

MARKSCHEME

May 2003

MATHEMATICAL STUDIES

Standard Level

Paper 2

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Paper 2 Markscheme

Instructions to Examiners

1 Method of marking

- (a) All marking must be done using a **red** pen.
- (b) Marks should be noted on candidates' scripts as in the markscheme:
 - show the breakdown of individual marks using the abbreviations *(M1), (A2) etc.*
 - write down each part mark total, indicated on the markscheme (for example, *[3 marks]*) – it is suggested that this be written at the end of each part, and underlined;
 - write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

- (i) penalise an error when it **first occurs**;
- (ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;
- (iii) award **M** marks for a correct method, and **A(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
$\$ 600 \times 1.02$	<i>MI</i>	Amount earned = $\$ 600 \times 1.02$	✓	<i>MI</i>
= $\$ 612$	<i>AI</i>	= $\$602$	×	<i>A0</i>
$\$ (306 \times 1.02) + (306 \times 1.04)$	<i>MI</i>	Amount = $301 \times 1.02 + 301 \times 1.04$	✓	<i>MI</i>
= $\$ 630.36$	<i>AI</i>	= $\$ 620.06$	✓	<i>AI(ft)</i>

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

- (i) **fewer** marks should be awarded at the discretion of the Examiner;
- (ii) marks awarded should be followed by “**(d)**” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

- (a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

- (i) a mark should be awarded followed by “**(d)**” (to indicate that these marks have been awarded at the **discretion** of the Examiner);
- (ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **METHOD 1**, **METHOD 2**, *etc.* Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

$$\begin{aligned} \text{Mean} &= 7906/134 && \text{(M1)} \\ &= 59 && \text{(A1)} \end{aligned}$$

OR

$$\text{Mean} = 59 \qquad \text{(G2)}$$

- (b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect ie, once the correct answer is seen, ignore further working.

- (c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , $1\cdot7$, $1,7$; different forms of vector notation such as \vec{u} , \overline{u} , \underline{u} ; $\tan^{-1} x$ for $\arctan x$.

5 Accuracy of Answers

There are two types of accuracy errors, incorrect level of accuracy, and rounding errors.

Unless the level of accuracy is specified in the question, candidates should be penalized **once only IN THE PAPER** for any accuracy error (**AP**). This could be an incorrect level of accuracy (**only applies to fewer than three significant figures**), or a rounding error. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy, or rounded incorrectly, maximum marks are **not** awarded, but on **all subsequent occasions** when accuracy errors occur, then maximum marks **are** awarded.

(a) Level of accuracy

- (i) In the case when the accuracy of the answer is **specified in the question** (for example: “find the size of angle A to the nearest degree”) the maximum mark is awarded **only if** the correct answer is given to the accuracy required.
- (ii) When the accuracy is **not** specified in the question, then the general rule applies:

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

However, if candidates give their answers to more than three significant figures, this is acceptable

(b) Rounding errors

Rounding errors should only be penalized at the **final answer** stage. This does **not** apply to intermediate answers, only those asked for as part of a question. Premature rounding which leads to incorrect answers should only be penalized at the answer stage.

Incorrect answers are wrong, and should not be considered under (a) or (b).

Examples

A question leads to the answer $4.6789\dots$

- 4.68 is the correct 3 s.f. answer.
- 4.7 , 4.679 are to the wrong level of accuracy : 4.7 should be penalised the first time this type of error occurs, but 4.679 is **not** penalized, as it has more than three significant figures.
- 4.67 is incorrectly rounded – penalise on the first occurrence.
- 4.678 is incorrectly rounded, but has more than the required accuracy, do **not** penalize.

Note: All these “incorrect” answers may be assumed to come from $4.6789\dots$, even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5 , 4.8 , and these should be penalised as being incorrect answers, not as examples of accuracy errors.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

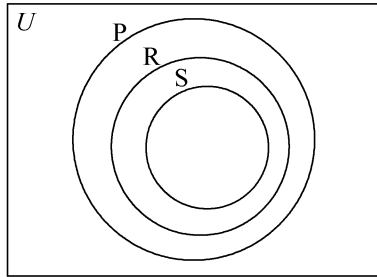
- (i) Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10 % of their overall mark.. Note this on the front cover.

- (ii) Calculator box not filled in.

Please apply a calculator penalty (*CP*) of 1 mark if this information is not provided. Note this on the front cover.

