a scale of 2 cm to represent each interval ($ 10.00) on the horizontal axis and 1 cm to represent 5 people on the vertical axis, draw and clearly label a frequency histogram which displays the above information. *[5 marks]*

(b) Answer the following questions:

(i) Which class is the modal class?

(ii) In which class is the median? *[2 marks]*

(c) Assuming these students spend the same amount on videos each month, find the probability that next month a student will spend an amount in the class interval:

(i) From $ 21 to $ 30 inclusive on video rentals.

(ii) $ 30 or less on video rentals.

(iii) From $ 41 to $ 60 on video rentals, given that they spent more than $ 20 on video rentals.

(iv) Not more than $ 60 on video rentals, given that they spent over $ 10 on video rentals. *[6 marks]*

4. *[Maximum mark: 14]*

The number of bacteria (y) present at any time is given by the formula:

$y = 15000\,e^{-0.25t}$, where t is the time in seconds and e $= 2.72$ correct to 3 s.f.

(a) Calculate the values of a, b and c to the nearest hundred in the table below:

Time in seconds (t)	0	1	2	3	4	5	6	7	8
Amount of bacteria (y) (nearest hundred)	a	11700	9100	7100	b	4300	3300	2600	c

[3 marks]

(b) On graph paper using 1 cm for each second on the horizontal axis and 1 cm for each thousand on the vertical axis, draw and label the graph representing this information. *[5 marks]*

(c) Using your graph, answer the following questions:

    (i) After how many seconds will there be 5000 bacteria? Give your answer correct to the nearest tenth of a second.

    (ii) How many bacteria will there be after 6.8 seconds? Give your answer correct to the nearest hundred bacteria.

   (iii) Will there be a time when there are no bacteria left? Explain your answer. *[6 marks]*

5.　　*[Maximum mark: 14]*

(a)　Uschi wants to mail a package to Singapore from the post office. She has two options. Option A has a fixed charge to mail the package plus a cost based on the weight of the package. These charges are expressed by the equation $A(x) = 6 + 3x$, where x is the weight of the package in kg and $A(x)$ is the total cost to mail the package in $.

　　　(i)　What is the fixed charge for mailing a package under Option A?

　　　(ii)　What would be the cost of mailing a package weighing 2.4 kg under Option A?　　　　　　　　　　　　　　　　　*[3 marks]*

(b)　The cost for Option B is partially shown in the graph below. The weight in kg is represented by the variable x.
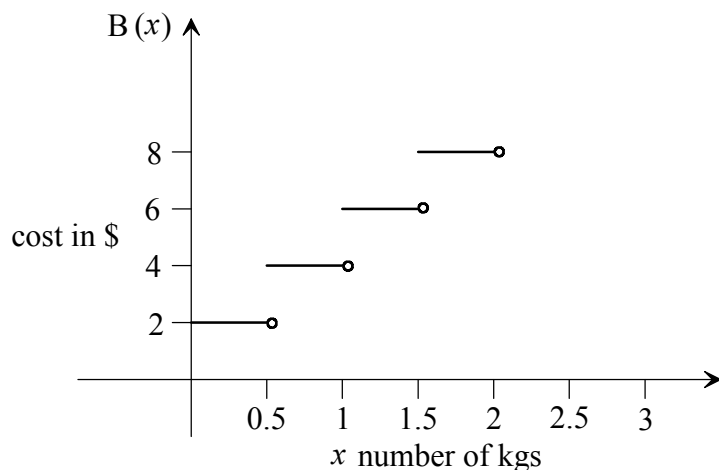
　　　(i)　The function $B(x)$ can be defined as follows for x varying between 0 and 1 kg.

$$B(x) = \begin{cases} 2 & \text{for} \quad 0 \le x < 0.5 \\ 4 & \text{for} \quad 0.5 \le x < 1 \end{cases}$$

　　　　For weights above 2 kg, the cost continues to increase in units of $ 2, following the same pattern as for lower weights.

