IBO

IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 1

Friday 8 November 2002 (afternoon)

1 hour

Name

Number

INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /90	TOTAL /90	TOTAL /90

0112

*Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

1. Let $f(x) = \sqrt{x^3}$. Find
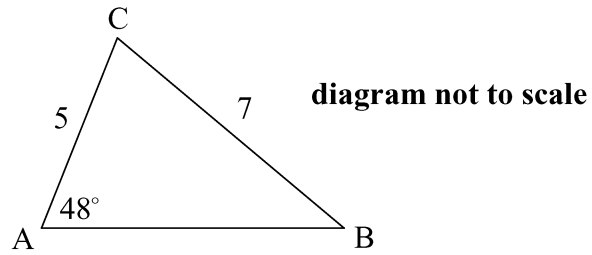
(a) $f'(x)$;

(b) $\int f(x)\,dx$.

Working:

Answers:

(a) _____

(b) _____

2. In triangle ABC, $AC = 5$, $BC = 7$, $\hat{A} = 48°$, as shown in the diagram.

C

5   7   **diagram not to scale**

48°

A   B

Find $\hat{B}$, giving your answer correct to the nearest degree.

Working:

Answer:

3.   Consider the function $f(x) = 2x^2 - 8x + 5$.

(a)   Express $f(x)$ in the form $a(x - p)^2 + q$, where $a, p, q \in \mathbb{Z}$.

(b)   Find the minimum value of $f(x)$.

Working:

Answers:

(a) _____

(b) _____

4.   Find the coefficient of x^3 in the expansion of $(2 - x)^5$.

Working:

Answer:

_____

5. Solve the equation $e^x = 5 - 2x$, giving your answer correct to **four** significant figures.

Working:

Answer:

6. Given that $\sin x = \dfrac{1}{3}$, where x is an acute angle, find the **exact** value of

(a) $\cos x$;

(b) $\cos 2x$.

Working:

Answers:

(a) _____

(b) _____

0 5 1 2

7. For events A and B, the probabilities are $P(A) = \dfrac{3}{11}$, $P(B) = \dfrac{4}{11}$.

Calculate the value of $P(A \cap B)$ if

(a) $P(A \cup B) = \dfrac{6}{11}$;

(b) events A and B are independent.

Working:

Answers:

(a) _____

(b) _____

0612

8. The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

Working:

Answer:

9. Three positive integers $a, b,$ and c, where $a < b < c$, are such that their median is 11, their mean is 9 and their range is 10. Find the value of a.

Working:

Answer:

0712

10. Consider the functions $f : x \mapsto 4(x-1)$ and $g : x \mapsto \dfrac{6-x}{2}$.

(a) Find g^{-1}.

(b) Solve the equation $(f \circ g^{-1})(x) = 4$.

Working:

Answers:

(a) _____

(b) _____

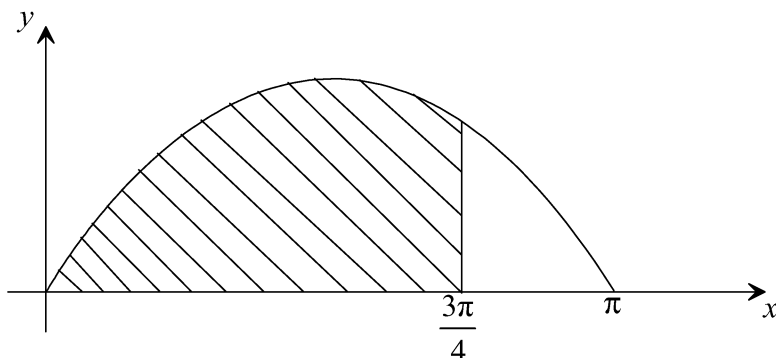**11.** Calculate the acute angle between the lines with equations

$$r = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{and} \quad r = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Working:

Answer:

_____

12. The diagram shows part of the curve $y = \sin x$. The shaded region is bounded by the curve and the lines $y = 0$ and $x = \dfrac{3\pi}{4}$.

