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IBO

M03/520/S(2)M+

MARKSCHEME

May 2003

MATHEMATICAL METHODS

Standard Level

Paper 2

23 pages

Paper 2 Markscheme

Instructions to Examiners

1 Method of marking

(a) All marking must be done using a **red** pen.

(b) Marks should be noted on candidates' scripts as in the markscheme:

- show the breakdown of individual marks using the abbreviations ***(M1)***, ***(A2)*** *etc*.
- write down each part mark total, indicated on the markscheme (for example, ***[3 marks]***) – it is suggested that this be written at the end of each part, and underlined;
- write down and circle the total for each question at the end of the question.

2 Abbreviations

The markscheme may make use of the following abbreviations:

M Marks awarded for **Method**

A Marks awarded for an **Answer** or for **Accuracy**

G Marks awarded for correct solutions, generally obtained from a **Graphic Display Calculator**, irrespective of working shown

R Marks awarded for clear **Reasoning**

AG **Answer Given** in the question and consequently marks are **not** awarded

3 Follow Through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks should be awarded. The procedures for awarding these marks require that all examiners:

(i) penalise an error when it **first occurs**;

(ii) **accept the incorrect answer** as the appropriate value or quantity to be used in all subsequent working;

(iii) award ***M*** marks for a correct method, and ***A*(ft)** marks if the subsequent working contains no further errors.

Follow through procedures may be applied repeatedly throughout the same problem.

The following illustrates a use of the **follow through** procedure:

Markscheme		Candidate's Script	Marking	
\$ 600 × 1.02	***M1***	Amount earned = \$ 600 × 1.02	√	***M1***
= \$ 612	***A1***	= \$602	×	***A0***
\$ (306 × 1.02) + (306 × 1.04)	***M1***	Amount = 301 × 1.02 + 301 × 1.04	√	***M1***
= \$ 630.36	***A1***	= \$ 620.06	√	***A1*(ft)**

Note that the candidate made an arithmetical error at line 2; the candidate used a correct method at lines 3, 4; the candidate's working at lines 3, 4 is correct.

However, if a question is transformed by an error into a **different, much simpler question** then:

(i) **fewer** marks should be awarded at the discretion of the Examiner;

(ii) marks awarded should be followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(iii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

4 Using the Markscheme

(a) This markscheme presents a particular way in which each question may be worked and how it should be marked. **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner which is consistent with the markscheme.

In this case:

(i) a mark should be awarded followed by "**(d)**" (to indicate that these marks have been awarded at the **discretion** of the Examiner);

(ii) a brief **note** should be written on the script explaining **how** these marks have been awarded.

Where alternative methods for complete questions are included, they are indicated by **Method 1**, **Method 2**, *etc*. Other alternative solutions, including graphic display calculator alternative solutions are indicated by **OR**. For example:

Mean = 7906/134 ***(M1)***
= 59 ***(A1)***

OR

Mean = 59 ***(G2)***

(b) Unless the question specifies otherwise, accept **equivalent forms**. For example: $\frac{\sin\theta}{\cos\theta}$ for $\tan\theta$.

On the markscheme, these equivalent numerical or algebraic forms will generally be written in brackets after the required answer. Paper setters will indicate the required answer, by allocating full marks at that point. Further working should be ignored, even if it is incorrect. For example: if candidates are asked to factorize a quadratic expression, and they do so correctly, they are awarded full marks. If they then continue and find the roots of the corresponding equation, do not penalize, even if those roots are incorrect ie, once the correct answer is seen, ignore further working.

(c) As this is an international examination, all **alternative forms of notation** should be accepted. For example: 1.7 , 1·7 , 1,7 ; different forms of vector notation such as $\vec{u}$, $\bar{u}$, $\underline{u}$; $\tan^{-1} x$ for arctan *x*.

5 Accuracy of Answers

There are two types of accuracy errors, incorrect level of accuracy, and rounding errors.

Unless the level of accuracy is specified in the question, candidates should be penalized **once only IN THE PAPER** for any accuracy error **(AP)**. This could be an incorrect level of accuracy **(only applies to fewer than three significant figures)**, or a rounding error. Hence, on the **first** occasion in the paper when a correct answer is given to the wrong degree of accuracy, or rounded incorrectly, maximum marks are **not** awarded, but on **all subsequent occasions** when accuracy errors occur, then maximum marks **are** awarded.

(a) **Level of accuracy**

(i) In the case when the accuracy of the answer is **specified in the question** (for example: "find the size of angle *A* to the nearest degree") the maximum mark is awarded **only if** the correct answer is given to the accuracy required.

(ii) When the accuracy is **not** specified in the question, then the general rule applies:

Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.

