# MATHEMATICAL STUDIES STANDARD LEVEL <br> PAPER 1 



Monday 5 May 2003 (afternoon)
1 hour

## INSTRUCTIONS TO CANDIDATES

- Write your candidate number in the box above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet e.g. Casio fx-9750G, Sharp EL-9600, Texas Instruments TI-85.

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Let $x=6.4 \times 10^{7}$ and $y=1.6 \times 10^{8}$.

Find
(a) $\frac{x}{y}$
(b) $y-2 x$,
giving your answers in the form $a \times 10^{k}$ where $1 \leq a<10$ and $k \in \mathbb{Z}$.

Working:

Answers:
(a)
(b)
2. The conversion formula for temperature from the Fahrenheit (F) to the Celsius (C) scale is given by $C=\frac{5(F-32)}{9}$.
(a) What is the temperature in degrees Celsius when it is $50^{\circ}$ Fahrenheit?

There is another temperature scale called the Kelvin (K) scale.
The temperature in degrees Kelvin is given by $K=C+273$.
(b) What is the temperature in Fahrenheit when it is zero degrees on the Kelvin scale?

## Working:

Answers:
(a)
(b) $\qquad$
3. Zog from the planet Mars wants to change some Martian Dollars (MD) into US Dollars (USD). The exchange rate is $1 \mathrm{MD}=0.412$ USD. The bank charges $2 \%$ commission.
(a) How many US Dollars will Zog receive if she pays 3500 MD ?

Zog meets Zania from Venus where the currency is Venusian Rupees (VR). They want to exchange money and avoid bank charges. The exchange rate is $1 \mathrm{MD}=1.63 \mathrm{VR}$.
(b) How many Martian Dollars, to the nearest dollar, will Zania receive if she gives Zog 2100 VR?

Working:

Answers:
(a)
(b) $\qquad$
4. An atlas gives the following information about the approximate population of some cities in the year 2000. The population of Nairobi has accidentally been left out.

| City | Population in Millions |
| :---: | :---: |
| Melbourne | 3.2 |
| Bangkok | 7.2 |
| Nairobi |  |
| Paris | 9.6 |
| São Paulo | 17.7 |
| Tokyo | 28.0 |
| Seattle | 2.1 |

The atlas tells us that the mean population for this group of cities is 10.01 million.
(a) Calculate the population of Nairobi.
(b) Which city has the median population value?

## Working:

Answers:
(a)
(b) $\qquad$
5. (a) The following truth table contains two entries which are incorrect, one in column three and one in column four. Circle the two incorrect entries.
(b) Fill in the two missing values in column five.
(c) Which one of the following words could you use to describe the statement represented by the values in the last column (number 6)?
(i) converse
(ii) tautology
(iii) inverse
(iv) contradiction
(v) contrapositive

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $p$ | $q$ | $p \wedge q$ | $\neg p$ | $p \vee q$ | $(p \vee q) \wedge(\neg p \wedge \neg q)$ |
| T | T | T | F | T | F |
| T | F | F | F |  | F |
| F | T | F | T | T | F |
| F | F | T | F |  | F |

Working:

Answer:
(c) $\qquad$
6. In the Venn diagram below, $A, B$ and $C$ are subsets of a universal set $U=\{1,2,3,4,6,7,8,9\}$.


List the elements in each of the following sets.
(a) $A \cup B$
(b) $A \cap B \cap C$
(c) $\left(A^{\prime} \cap C\right) \cup B$

## Working:

Answers:
(a)
(b) $\qquad$
(c) $\qquad$
7. The four diagrams below show the graphs of four different straight lines, all drawn to the same scale. Each diagram is numbered and c is a positive constant.





In the table below, write the number of the diagram whose straight line corresponds to the equation in the table.

| Equation | Diagram number |
| :---: | :---: |
| $y=\mathrm{c}$ |  |
| $y=-x+\mathrm{c}$ |  |
| $y=3 x+\mathrm{c}$ |  |
| $y=\frac{1}{3} x+\mathrm{c}$ |  |

8. The figure shows two adjacent triangular fields ABC and ACD where $\mathrm{AD}=30 \mathrm{~m}, \mathrm{CD}=80 \mathrm{~m}$, $\mathrm{BC}=50 \mathrm{~m} . \mathrm{AD} \mathrm{C}=60^{\circ}$ and $\mathrm{BA} \mathrm{C}=30^{\circ}$.


