



**MATHEMATICAL STUDIES**  
**STANDARD LEVEL**  
**PAPER 2**

Monday 12 November 2001 (morning)

2 hours

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**INSTRUCTIONS TO CANDIDATES**

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp *EL-9600*, Texas Instruments *TI-85*.

You are advised to start each new question on a new page. A correct answer with **no** indication of the method used will usually receive **no** marks. You are therefore advised to show your working. (If graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.)

**SECTION A**

Answer all **five** questions from this section.

1. [Maximum mark: 14]

- (i) The table below shows the percentage marks, to the nearest whole number, scored by candidates in an examination.

Marks (%)	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–100
Frequency	2	7	8	13	24	30	6	5	3	2

The following is the cumulative frequency table for the marks.

Marks (%)	Cumulative frequency
< 9.5	2
< 19.5	9
< 29.5	$s$
< 39.5	30
< 49.5	54
< 59.5	84
< 69.5	$t$
< 79.5	95
< 89.5	98
< 100	100

- (a) Calculate the values of  $s$  and of  $t$ .

[2 marks]

(This question continues on the following page)

(Question 1 (i) continued)

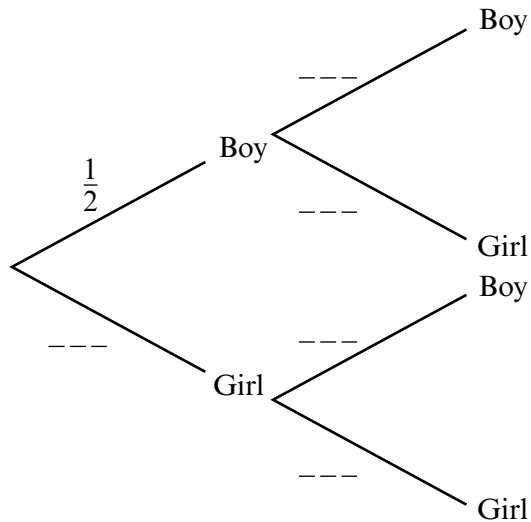
(b) Using a scale of 1 cm to represent 10 marks on the horizontal axis, and 1 cm to represent 10 candidates on the vertical axis, draw a cumulative frequency graph. [3 marks]

(c) Use your graph to estimate

- (i) the median mark;
- (ii) the lower quartile;
- (iii) the pass mark, if 40% of the candidates passed. [4 marks]

(ii) Let  $F$  be the set of all families that have exactly 2 children.

(a) Assuming  $P(\text{boy}) = P(\text{girl})$ , copy and complete the following tree diagram, for families with 2 children.



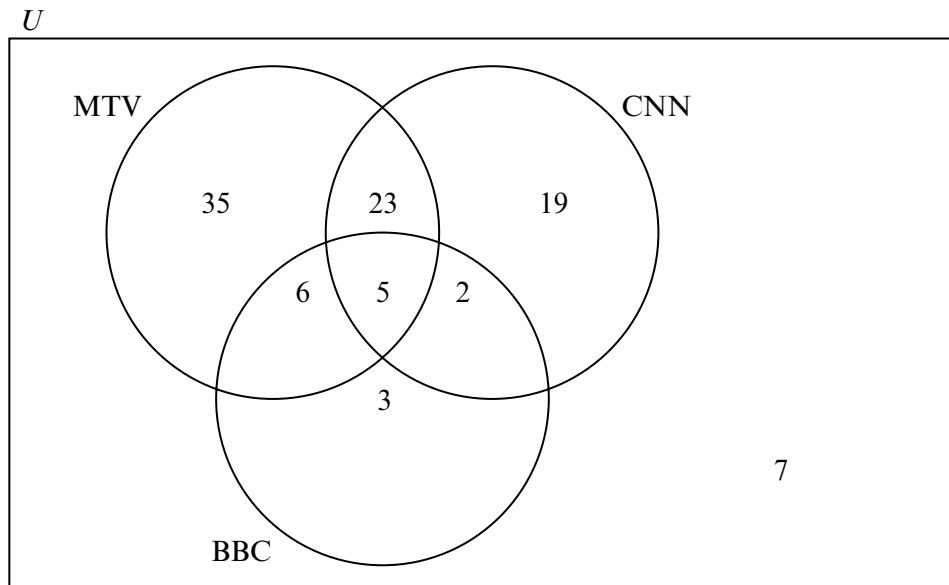
[2 marks]