However, if candidates give their answers to more than three significant figures, this is acceptable

(b) **Rounding errors**

Rounding errors should only be penalized at the **final answer** stage. This does **not** apply to intermediate answers, only those asked for as part of a question. Premature rounding which leads to incorrect answers should only be penalized at the answer stage.

Incorrect answers are wrong, and should not be considered under (a) or (b).

Examples

A question leads to the answer 4.6789….

- 4.68 is the correct 3 s.f. answer.
- 4.7, 4.679 are to the wrong level of accuracy : 4.7 should be penalised the first time this type of error occurs, but 4.679 is **not** penalized, as it has more than three significant figures.
- 4.67 is incorrectly rounded – penalise on the first occurrence.
- 4.678 is incorrectly rounded, but has more than the required accuracy, do **not** penalize.

Note: All these "incorrect" answers may be assumed to come from 4.6789..., even if that value is not seen, but previous correct working is shown. However, 4.60 is wrong, as is 4.5, 4.8, and these should be penalised as being incorrect answers, not as examples of accuracy errors.

6 Graphic Display Calculators

Many candidates will be obtaining solutions directly from their calculators, often without showing any working. They have been advised that they must use mathematical notation, not calculator commands when explaining what they are doing. Incorrect answers without working will receive no marks. However, if there is written evidence of using a graphic display calculator correctly, method marks may be awarded. Where possible, examples will be provided to guide examiners in awarding these method marks.

Calculator penalties

Candidates are instructed to write the make and model of their calculator on the front cover. Please apply the following penalties where appropriate.

(i) Illegal calculators

If candidates note that they are using an illegal calculator, please report this on a PRF, and deduct 10 % of their overall mark.. Note this on the front cover.

(ii) Calculator box not filled in.

Please apply a calculator penalty (***CP***) of 1 mark if this information is not provided. Note this on the front cover.

1. (a) Since the vertex is at $(3, 1)$

$h = 3$ ***(A1)***

$k = 1$ ***(A1)***

[2 marks]

(b) $(5, 9)$ is on the graph $\Rightarrow 9 = a(5-3)^2 + 1$ ***(M1)***

$= 4a + 1$ ***(A1)***

$\Rightarrow 9 - 1 = 4a = 8$ ***(A1)***

$\Rightarrow a = 2$ ***(AG)***

Note:	Award ***(M1)(A1)(A0)*** for using a reverse proof, *i.e.* substituting for *a*, *h*, *k* and showing that (5, 9) is on the graph.

[3 marks]

(c) $y = 2(x-3)^2 + 1$ ***(M1)***

$= 2x^2 - 12x + 19$ ***(AG)***

[1 mark]

(d) (i) Graph has equation $y = 2x^2 - 12x + 19$

$$\frac{dy}{dx} = 4x - 12$$ ***(A1)***

(ii) At point $(5, 9)$, gradient $= 4(5) - 12 = 8$ ***(A1)***

(iii) Equation: $y - 9 = 8(x - 5)$ ***(M1)(A1)***

$8x - y - 31 = 0$

OR

$9 = 8(5) + c$ ***(M1)***

$c = -31$

$y = 8x - 31$ ***(A1)***

[4 marks]

Total [10 marks]

2. (a) $\vec{OR} = \vec{PQ}$

$= \boldsymbol{q} - \boldsymbol{p}$

$= \begin{pmatrix} 10 \\ 1 \end{pmatrix} - \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ ***(A1)(A1)***

$= \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ ***(A1)***

[3 marks]

continued…

Question 2 continued

(b) $\cos \mathrm{O\hat{P}Q} = \dfrac{\vec{\mathrm{PO}} \cdot \vec{\mathrm{PQ}}}{|\vec{\mathrm{PO}}| \times |\vec{\mathrm{PQ}}|}$ ***(A1)***

$|\vec{\mathrm{PO}}| = \sqrt{(-7)^2 + (-3)^2} = \sqrt{58}$, $|\vec{\mathrm{PQ}}| = \sqrt{3^2 + (-2)^2} = \sqrt{13}$ ***(A1)(A1)***

$\vec{\mathrm{PO}} \cdot \vec{\mathrm{PQ}} = -21 + 6 = -15$ ***(A1)***

$\cos \mathrm{O\hat{P}Q} = \dfrac{-15}{\sqrt{58}\sqrt{13}} = \dfrac{-15}{\sqrt{754}}$ ***(AG)***

[4 marks]

(c) (i) Since $\mathrm{O\hat{P}Q} + \mathrm{P\hat{Q}R} = 180^\circ$ ***(R1)***

$\cos \mathrm{P\hat{Q}R} = -\cos \mathrm{O\hat{P}Q} \left(= \dfrac{15}{\sqrt{754}} \right)$. ***(AG)***