　　　　Define $B(x)$ for weights from 2 to 3 kg, writing your answer in the following way:

$$B(x) = \begin{cases} \underline{\quad} & \text{for} \quad \underline{\qquad} \\ \underline{\quad} & \text{for} \quad \underline{\qquad} \end{cases}$$

　　　(ii)　Find the cost of mailing a package weighing 1.6 kg, using Option B.　　*[5 marks]*

(This question continues on the following page)

(Question 5 continued)

    (c)   Answer the questions below based on the information about Options A and B. Show your method for each:

        (i)   If it cost Uschi $ 22.50 to mail a package using Option A, what was the weight of the package she mailed?

        (ii)   How much would it cost to mail this same package using Option B?

        (iii)   By considering appropriate values of $A(x) = 6 + 3x$ find a (non-zero) weight for which the cost of both options is the same. Explain your reasoning and state the cost.   *[6 marks]*

SECTION B

*Answer **one** question from this section.*

Matrices and Graph Theory

6. *[Maximum mark: 30]*

(i) The *Barundi Baking Company* has two sites, one in Denver and the other in Barcelona. Each site has three types of employee and, therefore, three weekly pay scales, one for each type of worker.

Those employees who work in administration earn $ 750 a week.
Those who work as office staff earn $ 350 a week.
Those who work in the factory earn $ 200 a week.

The number of workers in each section at each site is the following:

At the Denver site there are: 42 in Administration, 112 Office Staff and 316 Factory Workers.
At the Barcelona site there are: 22 in Administration, 56 Office Staff and 162 Factory Workers.

(a) Construct a 3×2 matrix A to show the number of workers of each type in each city. Label carefully.   *[2 marks]*

(b) (i) What would the matrix $C = \begin{pmatrix} 750 & 350 & 200 \end{pmatrix}$ represent?

(ii) What are the dimensions of this matrix?   *[2 marks]*

(c) Given that $CA = \begin{pmatrix} n & 68\,500 \end{pmatrix}$,

(i) Calculate the value of n.

(ii) Explain the significance of this value.   *[3 marks]*

(d) What is the total amount of the *Barundi Company*'s payroll in these two sites for four weeks?   *[3 marks]*

(This question continues on the following page)

(Question 6 continued)

(ii) Trains run on the following tracks between cities as shown on the map below:
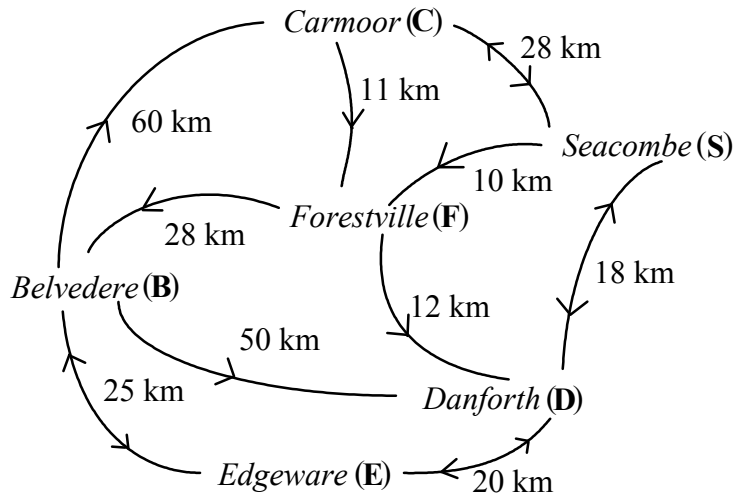
(a) Find the shortest path from *Belvedere* to *Seacombe*. *[1 mark]*

(b) Find the values of *a, b* and *c* in the adjacency matrix below which shows the routes between the cities.

$$
\begin{array}{c c c c c c c}
 & \mathbf{C} & \mathbf{B} & \mathbf{E} & \mathbf{D} & \mathbf{F} & \mathbf{S} \\
\mathbf{C} & 0 & 0 & 0 & 0 & 1 & 1 \\
\mathbf{B} & 1 & 0 & 1 & 1 & 0 & 0 \\
\mathbf{E} & 0 & 1 & 0 & 1 & 0 & 0 \\
\mathbf{D} & 0 & 0 & 1 & 0 & 0 & a \\
\mathbf{F} & 0 & 1 & c & 1 & 0 & b \\
\mathbf{S} & 1 & 0 & 0 & 1 & 1 & 0
\end{array}
$$

[3 marks]

(c) When it snows, all the tracks have to be cleared. Describe a trail that a snowplough could take to clear all the rail lines, where the snowplough would not go over any rail line more than once. *[3 marks]*

(This question continues on the following page)

(Question 6 continued)

(iii) In a recent study at the Onegin School, it was found that of the students who pass mathematics in their ninth year, 80 % pass mathematics in their tenth year. Of those who do not pass mathematics in their ninth year, 68 % do not pass in their tenth year either.