1. (i) (a)

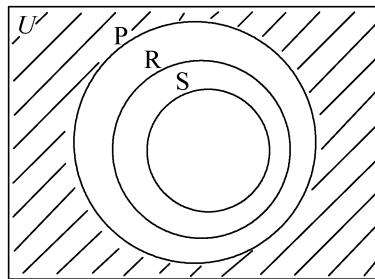


(A4)

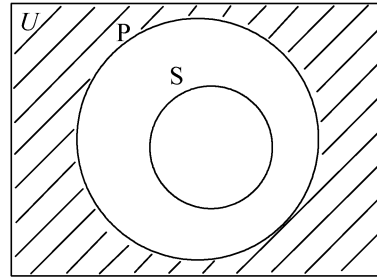
Note: Award (A1) for rectangle labelled with U, (A1) for R placed correctly with respect to S, (A1) for S placed correctly with respect to P, (A1) for R placed correctly with respect to P.

[4 marks]

(b) (i)

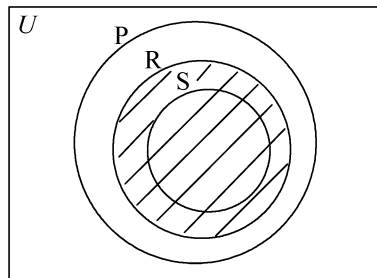


OR

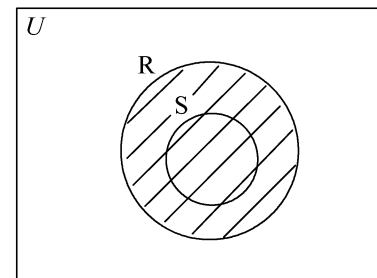


(A2)

(ii)



OR



(A2)

Note: Award (A2) for correct shading in each ft from circle placement in (a).

[4 marks]

(ii) (a) (i) $\neg q \Rightarrow \neg p$

(A1)

(ii) $q \Rightarrow (p \vee r)$

(A2)

Note: Award (A1) for $q \Rightarrow$ and (A1) for $(p \vee r)$ with the parentheses.

[3 marks]

(b) If Alex does not play the flute then it is not true that he is a scientist or from Uruguay.

OR

If Alex does not play the flute then he is neither a scientist nor from Uruguay.

(A3)

Note: Award (A1) if then correct, (A1) if antecedent correct, (A1) if consequent correct.

[3 marks]

continued...

Question 1 (ii) continued

(c)	$q \vee p$	$\neg(q \vee p)$	$\neg r \Rightarrow \neg(q \vee p)$
	T	F	T
	T	F	F
	T	F	T
	T	F	F
	T	F	T
	T	F	F
	F	T	T
	F	T	T
	(A1)	(A1)	(A1)

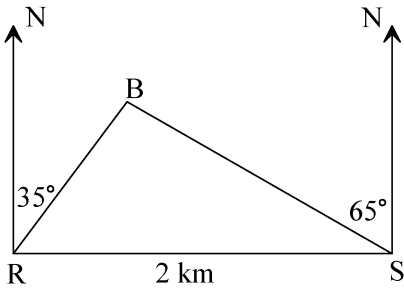
Not a logically valid argument.

(A1)

[4 marks]

Total [18 marks]

2. (a)

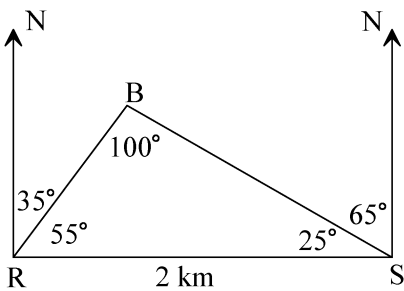


(A2)

Note: Award (A1) for 35° and (A1) for 65°.