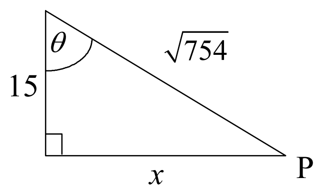
(ii) $\sin \mathrm{P\hat{Q}R} = \sqrt{1 - \left(\dfrac{15}{\sqrt{754}} \right)^2}$ ***(M1)***

$= \sqrt{\dfrac{529}{754}}$ ***(A1)***

$= \dfrac{23}{\sqrt{754}}$ ***(AG)***

OR

$\cos \theta = \dfrac{15}{\sqrt{754}}$



(M1)

therefore $x^2 = 754 - 225 = 529 \Rightarrow x = 23$ ***(A1)***

$\Rightarrow \sin \theta = \dfrac{23}{\sqrt{754}}$ ***(AG)***

Note: Award ***(A1)(A0)*** for the following solution. $\cos \theta = \dfrac{15}{\sqrt{754}} \Rightarrow \theta = 56.89^\circ$ $\Rightarrow \sin \theta = 0.8376$ $\dfrac{23}{\sqrt{754}} = 0.8376 \Rightarrow \sin \theta = \dfrac{23}{\sqrt{754}}$

continued…

Question 2 (b) continued

(iii) Area of OPQR $= 2$ (area of triangle PQR) ***(M1)***

$= 2 \times \frac{1}{2} \left| \vec{PQ} \right| \times \left| \vec{QR} \right| \times \sin P\hat{Q}R$ ***(A1)***

$= 2 \times \frac{1}{2} \sqrt{13}\sqrt{58}\, \frac{23}{\sqrt{754}}$ ***(A1)***

$= 23$ sq units. ***(A1)***

OR

Area of OPQR $= 2$ (area of triangle OPQ) ***(M1)***

$= 2\left|\left(\frac{1}{2}\right)(7\times 1 - 3\times 10)\right|$ ***(A1)(A1)***

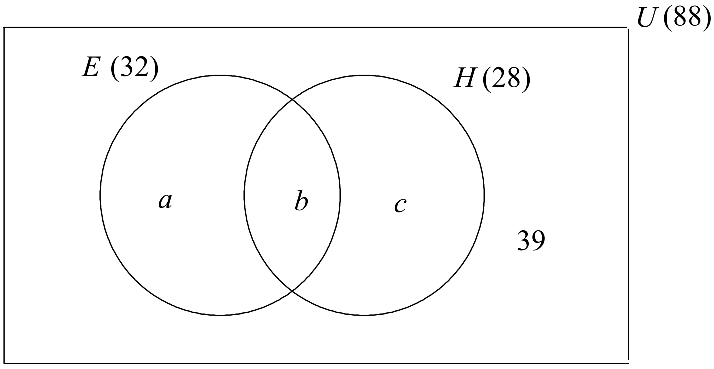
$= 23$ sq units ***(A1)***

Notes: Other valid methods can be used.
Award final ***(A1)*** for the **integer** answer.

[7 marks]

Total [14 marks]

3. (a)



$n(E \cup H) = a + b + c = 88 - 39 = 49$ ***(M1)***

$n(E \cup H) = 32 + 28 - b = 49$

$60 - 49 = b = 11$ ***(A1)***

$a = 32 - 11 = 21$ ***(A1)***

$c = 28 - 11 = 17$ ***(A1)***

Note: Award ***(A3)*** for correct answers with no working.

[4 marks]

continued…

Question 3 continued

(b) (i) $P(E \cap H) = \frac{11}{88} = \frac{1}{8}$ ***(A1)***

(ii) $P(H'|E) = \frac{P(H' \cap E)}{P(E)} = \frac{\frac{21}{88}}{\frac{32}{88}}$ ***(M1)***

$= \frac{21}{32} (= 0.656)$ ***(A1)***

OR

Required probability $= \frac{21}{32}$ ***(A1)(A1)***

[3 marks]

(c) (i) P(none in economics) $= \frac{56 \times 55 \times 54}{88 \times 87 \times 86}$ ***(M1)(A1)***

$= 0.253$ ***(A1)***

Note:	Award ***(M0)(A0)(A1)*(ft)** for $\left(\frac{56}{88}\right)^3 = 0.258$.
	Award no marks for $\frac{56 \times 55 \times 54}{88 \times 88 \times 88}$.