(a) Write down the values of p, q, r and s in the matrix below which express the above data:

tenth year students

pass   do not pass

ninth year students   pass   do not pass

$$\begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

[4 marks]

(b) This year there are 300 ninth year students who passed mathematics, and 75 who did not pass mathematics.

(i) Assuming no one leaves the school, how many of these students will be passing mathematics after their tenth year?

(ii) How many students will **not** be passing mathematics after their tenth year?

[4 marks]

(iv) The following is a payoff matrix for Bonnie and Clyde which shows Bonnie's winnings:

Clyde

L   M   N

Bonnie
$$\begin{array}{c} P \\ Q \\ R \end{array} \begin{pmatrix} 3 & 5 & -6 \\ -2 & 1 & 2 \\ 0 & 4 & -1 \end{pmatrix}$$

(a) If Bonnie plays row **Q** and Clyde plays column **L**, what will be the outcome?

[1 mark]

A "play safe" strategy is one which minimizes losses.

(b) Which row should Bonnie choose to "play safe"?

[1 mark]

(c) Which column should Clyde choose to "play safe"?

[1 mark]

(d) If both players "play safe", who will win and how much?
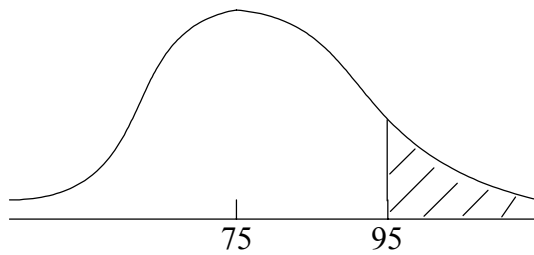
[2 marks]

Further Statistics and Probability

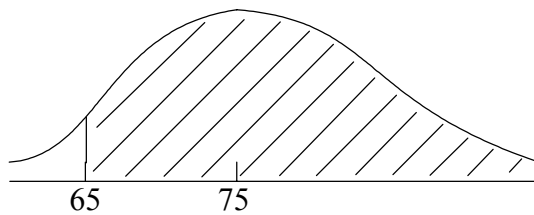**7.** *[Maximum mark: 30]*

    (i)   A set of 1000 test scores is normally distributed, with a mean of 75 and a standard deviation of 10.

        (a)   Calculate the probability which is represented by each of the following diagrams, giving your answers to 3 decimal places.
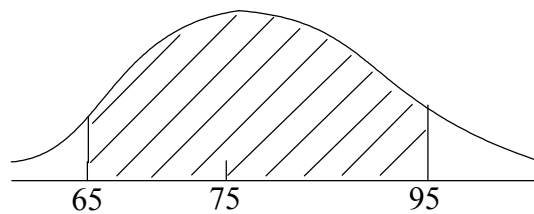
            (i)

            (ii)

            (iii)

[6 marks]

        (b)   Out of the thousand students, how many received test results higher than 87? *[3 marks]*

(This question continues on the following page)

(Question 7 continued)

(ii)   In the small town of *Joinville*, population 1000, an election was held. The results were as follows:

	Urban Voters	Rural Voters
Candidate A	295	226
Candidate B	313	166

In (a) to (c) below we will use a chi-squared test to decide whether the choice of candidate depends on where the voter lives.

Null Hypothesis H_0 :   The choice of candidate is independent of where the voter lives.

(a)   (i)   Write down the alternative hypothesis.

(ii)   Use the information above to fill in *a* and *b* in the table below.

Cell	f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$
1	295	317	–22	484
2	226	204	22	484
3	313	291	22	484
4	166	*a*	*b*	484

[3 marks]

(b)   (i)   Calculate the chi-squared statistic.

