N02/530/S(2)

MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2

Monday 11 November 2002 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive ***no*** *marks.*

SECTION A

Answer all ***five*** *questions from this section.*

1. *[Maximum mark: 12]*

(i) The weight in kilograms of 12 students in a class are as follows.

63 76 99 65 63 51 52 95 63 71 65 83

(a) State the mode. *[1 mark]*

(b) Calculate

(i) the mean weight;

(ii) the standard deviation of the weights. *[2 marks]*

When one student leaves the class, the mean weight of the remaining 11 students becomes 70 kg.

(c) Find the weight of the student who left. *[2 marks]*

(ii) All 400 students in a school are weighed.

The following table shows the distribution of their weights.

Weight of student (w)	Frequency	Frequency density
$40 \le w < 60$	80	s
$60 \le w < 70$	120	12
$70 \le w < 80$	110	t
$80 \le w < 100$	90	4·5

(a) Calculate the values of s and t. *[2 marks]*

(b) Taking a scale of 1 cm to represent 10 kg on the x-axis and 1 cm to represent a frequency density of 2 on the y-axis, represent this information on a **frequency density** histogram. *[3 marks]*

(c) What is the probability that a student weighs less than 70 kg? *[2 marks]*

2. *[Maximum mark: 15]*

Bob rents out boats on a lake. He has two types of boats, kayaks and canoes. Every day he always rents at least 10 kayaks and at least 25 canoes.

The maximum number of boats that can sail on the lake at any time is 60.

Bob rents out at least twice as many canoes as kayaks.

Let x be the number of kayaks rented per day, and y be the number of canoes rented per day.

(a) Explain why $y \geq 2x$. *[1 mark]*

(b) Write down three more inequalities to represent the above information. *[3 marks]*

(c) Using a scale of 2 cm to represent 10 kayaks on the x-axis, and 2 cm to represent 10 canoes on the y-axis, draw graphs to represent the four inequalities. *[5 marks]*

(d) (i) Shade the region (R) which satisfies all four inequalities.

(ii) Write down the coordinates of the vertices of this region. *[3 marks]*

Bob charges GBP 25 per day for a kayak and GBP 40 per day for a canoe.

(e) Write down an equation to represent the amount (A) that Bob charges in one day. *[1 mark]*

(f) Calculate the maximum amount that Bob can charge in one day. *[2 marks]*

3. *[Maximum mark: 14]*

On Vera's 18^{th} birthday she was given an allowance from her parents. She was given the following choices.

Choice A	\$ 100 every month of the year.
Choice B	A fixed amount of \$ 1100 at the beginning of the year, to be invested at an interest rate of 12 % per annum, compounded monthly.
Choice C	\$ 75 the first month and an increase of \$ 5 every month thereafter.
Choice D	\$ 80 the first month and an increase of 5 % every month.

(a) Assuming that Vera does not spend any of her allowance during the year, calculate, for each of the choices, how much money she would have at the end of the year. *[8 marks]*

(b) Which of the choices do you think that Vera should choose? Give a reason for your answer. *[2 marks]*

(c) On her 19^{th} birthday Vera invests \$ 1200 in a bank that pays interest at r % per annum compounded annually. Vera would like to buy a scooter costing \$ 1452 on her 21^{st} birthday. What rate will the bank have to offer her to enable her to buy the scooter? *[4 marks]*

4. *[Maximum mark: 14]*

The sides of a triangular lake are defined by the vectors $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{BC}$, where $\overrightarrow{OB} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

(a) (i) On graph paper, with a scale of 1 cm for each unit on both axes, draw the lake OBC, taking O as the origin.

(ii) Write down vector $\overrightarrow{CB}$ in component form.

(iii) Calculate $|\overrightarrow{CB}|$. *[6 marks]*

A path follows the edges of the lake.