(ii) P(at least one) $= 1 - 0.253$ ***(M1)***

$= 0.747$ ***(A1)***

OR

$3\left(\frac{32}{88} \times \frac{56}{87} \times \frac{55}{86}\right) + 3\left(\frac{32}{88} \times \frac{31}{87} \times \frac{56}{86}\right) + \frac{32}{88} \times \frac{31}{87} \times \frac{30}{86}$ ***(M1)***

$= 0.747$ ***(A1)***

[5 marks]

Total [12 marks]

4. (a) (i) When $t = 0$, $v = 50 + 50\mathrm{e}^{0}$ ***(A1)***

$= 100\ \mathrm{m\,s^{-1}}$ ***(A1)***

(ii) When $t = 4$, $v = 50 + 50\mathrm{e}^{-2}$ ***(A1)***

$= 56.8\ \mathrm{m\,s^{-1}}$ ***(A1)***

[4 marks]

(b) $v = \dfrac{\mathrm{d}s}{\mathrm{d}t} \Rightarrow s = \int v\,\mathrm{d}t$

$\int_0^4 \left(50 + 50\mathrm{e}^{-0.5t}\right)\mathrm{d}t$ ***(A1)(A1)(A1)***

Note: Award ***(A1)*** for each limit in the correct position and ***(A1)*** for the function.

[3 marks]

(c) Distance travelled in 4 seconds $= \int_0^4 \left(50 + 50\mathrm{e}^{-0.5t}\right)\mathrm{d}t$

$= \left[50t - 100\mathrm{e}^{-0.5t}\right]_0^4$ ***(A1)***

$= \left(200 - 100\mathrm{e}^{-2}\right) - \left(0 - 100\mathrm{e}^{0}\right)$

$= 286\ \mathrm{m}$ (3 s.f.) ***(A1)***

Note: Award first ***(A1)*** for $[50t - 100\mathrm{e}^{-0.5t}]$, *i.e.* limits not required.

OR

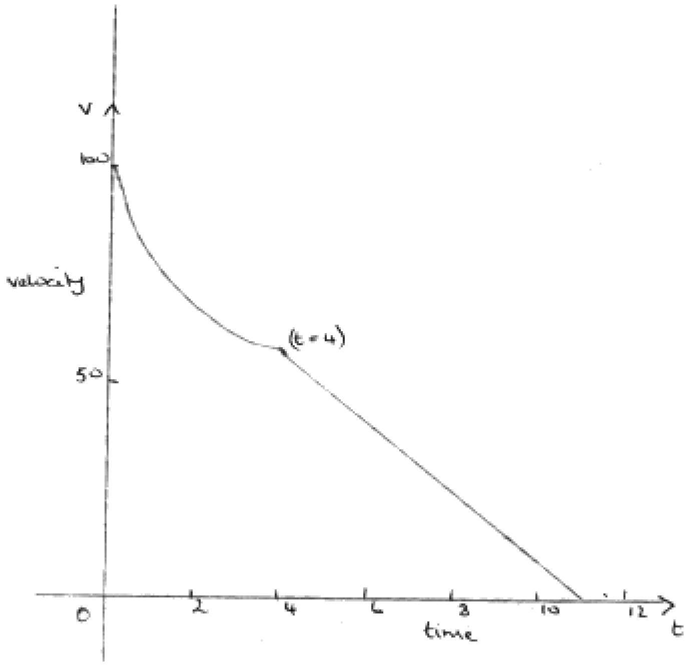
Distance travelled in 4 seconds $= 286\ \mathrm{m}$ (3 s.f.) ***(G2)***

[2 marks]

continued...

(Question 4 continued)

(d)



Notes:	Award ***(A1)*** for the exponential part, ***(A1)*** for the straight line through (11, 0), Award ***(A1)*** for indication of time on x-axis **and** velocity on y-axis, ***(A1)*** for scale on x-axis **and** y-axis. Award ***(A1)*** for marking the point where $t = 4$.

[5 marks]

(e) Constant rate $= \dfrac{56.8}{7}$ ***(M1)***

$= 8.11\,\text{m}\,\text{s}^{-2}$ ***(A1)***

Note: Award ***(M1)(A0)*** for -8.11.

[2 marks]

(f) distance $= \dfrac{1}{2}(7)(56.8)$ ***(M1)***

$= 199\text{ m}$ ***(A1)***

Note: Do not award **ft** in parts (e) and (f) if candidate has not used a straight line for $t = 4$ to $t = 11$ or if they continue the exponential beyond $t = 4$.