(b) What is the probability that a family chosen at random from  $F$  has exactly

- (i) 2 boys?
- (ii) 2 boys, if it is known that the first child is a boy?
- (iii) 2 boys, if it is known that there is a boy in the family? [3 marks]

2. [Maximum mark: 13]

- (i) 100 students were asked which television channel (MTV, CNN or BBC) they had watched the previous evening. The results are shown in the Venn diagram below.



From the information in the Venn diagram, write down the number of students who watched

- (a) both MTV and BBC;
- (b) MTV or BBC;
- (c) CNN and BBC but not MTV;
- (d) MTV or CNN but not BBC.

[4 marks]

- (ii) Let  $p$  and  $q$  be the statements

$p$ : you watch the music TV channel

$q$ : you like music

- (a) Consider the following logic statement.

*If you watch the music TV channel then you like music.*

- (i) Write down in words the inverse of the statement.
- (ii) Write down in words the converse of the statement.

[4 marks]

(This question continues on the following page)

*(Question 2 (ii) continued)*

(b) Construct truth tables for the following statements:

(i)  $p \Rightarrow q$

(ii)  $\neg p \Rightarrow \neg q$

(iii)  $p \vee \neg q$

(iv)  $\neg p \wedge q$

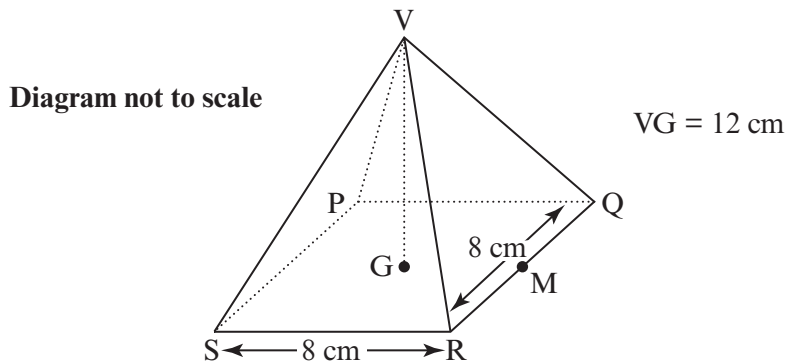
*[4 marks]*

(c) Which of the statements in part (b) are logically equivalent?

*[1 mark]*

3. [Maximum mark: 15]

- (i) In the diagram below, PQRS is the square base of a solid right pyramid with vertex V. The sides of the square are 8 cm, and the height VG is 12 cm. M is the midpoint of [QR].



- (a) (i) Write down the length of [GM].  
 (ii) Calculate the length of [VM]. [2 marks]
- (b) Find
- (i) the total surface area of the pyramid;  
 (ii) the angle between the face VQR and the base of the pyramid. [4 marks]

- (ii) The table below shows the deposits, in Australian dollars (AUD), made by Vicki in an investment account on the **first** day of each month for the first four months in 1999. The interest rate is 0.75% **per month** compounded monthly. The interest is added to the account at the **end** of each month.

Month	Deposit (AUD)
January	600
February	1300
March	230
April	710

- (a) Show that the amount of money in Vicki's account at the **end** of February is AUD 1918.78. [3 marks]
- (b) Calculate the amount of Australian dollars in Vicki's account at the **end** of April. [2 marks]

(This question continues on the following page)

(Question 3 (ii) continued)

Vicki makes no withdrawals or deposits after 1st April 1999.