[2 marks]

(b)



(i) $\hat{RBS} = 100^\circ$

(A1)

(ii) Correct measure of \hat{RSB} is 25°

(A1)

Using sine rule, $\frac{RB}{\sin 25^\circ} = \frac{2}{\sin 100^\circ}$

(M1)

$RB = 0.858$

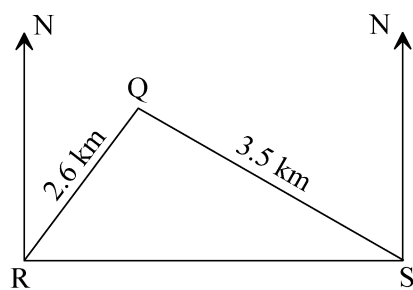
(A1)

$= 0.9 \text{ km or } 900 \text{ m}$

(A1)

[5 marks]

(c)



$2^2 = 2.6^2 + 3.5^2 - 2(2.6)(3.5)\cos Q$

(M1)(A1)

$\hat{Q} = 34.4^\circ$

(A1)

$= 34^\circ \text{ (to nearest degree)}$

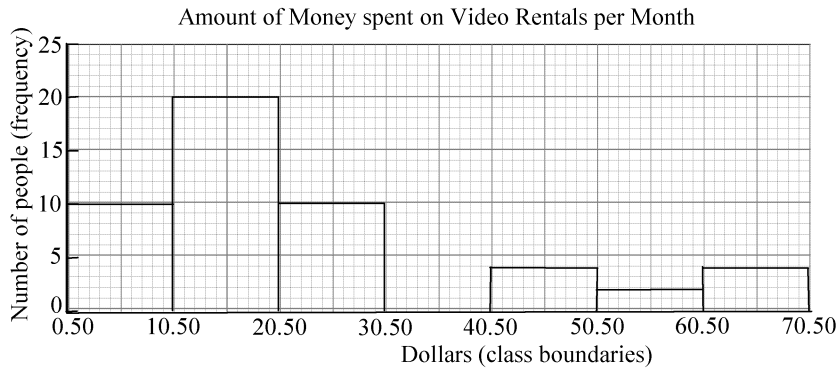
(A1)

Note: Award (M1) for using cosine rule, (A1) for correct substitution. So, for example, $2^2 = 2.6^2 + 3.5^2 + 2(2.6)(3.5)\cos Q$ is awarded (M1)(A0)(A1)(ft)(A1)(ft).

[4 marks]

Total [11 marks]

3. (a)



(A5)

Notes: Award (A1) for correct scales, (A1) for bars connected (where possible), (A1) for axes labelled correctly, (A2) for bars all correctly drawn. If one bar is incorrect, award (A1), if 2 or more bars are incorrect, award (A0). 30.50 – 40.50 counts as a bar, height zero. Also, allow horizontal scale to be 10, 20, 30, etc, with bars moved 0.50 to the right.

[5 marks]

(b) (i) 11 – 20 (accept 10.50 – 20.50)

(A1)

(ii) 11 – 20 (accept 10.50 – 20.50)

(A1)

[2 marks]

(c) (i) $\frac{10}{50} = \frac{1}{5}$ or 20 %

(A1)

(ii) $\frac{40}{50} = \frac{4}{5}$ or 80 %

(A1)

(iii) $\frac{6}{20} = \frac{3}{10}$ or 30 %

(A2)

Note: Award (A1) for correct numerator and (A1) for correct denominator.

(iv) $\frac{36}{40} = \frac{9}{10}$ or 90 %

(A2)

Note: Award (A1) for correct numerator and (A1) for correct denominator.

[6 marks]

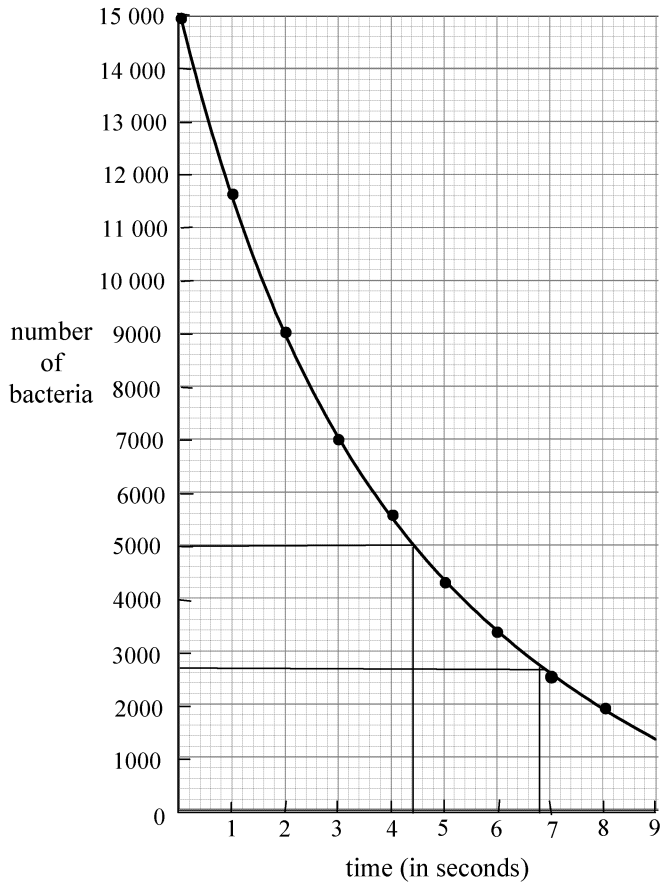
Total [13 marks]