[2 marks]

Total [18 marks]

5. (a) Sine rule $\frac{\text{PR}}{\sin 35} = \frac{9}{\sin 120}$ ***(M1)(A1)***

$$\text{PR} = \frac{9 \sin 35}{\sin 120}$$

$$= 5.96 \text{ km}$$ ***(A1)***

[3 marks]

(b) **EITHER**

Sine rule to find PQ

$$\text{PQ} = \frac{9 \sin 25}{\sin 120}$$ ***(M1)(A1)***

$$= 4.39 \text{ km}$$ ***(A1)***

OR

Cosine rule: $\text{PQ}^2 = 5.96^2 + 9^2 - (2)(5.96)(9)\cos 25$ ***(M1)(A1)***

$$= 19.29$$

$$\text{PQ} = 4.39 \text{ km}$$ ***(A1)***

Time for Tom $= \frac{4.39}{8}$ ***(A1)***

Time for Alan $= \frac{5.96}{a}$ ***(A1)***

Then $\frac{4.39}{8} = \frac{5.96}{a}$ ***(M1)***

$$a = 10.9$$ ***(A1)***

[7 marks]

(c) $\text{RS}^2 = 4\text{QS}^2$ ***(A1)***

$4\text{QS}^2 = \text{QS}^2 + 81 - 18 \times \text{QS} \times \cos 35$ ***(M1)(A1)***

$3\text{QS}^2 + 14.74\text{QS} - 81 = 0$ (**or** $3x^2 + 14.74x - 81 = 0$) ***(A1)***

$\Rightarrow \text{QS} = -8.20$ **or** $\text{QS} = 3.29$ ***(G1)***

therefore $\text{QS} = 3.29$ ***(A1)***

OR

$$\frac{\text{QS}}{\sin \text{S}\hat{\text{R}}\text{Q}} = \frac{2\text{QS}}{\sin 35}$$ ***(M1)***

$$\Rightarrow \sin \text{S}\hat{\text{R}}\text{Q} = \frac{1}{2}\sin 35$$ ***(A1)***

$$\text{S}\hat{\text{R}}\text{Q} = 16.7^\circ$$ ***(A1)***

Therefore, $\text{Q}\hat{\text{S}}\text{R} = 180 - (35 + 16.7)$

$$= 128.3^\circ$$ ***(A1)***

$$\frac{9}{\sin 128.3} = \frac{\text{QS}}{\sin 16.7} \left(= \frac{\text{SR}}{\sin 35} \right)$$ ***(M1)***

$$\text{QS} = \frac{9 \sin 16.7}{\sin 128.3} \left(= \frac{9 \sin 35}{2 \sin 128.3} \right)$$

$$= 3.29$$ ***(A1)***

[6 marks]

Total [16 marks]

6. (i) $X \sim \mathrm{N}\left(80, 8^2\right)$

(a) $\mathrm{P}(X < 72) = \mathrm{P}(Z < -1)$ ***(M1)***

$= 1 - 0.8413$

$= 0.159$ ***(A1)***

OR

$\mathrm{P}(X < 72) = 0.159$ ***(G2)***

[2 marks]

(b) (i) $\mathrm{P}(72 < X < 90) = \mathrm{P}(-1 < Z < 1.25)$ ***(M1)***

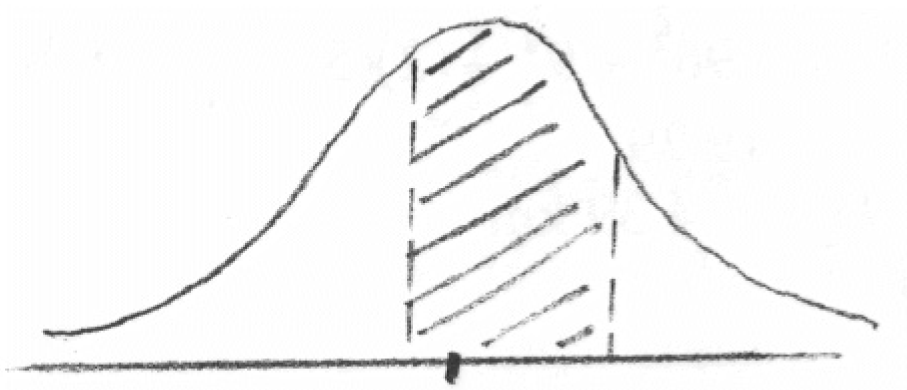
$= 0.3413 + 0.3944$ ***(A1)***

$= 0.736$ ***(A1)***

OR

$\mathrm{P}(72 < X < 90) = 0.736$ ***(G3)***

(ii)



(A1)(A1)

Note:	Award ***(A1)*** for a normal curve and ***(A1)*** for the shaded area, which should not be symmetrical.

[5 marks]

(c) 4 % fail in less than *x* months

$\Rightarrow x = 80 - 8 \times \Phi^{-1}(0.96)$ ***(M1)***

$= 80 - 8 \times 1.751$ ***(A1)***

$= 66.0$ months ***(A1)***

OR

$x = 66.0$ months ***(G3)***

[3 marks]

continued...

(Question 6 continued)

(d) (i) $H_0:$ the mean lifetime is 80 months ***(A1)***

$H_1:$ the mean lifetime is less than 80 months ***(A1)***

(ii) Sample of 100.