(ii)   Write the number of degrees of freedom.

(iii)   At the 5 % confidence level, state the chi-squared critical value.   *[5 marks]*

(c)   (i)   Hence, state your conclusion.

(ii)   Explain why you reached this conclusion.   *[2 marks]*

(This question continues on the following page)

Turn over

(Question 7 continued)

(iii) The following are the results of a survey of the scores of 10 people on both a mathematics (x) and a science (y) aptitude test:

Student	Mathematics (x)	Science (y)	
1	90	85	
2	38	60	
3	58	78	$\bar{x} = 73$
4	85	70	$\bar{y} = 78$
5	73	65	
6	82	71	
7	56	80	$S_x = 16.7$
8	73	90	$S_y = 10.8$
9	95	96	$S_{xy} = 100.1$
10	80	85	

(a) Copy the graph below on graph paper and fill in the missing points for students 7–10 on the graph.
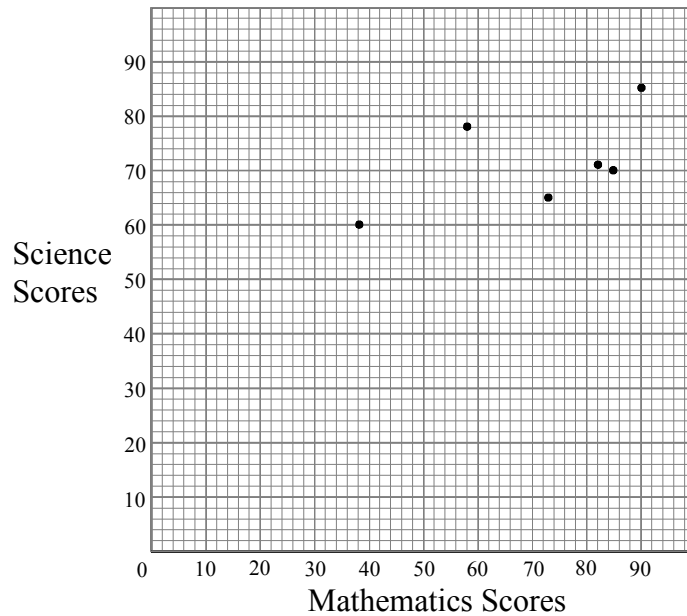
[4 marks]

(b) Plot the point M $(\bar{x}, \bar{y})$ on the graph.

[1 mark]

(c) Find the equation of the regression line of y on x in the form $y = ax + b$.

[2 marks]

(d) Graph this line on the above graph.

[2 marks]

(e) Given that a student receives an 88 on the mathematics test, what would you expect this student's science score to be? Show how you arrived at your result.

[2 marks]

Introductory Differential Calculus

8. *[Maximum mark: 30]*

   (i) Consider the function $f(x) = x^3 - 4x^2 - 3x + 18$

      (a) (i) Find $f'(x)$.

           (ii) Find the coordinates of the maximum and minimum points of the function. *[10 marks]*

      (b) Find the values of $f(x)$ for a and b in the table below:

x	−3	−2	−1	0	1	2	3	4	5
$f(x)$	−36	a	16	b	12	4	0	6	28

[2 marks]

      (c) Using a scale of 1 cm for each unit on the *x*-axis and 1 cm for each 5 units on the *y*-axis, draw the graph of $f(x)$ for $-3 \le x \le 5$. Label clearly. *[5 marks]*

      (d) The gradient of the curve at any particular point varies. Within the interval $-3 \le x \le 5$, state all the intervals where the gradient of the curve at any particular point is

           (i) negative.

           (ii) positive. *[3 marks]*

   (ii) The acceleration $a(t)$ in m s^{-2} of a vehicle is given by $a(t) = 2t - 3$, where t is the time in seconds.

      (a) Is this vehicle accelerating or decelerating when $t = 3$? Explain why. *[3 marks]*

      (b) At what value of t is the vehicle neither accelerating nor decelerating? *[2 marks]*

      (c) After 2 seconds, the velocity of the vehicle is 6 m s^{-1}.

           (i) Find $v(t)$, the velocity of the vehicle, in terms of t.

           (ii) How fast is the vehicle moving after four seconds? *[5 marks]*