Note:
Diagram not drawn to scale
(a) Using triangle ACD calculate the length AC.
(b) Calculate the size of ABC .

Working:

Answers:
(a)
(b) $\qquad$
9. A group of 15 students was given a test on mathematics. The students then played a computer game. The diagram below shows the scores on the test and the game.


The mean score on the mathematics test was 56.9 and the mean score for the computer game was 45.9. The point M has coordinates (56.9, 45.9).
(a) Describe the relationship between the two sets of scores.

A straight line of best fit passes through the point $(0,69)$.
(b) On the diagram draw this straight line of best fit.

Jane took the tests late and scored 45 at mathematics.
(c) Using your graph or otherwise, estimate the score Jane expects on the computer game, giving your answer to the nearest whole number.

## Working:

## Answers:

(a) $\qquad$
(c) $\qquad$
10. A machine can make either boxes of screws or boxes of bolts. It can run for a maximum of 80 hours per week. Each box takes one hour to produce. The machine must make at least 20 boxes of screws per week. The number of boxes of bolts, $y$, must be at least as big as the number of boxes of screws, $x$.

This information is shown in the diagram below.

(a) Write down the equation of the line PQ .

The profit for a box of screws is $\$ 40$ and for a box of bolts is $\$ 60$.
(b) How many boxes of screws and how many boxes of bolts should be produced in order to achieve maximum profit?

Working:

Answers:
(a)
(b) $\qquad$
11. In the diagram below, $L, M$ and $N$ are the mid-points of the sides $P Q, P S$ and $Q R$ respectively.


The sum of vectors $\overrightarrow{P Q}, \overrightarrow{Q R}$ and $\overrightarrow{R P}$ is zero, i.e. $\overrightarrow{P Q}+\overrightarrow{Q R}+\overrightarrow{R P}=0$.
(a) Write down a sum of four vectors which is zero.
(b) Find an expression for the vector $\overrightarrow{L N}$ in terms of $\overrightarrow{P Q}$ and $\overrightarrow{Q R}$.

Working:

Answers:
(a)
(b)
12. The diagram shows a chain hanging between two hooks $A$ and $B$.

The points A and B are at equal heights above the ground. P is the lowest point on the chain. The ground is represented by the $x$-axis. The $x$-coordinate of A is -2 and the $x$-coordinate of B is 2 . Point P is on the $y$-axis.

The shape of the chain is given by $y=2^{x}+2^{-x}$ where $-2 \leq x \leq 2$

(a) Calculate the height of the point P .
(b) Find the range of $y$. Write your answer as an interval or using inequality symbols.

## Working:

Answers:
(a) $\qquad$
(b) $\qquad$
13. Mario has spent $\$ 40000$ to buy some land. The land increases in value by $5 \%$ each year.
(i) What is the value of the land after the end of five years?

At the end of five years, Mario sells the land. He pays $1 \%$ tax on the sale and spends the rest of the money on a car. The car loses value at a rate of $\$ 2500$ every year.
(ii) How much tax does Mario pay?
(iii) How much is the car worth five years after Mario buys it?

Working:

Answers:
(i) $\qquad$
(ii)
(iii)
14. The figure below shows part of the graph of a quadratic function $y=a x^{2}+4 x+c$.

(a) Write down the value of $c$.
(b) Find the value of $a$.
(c) Write the quadratic function in its factorised form.

## Working:

Answers:
(a)
(b)
(c) $\qquad$
15. The figure below shows a hexagon with sides all of length 4 cm and with centre at O . The interior angles of the hexagon are all equal.


The interior angles of a polygon with $n$ equal sides and $n$ equal angles (regular polygon) add up to $(n-2) \times 180^{\circ}$.
(a) Calculate the size of angle ABC .
(b) Given that $\mathrm{OB}=\mathrm{OC}$, find the area of the triangle OBC .
(c) Find the area of the whole hexagon.

Working:

Answers:
(a)
(b) $\qquad$
(c) $\qquad$