$\text{SE} = \dfrac{8}{\sqrt{100}} = 0.8$ ***(A1)***

A one-tailed test is used.

The critical region for a one-tailed test at the 5 % level is

$\mu < -1.645\sigma$

i.e. $Z < -1.645$ ***(A1)***

Here $Z = -\dfrac{1.5}{0.8} = -1.875$ ***(A1)***

Since this is in the critical region, we reject H_0, and conclude that the newspaper was justified in its statement. ***(AG)***

OR

$\text{P}(\bar{X} < 78.5) = \text{P}\left(Z < -\dfrac{1.5}{0.8} = -1.875\right)$ ***(A1)***

$= 1 - 0.9696$

$= 0.0304$ ***(A1)***

Since 0.0304 < 0.05 (5 %) we reject H_0, and conclude that the newspaper was justified in its statement. ***(AG)***

[5 marks]

(Question 6 continued)

(ii) (a) $y = 81.0x + 1370$ ***(G1)(G1)***

[2 marks]

(b) Let the least number sold to make a profit be *a*.
Then income $= 120a$ ***(A1)***
Production costs $= 81.0a + 1370$ ***(A1)***
Thus $120a > 81a + 1370$ ***(A1)***
$$a > \frac{1370}{120 - 81}$$
$a > 35.1$ ***(A1)***
Hence, to make a profit, the factory must produce at least 36 chairs each week. ***(A1)***

[5 marks]

(iii) Observed frequencies

	Australia	**Europe**
Top Ten finish	6	6
Outside Top Ten	11	7

(a) Expected frequencies

	Australia	**Europe**
Top Ten finish	6.8	5.2
Outside Top Ten	10.2	7.8

(A2)

Note: Award ***(A2)*** for all four correct, ***(A1)*** for three correct.

[2 marks]

(b) $$\chi^2 = \frac{(6-6.8)^2}{6.8} + \frac{(6-5.2)^2}{5.2} + \frac{(11-10.2)^2}{10.2} + \frac{(7-7.8)^2}{7.8}$$ ***(A1)***
$= 0.0941 + 0.1231 + 0.0627 + 0.0821$
$= 0.362$ ***(A1)***

OR

$\chi^2 = 0.362$ ***(G2)***

[2 marks]

continued...

(Question 6 (iii) continued)

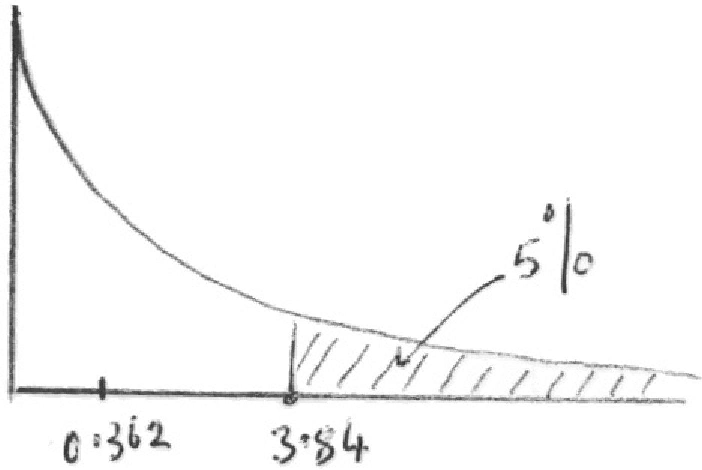
(c) (i) $\chi_1^2(5\,\%) = 3.84$ ***(A1)***

since the value of $\chi^2(0.362)$ is considerably less than 3.84, we conclude that the golfer's performance is not affected by where the tournament is played. ***(R1)***

OR

$p = 0.547 > 0.05$, so we conclude that the golfer's performance is not affected by where the tournament is played. ***(A1)*** ***(R1)***

(ii)



Note: Award ***(A1)*** for the graph and ***(A1)*** for the critical region.

[4 marks]

Total [30 marks]

7. (i) (a) (i) $\cos\left(-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}},\ \sin\left(-\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$ ***(A1)***

therefore $\cos\left(-\frac{\pi}{4}\right)+\sin\left(-\frac{\pi}{4}\right)=0$ ***(AG)***

(ii) $\cos x+\sin x=0 \Rightarrow 1+\tan x=0$

$\Rightarrow \tan x=-1$ ***(M1)***

$x=\frac{3\pi}{4}$ ***(A1)***

Note: Award ***(A0)*** for 2.36.