Given that $\sin\dfrac{3\pi}{4} = \dfrac{\sqrt{2}}{2}$ and $\cos\dfrac{3\pi}{4} = -\dfrac{\sqrt{2}}{2}$, calculate the **exact** area of the shaded region.

Working:

Answer:

0912

13. $1000 is invested at 15 % per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed $ 3000.

Working:

Answer:

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14. Consider the trigonometric equation $2\sin^2 x = 1 + \cos x$.

(a) Write this equation in the form $f(x) = 0$, where $f(x) = a\cos^2 x + b\cos x + c$, and $a, b, c \in \mathbb{Z}$.

(b) Factorize $f(x)$.
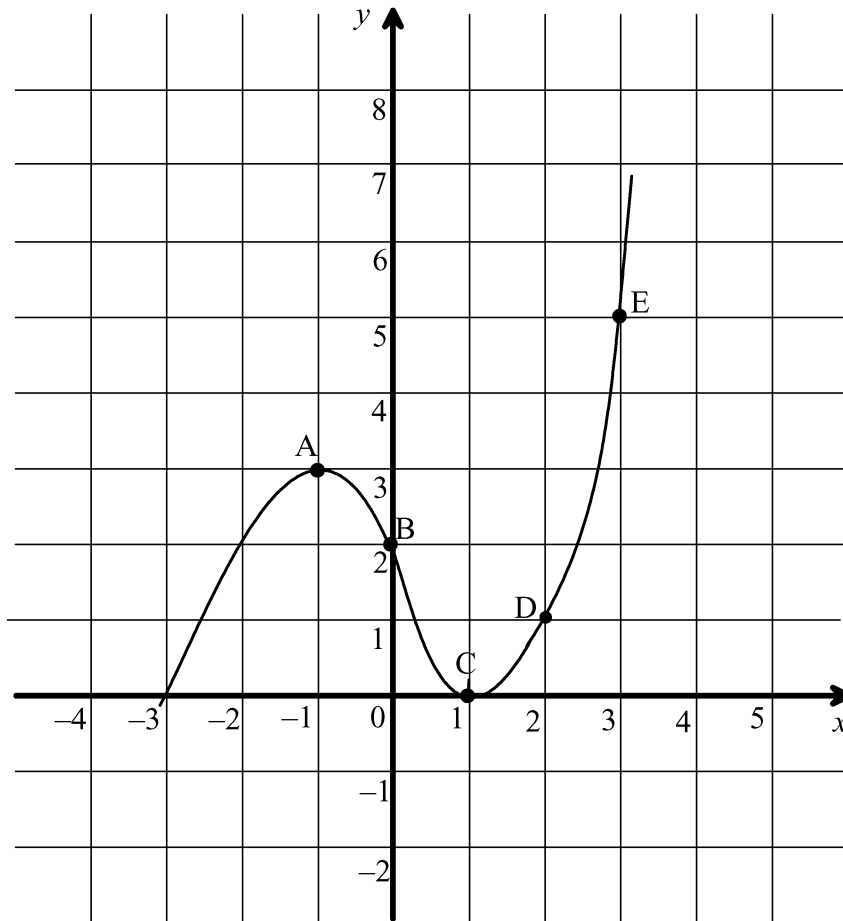
(c) Solve $f(x) = 0$ for $0° \leq x \leq 360°$.

Working:

Answers:

(a) _____

(b) _____

(c) _____

1112

15. The sketch shows part of the graph of $y = f(x)$ which passes through the points A$(-1, 3)$, B$(0, 2)$, C$(1, 0)$, D$(2, 1)$ and E$(3, 5)$.

A second function is defined by $g(x) = 2f(x-1)$.

(a) Calculate $g(0)$, $g(1)$, $g(2)$ and $g(3)$.

(b) On the same axes, sketch the graph of the function $g(x)$.

Working:

Answer:

(a) _____

_____