(b) Given that $|\overrightarrow{OB}| = 6.32$ km and $|\overrightarrow{OC}| = 5$ km,

(i) write down the total length of the path;

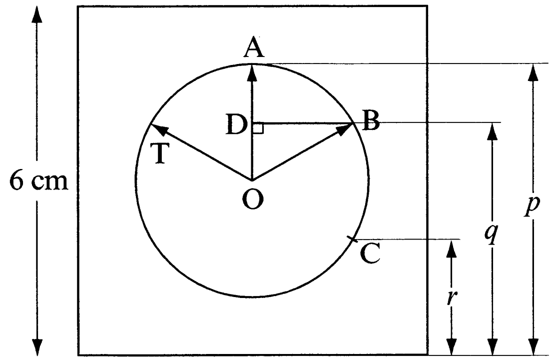
(ii) calculate the size of angle BOC ;

(iii) calculate the area of the lake. *[6 marks]*

(c) A hawk is hovering at a height of 800 **metres** directly above vertex C. The coordinates of the hawk are (–3, 4, 0·8). The hawk spots a fish in the lake at the point (–2, 2, 0). How far is the hawk from the fish? *[2 marks]*

5. *[Maximum mark: 15]*

The diagram below represents a stopwatch. This is a circle, centre O, inside a square of side 6 cm, also with centre O. The stopwatch has a minutes hand and a seconds hand. The seconds hand, with end point T, is shown in the diagram, and has a radius of 2 cm.



(a) When T is at the point A, the shortest distance from T to the base of the square is p. Calculate the value of p. *[2 marks]*

(b) In 10 seconds, T moves from point A to point B. When T is at the point B, the shortest distance from T to the base of the square is q. Calculate

(i) the size of angle AOB ;

(ii) the distance OD ;

(iii) the value of q. *[5 marks]*

(c) In another 10 seconds, T moves from point B to point C. When T is at the point C, the shortest distance from T to the base of the square is r. Calculate the value of r. *[4 marks]*

Let d be the shortest distance from T to the base of the square, when the seconds hand has moved through an angle θ. The following table gives values of d and θ.

Angle θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
Distance d	p	4.7	q	3	r	1.3	1	1.3	r	3	q	4.7	p

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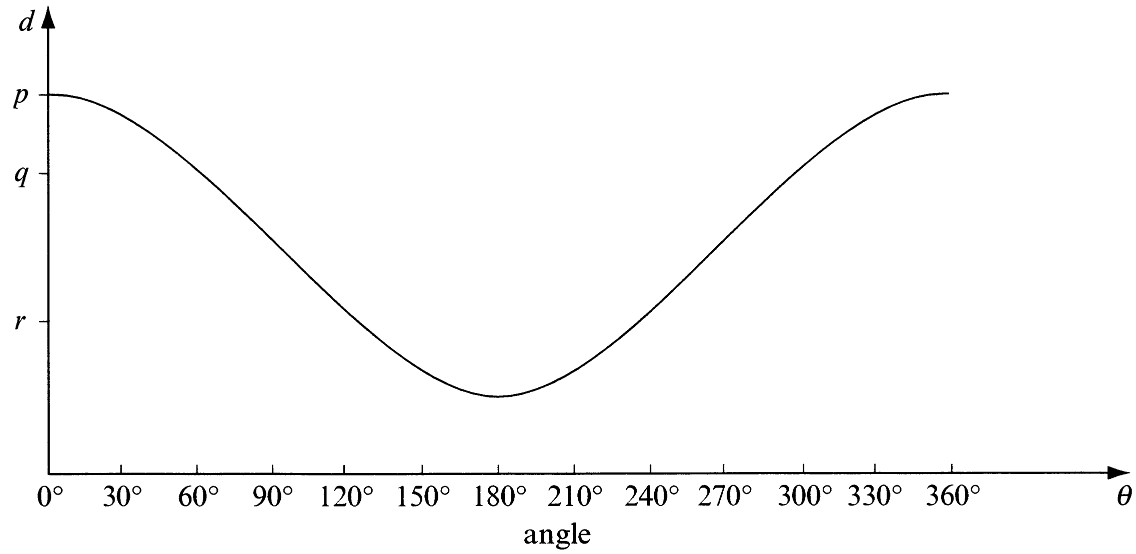
(Question 5 continued)

The graph representing this information is as follows.