OR

$x=\frac{3\pi}{4}$ ***(G2)***

[3 marks]

(b) $y=e^x(\cos x+\sin x)$

$\frac{dy}{dx}=e^x(\cos x+\sin x)+e^x(-\sin x+\cos x)$ ***(M1)(A1)(A1)***

$=2e^x\cos x$

[3 marks]

(c) $\frac{dy}{dx}=0$ for a turning point $\Rightarrow\ 2e^x\cos x=0$ ***(M1)***

$\Rightarrow \cos x=0$ ***(A1)***

$\Rightarrow x=\frac{\pi}{2}\Rightarrow a=\frac{\pi}{2}$ ***(A1)***

$y=e^{\frac{\pi}{2}}\left(\cos\frac{\pi}{2}+\sin\frac{\pi}{2}\right)=e^{\frac{\pi}{2}}$

$b=e^{\frac{\pi}{2}}$ ***(A1)***

Note: Award ***(M1)(A1)(A0)(A0)*** for $a=1.57$, $b=4.81$.

[4 marks]

(d) At D, $\frac{d^2y}{dx^2}=0$ ***(M1)***

$2e^x\cos x-2e^x\sin x=0$ ***(A1)***

$2e^x(\cos x-\sin x)=0$

$\Rightarrow \cos x-\sin x=0$ ***(A1)***

$\Rightarrow x=\frac{\pi}{4}$ ***(A1)***

$\Rightarrow y=e^{\frac{\pi}{4}}\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)$ ***(A1)***

$=\sqrt{2}e^{\frac{\pi}{4}}$ ***(AG)***

[5 marks]

continued…

(Question 7 (i) continued)

(e) Required area $= \int_0^{\frac{3\pi}{4}} e^x(\cos x + \sin x)\,dx$ ***(M1)***

$= 7.46$ sq units ***(G1)***

OR

Area $= 7.46$ sq units ***(G2)***

Note: Award ***(M1)(G0)*** for the answer 9.81 obtained if the calculator is in degree mode.

[2 marks]

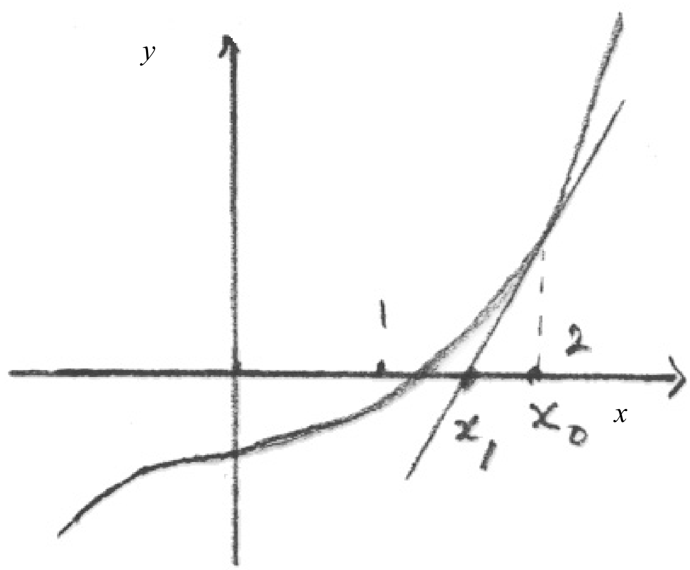
(ii) (a) $x_1 = 2 - \dfrac{\frac{x^3}{3} - \frac{x^2}{2} + x - 1}{x^2 - x + 1}$ (may be implied) ***(M1)***

$= 1.444444$ ***(A1)***

$x_2 = 1.197299$ ***(A1)***

[3 marks]

(b)



Note: Award ***(A1)*** for the diagram, ***(A1)*** for $x_0 = 2$, ***(A1)*** for the tangent at $x = 2$, ***(A1)*** for the tangent meeting the x-axis at x_1.

[4 marks]

(c) 5 iterations required (accept 4, 5, 6) ***(A1)***

[1 mark]

continued...

(Question 7(ii) continued)

(d) $\frac{x^3}{3} - \frac{x^2}{2} + x - 1 = 0$

$x^3 = 3\left(\frac{x^2}{2} - x + 1\right)$ ***(M1)***

$x_{n+1} = \sqrt[3]{3\left(\frac{x_n^2}{2} - x_n + 1\right)}$ ***(AG)***

[1 mark]

(e) (i) $x_0 = 2$

$x_1 = 1.442250$ ***(G2)***

$x_1 = 1.214947$ ***(G1)***

(ii) This method requires 8 iterations
(accept 7, 8, 9 corresponding with answer in (c)). ***(A1)***

[4 marks]

Total [30 marks]

8. (i) $\begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}\boldsymbol{X} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\boldsymbol{X} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$

$\begin{pmatrix} 4 & 1 \\ -5 & 7 \end{pmatrix}\boldsymbol{X} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ ***(M1)***

Pre-multiply by inverse of $\begin{pmatrix} 4 & 1 \\ -5 & 7 \end{pmatrix}$ ***(M1)***

$\boldsymbol{X} = \frac{1}{33}\begin{pmatrix} 7 & -1 \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ ***(A1)(A1)***

Note: Award ***(A1)*** for determinant, ***(A1)*** for matrix $\begin{pmatrix} 7 & -1 \\ 5 & 4 \end{pmatrix}$.