- (c) How much money is in Vicki’s account at the end of December 1999? [2 marks]

From 1st January 2000 the bank applies a new interest rate of 3.5% **per annum** compounded annually.

- (d) In how many full years after December 1999 will Vicki’s investment first exceed AUD 3300? [2 marks]

4. [Maximum mark: 14]

A rectangle has dimensions  $(5 + 2x)$  metres and  $(7 - 2x)$  metres.

- (a) Show that the area,  $A$ , of the rectangle can be written as  $A = 35 + 4x - 4x^2$  [1 mark]  
 (b) The following is the table of values for the function  $A = 35 + 4x - 4x^2$ .

$x$	-3	-2	-1	0	1	2	3	4
$A$	-13	$p$	27	35	$q$	$r$	11	$s$

- (i) Calculate the values of  $p$ ,  $q$ ,  $r$  and  $s$ .  
 (ii) On graph paper, using a scale of 1 cm for 1 unit on the  $x$ -axis and 1 cm for 5 units on the  $A$ -axis, plot the points from your table and join them up to form a smooth curve. [6 marks]  
 (c) Answer the following, using your graph or otherwise.  
 (i) Write down the equation of the axis of symmetry of the curve.  
 (ii) Find one value of  $x$  for a rectangle whose area is  $27 \text{ m}^2$ .  
 (iii) Using this value of  $x$ , write down the dimensions of the rectangle. [4 marks]  
 (d) (i) On the same graph, draw the line with equation  $A = 5x + 30$ .  
 (ii) Hence or otherwise, solve the equation  $4x^2 + x - 5 = 0$ . [3 marks]

5. [Maximum mark: 14]

(i) The vectors  $i$ ,  $j$  are unit vectors in the directions East and North respectively.

(a) Let  $a = 3i - 4j$ .

(i) Draw a diagram to represent the information given above, marking the vector  $a$  clearly on the diagram.

(ii) Find

(a) the magnitude of  $a$ ;

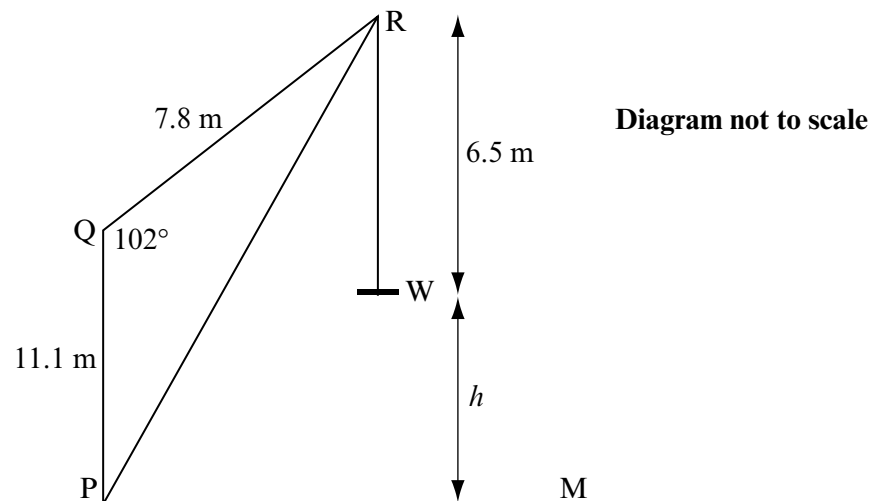
(b) the angle between  $a$  and  $j$ .

[4 marks]

(b) Express, in terms of  $i$  and  $j$ , the vector  $b$  of magnitude 80 units in the direction  $300^\circ$  clockwise from North.

[3 marks]

(ii) The diagram below shows a crane PQR that carries a flat box W. (PQ) is vertical, and the floor (PM) is horizontal.