4. (a) $a = 15000$
 $b = 5500$
 $c = 2000$

(AI)
(AI)
(AI)

[3 marks]

(b)



Note: Award (AI) for axes correctly labelled, (AI) for correct scales, (AI) for smooth curve, (A2) for all points correctly plotted, (AI) for at least 4 points correct.

[5 marks]

Question 4 continued

- (c) (i) 4.4 secs *(MI)(AI)*

Note: Award *(MI)(AI)(ft)* from graph (see (b)) or *(AI)* if correct and no line seen.

- (ii) 2700 bacteria (± 200 bacteria) *(MI)(AI)*

Note: Award *(MI)(AI)(ft)* from graph (see (b)) or *(AI)* if correct and no line seen.

- (iii) No – theoretically, the curve never touches the horizontal axis
(or any answer to suggest that the horizontal axis is an asymptote). *(AI)(RI)*

Note: Award *(AI)(RI)* for any time over 39 seconds with a reasonable explanation
(when the number of bacteria is less than one).
Award *(A0)(R0)* for a yes or no with no explanation.
Do not award *(AI)* if *(RI)* is not awarded.

[6 marks]

Total [14 marks]

5. (a) (i) Fixed charge is \$ 6. (AI)
- (ii) $A(x) = 6 + 3x$
 $= 6 + 3(2.4)$ (AI)
 $= \$13.20$ (do not accept \$ 13.2). (AI)

[3 marks]

- (b) (i) $B(x) = \begin{cases} 10 & \text{for } 2 \leq x < 2.5 \\ 12 & \text{for } 2.5 \leq x < 3 \end{cases}$ (A3)

Note: Award (AI) for both \$ correct, (AI) for each interval correct.

- (ii) $x = 1.6 \Rightarrow 1.5 \leq x < 2$ (M1)
cost = \$ 8 (AI)

OR

- cost = \$ 8 (A2)

[5 marks]

- (c) (i) $22.50 = 6 + 3x$ (M1)
 $x = 5.5$
5.5 kg (AI)

- (ii) $5 \leq x < 5.5$ 22
 $5.5 \leq x < 6$ 24
\$ 24 (AI)

- (iii) 4.0 kg (AI)
The cost is \$ 18 for both options, (AI)
because at 4.0 kg Option A cost $\$6 + 3 \times 4 = \18
and Option B costs \$ 18 also. (R1)

OR

- $4\frac{2}{3}$ kg (AI)

- The cost is \$ 20 for both options (AI)

because Option A costs $6 + 3 \times 4\frac{2}{3} = 20$

- Option B costs \$ 20 (R1)

[6 marks]

Total [14 marks]

6. (i) (a)

	Denver	Barcelona
Administration	42	22
Office Staff	112	56
Factory Workers	316	162

(A2)

Note: Award (AI) for correct labels, (AI) for correct entries.
Award (A0) for 2×3 matrix.

[2 marks]

(b) (i) The pay scales for the 3 categories of workers (AI)

(ii) 1×3 (AI)

[2 marks]

(c) (i) $n = 42(750) + 112(350) + 316(200)$ (MI)
 $= \$133\,900$ (accept \$134 000) (AI)

OR

$n = \$133\,900$ (accept \$134 000) (A2)

(ii) The amount paid for weekly salaries at the Denver site (AI)

[3 marks]

(d) $68\,500 + 133\,900 = 202\,400$ (MI)
 $202\,400(4) = \$809\,600$ (accept \$ 810 000) (MI)(AI)

Note: Award (MI) for adding, (MI) for multiplying.