$= \frac{1}{33}\begin{pmatrix} 28 & 59 \\ 20 & 28 \end{pmatrix}$ ***(A1)(A1)(A1)(A1)***

$\left(\Rightarrow a = \frac{28}{33}, b = \frac{59}{33}, c = \frac{20}{33}, d = \frac{28}{33}\right)$

OR

$\begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 3a + c & 3b + d \\ -5a + 6c & -5b + 6d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ ***(A1)***

$4a + c = 4$

$-5a + 7c = 0$ ***(A1)***

$4b + d = 8$

$-5b + 7d = -3$ ***(A1)***

Note: Award ***(A1)*** for each pair of equations. Allow **ft** from their equations.

$a = \frac{28}{33}, b = \frac{59}{33}, c = \frac{20}{33}, d = \frac{28}{33}$ ***(A1)(A1)(A1)(A1)***

Note: Award ***(A0)(A0)(A1)(A1)*** if the final answers are given as decimals *i.e.* 0.848, 1.79, 0.606, 0.848.

[8 marks]

continued...

(Question 8 continued)

(ii) (a) $\boldsymbol{P}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}3\\6\end{pmatrix}$ ***(A1)***

[1 mark]

(b) (i) $\boldsymbol{P}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}8\\8\end{pmatrix}$ ***(A1)***

(ii) $\boldsymbol{P}\begin{pmatrix}3\\-2\end{pmatrix}=\begin{pmatrix}9\\-6\end{pmatrix}$ ***(A1)***

[2 marks]

(c) Since $\boldsymbol{P}$ is a linear transformation, $\begin{pmatrix}0\\0\end{pmatrix}$ is invariant. ***(A1)***

And since $\begin{pmatrix}1\\1\end{pmatrix}\rightarrow\begin{pmatrix}8\\8\end{pmatrix}$ the line $y=x$ is invariant. ***(A1)(A1)***

Similarly, the line $2x+3y=0$ is also invariant. ***(A1)(A1)***

OR

$y=x$ ***(A3)***

$2x+3y=0$ ***(A2)***

[5 marks]

(d) The only point which is invariant under $\boldsymbol{P}$ is the point of intersection of the two invariant lines, *i.e.* the origin. ***(R1)***

[1 mark]

(iii) (a) $\tan\theta=\frac{12}{5}$

$\Rightarrow \sin\theta=\frac{12}{13}$ and $\cos\theta=\frac{5}{13}$ ***(A1)(A1)***

$$\Rightarrow \boldsymbol{R}=\begin{pmatrix}\frac{5}{13} & -\frac{12}{13}\\ \frac{12}{13} & \frac{5}{13}\end{pmatrix}$$ ***(A1)(A1)***

[4 marks]

(b) $\boldsymbol{T}\begin{pmatrix}5\\1\end{pmatrix}=\begin{pmatrix}5\\1\end{pmatrix}$ ***(A1)***

[1 mark]

(c) $\boldsymbol{T}\begin{pmatrix}x\\y\end{pmatrix}=\boldsymbol{R}\begin{pmatrix}x\\y\end{pmatrix}+\begin{pmatrix}h\\k\end{pmatrix}$

$$\begin{pmatrix}\frac{5}{13} & -\frac{12}{13}\\ \frac{12}{13} & \frac{5}{13}\end{pmatrix}\begin{pmatrix}5\\1\end{pmatrix}+\begin{pmatrix}h\\k\end{pmatrix}=\begin{pmatrix}5\\1\end{pmatrix}$$ ***(M1)***

$1+h=5 \quad 5+k=1$ ***(A1)***

$h=4,\ k=-4$ ***(A1)(A1)***

[4 marks]

continued...

(Question 8 (iii) continued)

(d) ***S*** is the translation ***(A1)***

with vector $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$ ***(A1)***

OR

translation which maps $(0, 0)$ to ***(A1)***

$(4, -4)$ ***(A1)***

[2 marks]

(e) Since ***T*** is an isometry, length of $PQ' =$ length of PQ, $P(5,1)$ $Q(17, -4)$. ***(M1)***

$PQ' = \sqrt{12^2 + (-5)^2} = 13$ ***(A1)***

OR

$Q(17, -4) \to Q' \frac{1}{13}(185, 132)$ ***(A1)***

$PQ' = 13$ ***(A1)***

[2 marks]

Total [30 marks]