The equation of this graph can be written in the form $d = c + k\cos(\theta)$.

(d) Find the values of c and k . *[4 marks]*

SECTION B

*Answer **one** question from this section.*

6. *[Maximum mark: 30]*

(i) Consider the following matrices:

$$\boldsymbol{A}=\begin{pmatrix}2 & x\\ y & 1\end{pmatrix} \qquad \boldsymbol{B}=\begin{pmatrix}-3 & 5\\ -6 & 2\\ 0 & -1\end{pmatrix} \qquad \boldsymbol{C}=\begin{pmatrix}7 & 4\\ -3 & -1\\ -6 & 2\end{pmatrix}$$

$$\boldsymbol{D}=\begin{pmatrix}p & -6\\ 3 & q\end{pmatrix} \qquad \boldsymbol{E}=\begin{pmatrix}2m & 4\\ 8 & m\end{pmatrix} \qquad \boldsymbol{F}=\begin{pmatrix}n & 3\\ 3 & -n\end{pmatrix}$$

(a) Calculate $\boldsymbol{B}-2\boldsymbol{C}$. *[2 marks]*

(b) If $2\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}=\boldsymbol{D}$, find the values of x, y, p and q. *[5 marks]*

(c) If $\boldsymbol{E}$ is a singular matrix, find the values of m. *[3 marks]*

(d) If $\boldsymbol{F}^2=9\boldsymbol{I}$ where $\boldsymbol{I}$ is the 2×2 identity matrix, show that $n=0$. *[3 marks]*

(ii) The following adjacency matrix $\boldsymbol{M}$ for a directed graph shows the number of roads connecting 4 towns P, Q, R and S.

From \ To	P	Q	R	S
P	2	2	1	2
Q	1	0	0	0
R	0	0	0	1
S	2	1	1	1

(a) Draw the directed graph to represent the above information. *[4 marks]*

(b) Given that $\boldsymbol{M}^2=\begin{pmatrix}a & 6 & 4 & 7\\ 2 & 2 & 1 & 2\\ 2 & 1 & 1 & 1\\ 7 & b & 3 & 6\end{pmatrix}$

calculate the values of a and b. *[4 marks]*

(c) How many ways can you travel from P to S in two stages? *[1 mark]*

(This question continues on the following page)

(Question 6 continued)

(iii) 200 students were asked how often they visited the cinema.

Of those students who had visited the cinema last weekend, 80 % intended to go next weekend.

Of those who had not visited the cinema last weekend, 30 % intended to go next weekend.

(a) Copy and complete the matrix, $\boldsymbol{T}$, below to represent the given information.

		last weekend Cinema	last weekend Not cinema
next weekend	Cinema	0.8	
	Not cinema		

[3 marks]

Of the 200 students, 60 % had visited the cinema last weekend.

(b) How many students visited the cinema last weekend? *[1 mark]*

(c) Calculate the number of students who will visit the cinema next weekend. *[2 marks]*

(d) Given that $\boldsymbol{T}^2 = \begin{pmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{pmatrix}$

calculate the number of students who will not visit the cinema two weekends from now. *[2 marks]*

7. *[Maximum mark: 30]*

(i) A slimming club has 500 members.

(a) It is believed that the amount of weight (w kg) lost per year is normally distributed with a mean of 18.5 kg and a standard deviation of 5 kg. If this is true, calculate the number of members who lost

(i) 20 kg or more in one year;

(ii) less than 15 kg in one year. *[6 marks]*

The table below shows the expected frequencies of the weight loss w.