[3 marks]

(ii) (a) **B-E-D-S** (accept 63 kms) (AI)

[1 mark]

(b) $a = 1$ (AI)

$b = 0$ (AI)

$c = 0$ (AI)

[3 marks]

(c) **S-F-D-S-C-F-B-D-E-B-C** (A3)

Note: Award (AI) for starting at S or C, (AI) for ending at S or C,
(AI) for any correct path in between.

[3 marks]

Question 6 continued

- (iii) (a) $p = 0.8$ (AI)
- $q = 0.2$ (AI)
- $r = 0.32$ (AI)
- $s = 0.68$ (AI)

Note: Answers may also be percentages.

[4 marks]

- (b) (i) $300(0.8) + 75(0.32)$ (M1)
- $= 264$ (A1)
- OR**
- 264 (A2)
- (ii) $300(0.2) + 75(0.68)$ (M1)
- $= 111$ (A1)
- OR**
- $375 - 264$ (M1)
- $= 111$ (A1)
- OR**
- 111 (A2)

[4 marks]

- (iv) (a) Bonnie will lose 2 pts (or Clyde will win 2 pts). (A1)
- [1 mark]
- (b) Row R (A1)
- [1 mark]
- (c) Column N (A1)
- [1 mark]
- (d) Clyde will win 1 pt (or Bonnie will lose 1 pt). (A2)

Note: Award (AI) for Clyde wins, and (AI) for 1 point.

[2 marks]

Total [30 marks]

7. (i) (a) (i) $z = \frac{95 - 75}{10}$ (M1)
 $= 2.0$
 $P(z > 2.0) = 1 - 0.977$
 $= 0.023$ (A1)

OR

$P(z > 2.0) = 0.023$ (G2)

(ii) $z = \frac{65 - 75}{10}$ (M1)
 $= -1.0$
 $P(z > -1.0) = 0.841$ (A1)

OR

$P(z > -1.0) = 0.841$ (G2)

(iii) $P(-1.0 < z < 2.0)$ (M1)
 $= 0.819$ (accept 0.818 also) (A1)

OR

$0.841 - 0.023 = 0.818$ (M1)(A1)

[6 marks]

(b) $\frac{87 - 75}{10} = 1.2$ (M1)
 $0.1151(1000) =$ (M1)
 $= 115$ students (A1)

[3 marks]

- (ii) (a) (i) Alternative hypothesis: Choice of candidates depends on voter location. (A1)
(ii) **a** = 188 (A1)
b = -22 (A1)

[3 marks]

(b) (i) $\chi^2_{\text{calc}} = \frac{484}{317} + \frac{484}{204} + \frac{484}{291} + \frac{484}{188}$ (M1)
 $= 8.14$ (accept 8.13) (A1)

Note: Award (G2) for 7.97.

(ii) $\nu = 1$ (A1)

(iii) $\chi^2(0.95, 1) =$ (M1)
 $= 3.84$ (A1)

[5 marks]

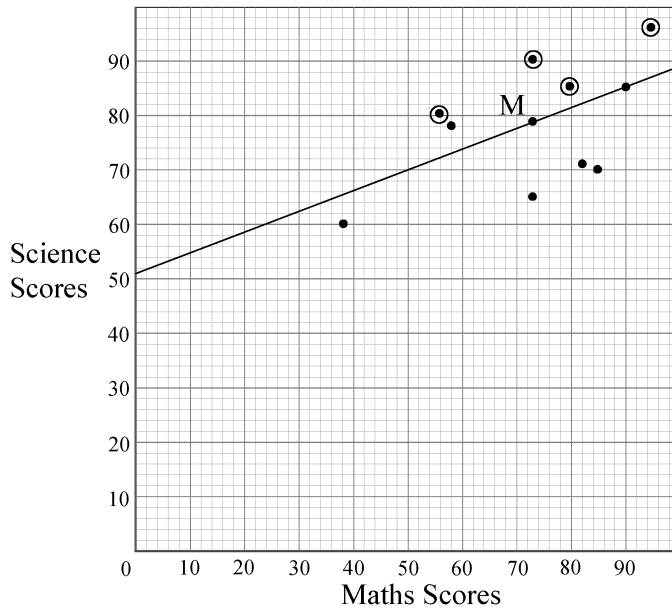
- (c) (i) Where voters live does affect how they vote. (A1)
(ii) $\chi^2_{\text{calc}} > \chi^2$ or $8.14 > 3.84$ so we reject the null hypothesis. (R1)

[2 marks]

continued...

Question 7 continued

(iii) (a)



(A4)

Note: Award (A1) for each circled point plotted correctly.

[4 marks]

(b) Point M plotted correctly.

(A1)

[1 mark]

(c) $y = 0.359x + 51.8$

(A1)(A1)

[2 marks]

(d) For reasonable line of best fit.

(A2)

Note: Award (A1) for going through M, (A1) for y intercept anywhere from 50 to 54 or ft from (c). Extrapolate line to y-axis is necessary.

[2 marks]

(e) $y = 0.359 \times 88 + 51.8$

(M1)

$y = 83$

(A1)

OR

$y = 83(\pm 2)$ if read from the graph and method is shown.

(A2)

[2 marks]

Total [30 marks]

8. (i) (a) (i) $f'(x) = 3x^2 - 8x - 3$ (AI)
- (ii) $3x^2 - 8x - 3 = 0$ (MI)
- $(3x+1)(x-3) = 0$ (AI)
- $x = -\frac{1}{3}$ (AI)
- $x = 3$ (AI)

Note: Alternatively, award (GI) for 1 correct answer, (G3) for both.

$$f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18$$

= 18.5 (AI)

$$f(3) = (3)^3 - 4(3)^2 - 3(3) + 18$$

= 0 (AI)

Points are $\left(-\frac{1}{3}, 18.5\right)$ and $(3, 0)$ (AI)

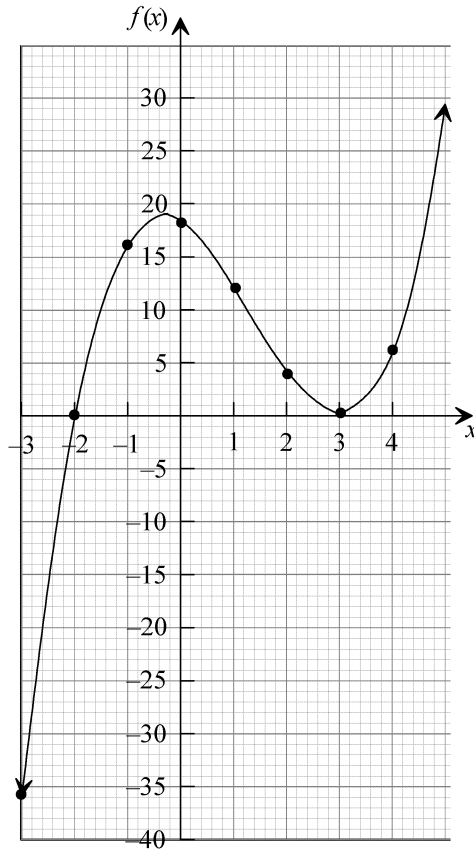
[10 marks]

- (b) $a = 0$ (AI)
- $b = 18$ (AI)

[2 marks]

Question 8 (i) continued

(c)



(A5)

Note: Award (A1) for scales and axes labelled correctly,
(A1)(A1) for maximum and minimum placed correctly,
(A1) for smooth curve, (A1) for all points plotted correctly.

[5 marks]

(d) (i) $\left(-\frac{1}{3}, 3\right)$ or $-\frac{1}{3} < x < 3$ all parts correct (A1)

(ii) $\left(-3, -\frac{1}{3}\right)$ or $-3 < x < -\frac{1}{3}$ (allow $x < -\frac{1}{3}$) (A1)

and $(3, 5)$ (allow $(x > 3)$) ft from error in (i). (A1)

[3 marks]

Question 8 continued

- (ii) (a) $a(t) = 2(3) - 3$ (M1)
 $= 3$
accelerating because 3 is positive. (A1)(R1)

[3 marks]

- (b) $2t - 3 = 0$ (M1)
 $t = \frac{3}{2}$ (or 1.5 sec). (A1)

[2 marks]

- (c) (i) $a(t) = 2t - 3$
 $v(t) = t^2 - 3t + c$ (M1)
 $6 = 2^2 - 3(2) + c$
 $c = 8$ (A1)
 $v(t) = t^2 - 3t + 8$ (must be written down). (A1)

- (ii) $v(t) = 4^2 - 3(4) + 8$ (A1)
 $= 12 \text{ ms}^{-1}$ (A1)

[5 marks]

Total [30 marks]