Weight lost (kg)	Expected frequency
$0 \le w < 10$	22
$10 \le w < 15$	a
$15 \le w < 20$	188
$20 \le w < 25$	b
$25 \le w < 30$	43
$30 \le w < 40$	5

(b) Using your answers from part (a), or otherwise, calculate the values of a and b. *[2 marks]*

At the end of the year all 500 members are weighed to determine whether the results are in fact normally distributed with mean 18.5 kg and standard deviation 5 kg. The results are shown in the table below.

Weight lost	Observed frequency
$0 \le w < 10$	24
$10 \le w < 15$	70
$15 \le w < 20$	131
$20 \le w < 25$	164
$25 \le w < 30$	83
$30 \le w < 40$	28

(This question continues on the following page)

(Question 7 continued)

(c) A chi-squared test, at the 5 % level of significance, is performed to investigate the results.

(i) State the null hypothesis.

(ii) State the number of degrees of freedom.

(iii) Calculate the chi-squared value.

(iv) Write down the critical value of chi-squared.

(v) State whether you would accept or reject the null hypothesis and give a reason for your answer. *[8 marks]*

(ii) A shopkeeper wanted to investigate whether or not there was a correlation between the prices of food 10 years ago in 1992, with their prices today. He chose 8 everyday items and the prices are given in the table below.

	sugar	milk	eggs	rolls	tea bags	coffee	potatoes	flour
1992 price	$ 1.44	$ 0.80	$ 2.16	$ 1.80	$ 0.92	$ 3.16	$ 1.32	$ 1.12
2002 price	$ 2.20	$ 1.04	$ 2.64	$ 3.00	$ 1.32	$ 2.28	$ 1.92	$ 1.44

(a) Calculate the mean and the standard deviation of the prices

(i) in 1992;

(ii) in 2002. *[4 marks]*

(b) (i) Given that $s_{xy} = 0.3104$, calculate the correlation coefficient.

(ii) Comment on the relationship between the prices. *[4 marks]*

(c) Find the equation of the line of best fit in the form $y = mx + c$. *[3 marks]*

(d) What would you expect to pay now for an item costing $ 2.60 in 1992? *[1 mark]*

(e) Which item would you omit to increase the correlation coefficient? *[2 marks]*

8. *[Maximum mark: 30]*

(i) Consider the function $f(x) = 2x^3 - 3x^2 - 12x + 5$.

(a) (i) Find $f'(x)$.

(ii) Find the gradient of the curve $f(x)$ when $x = 3$. *[4 marks]*

(b) Find the x-coordinates of the points on the curve where the gradient is equal to -12 . *[3 marks]*

(c) (i) Calculate the x-coordinates of the local maximum and minimum points.

(ii) Hence find the coordinates of the local minimum. *[6 marks]*

(d) For what values of x is the value of $f(x)$ increasing? *[2 marks]*

(ii) The distance s metres run by an athlete in t minutes is given by the formula

$$s(t) = 250t + 5t^2 - 0.06t^3, \; 0 \le t \le 70 .$$

(a) Calculate the distance run after 50 minutes. *[1 mark]*

(b) (i) Show that 50 minutes and 1 second may be written as $50\frac{1}{60}$ minutes.

(ii) Calculate the distance run after $50\frac{1}{60}$ minutes.

(iii) Calculate the value of $\dfrac{s(50\frac{1}{60}) - s(50)}{\frac{1}{60}}$. *[5 marks]*

(c) (i) Express the velocity of the athlete in terms of t .

(ii) Calculate the velocity when $t = 50$. *[3 marks]*

(d) Explain why your answers to parts b(iii) and c(ii) are very close in value. *[2 marks]*

(iii) The gradient function of a curve is given by $g'(x) = x^2 - 2x + 5$.

The curve $g(x)$ passes through the point (3, 5) . Find the equation of $g(x)$. *[4 marks]*