Given that  $PQ = 11.1$  m,  $QR = 7.8$  m,  $\widehat{PQR} = 102^\circ$  and  $RW = 6.5$  m, calculate

(a)  $PR$ ;

[2 marks]

(b) angle  $PRQ$ ;

[2 marks]

(c) the height,  $h$ , of  $W$  above  $(PM)$ .

[3 marks]



**SECTION B**

Answer **one** question from this section.

**Matrices and Graph Theory**

6. [Maximum mark: 30]

- (a) A building contractor accepted orders for 5 ranch style houses, 7 cape cod style houses and 12 colonial style houses. The amounts of four kinds of raw materials required to build each style of house, expressed in convenient units, are as follows:

**Ranch:** 5 units of steel, 20 units of wood, 16 units of glass and 7 units of paint.

**Cape Cod:** 12 units of glass, 18 units of wood, 7 units of steel and 9 units of paint.

**Colonial:** 5 units of paint, 25 units of wood, 8 units of glass and 6 units of steel.

The contractor wants to use matrices to compute the amount of each raw material he should obtain in order to fulfil his contract.

- (i) Write down two matrices the contractor can use.
- (ii) Use the matrices in part (i) to calculate the amount of glass the contractor should order. [5 marks]

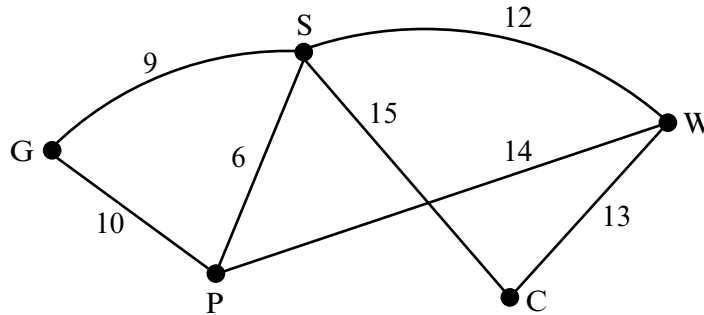
Steel costs \$150 per unit, wood costs \$80 per unit, glass costs \$50 per unit and paint costs \$10 per unit. The contractor again uses matrices to calculate the price that he will have to pay for each raw material.

- (iii) Write down a cost matrix that the contractor can use to continue his calculations.
- (iv) Carry out a matrix multiplication to find the total cost of raw materials for a Colonial style house. [3 marks]

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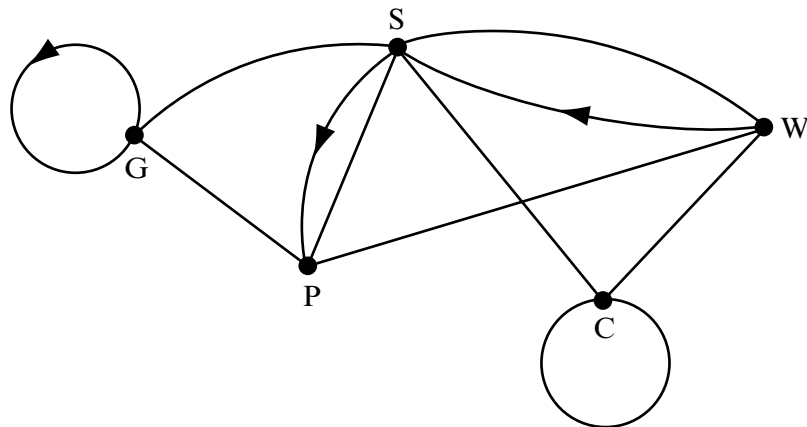
(Question 6 continued)

- (b) The contractor lives in town C and the factories for paint, glass, steel and wood are in towns P, G, S and W respectively. The shortest distances, in kilometres, between the towns are shown in the diagram below.



The contractor wants to collect raw materials from each factory, starting and finishing at C. He must however collect the material by driving along each of the roads he uses only once and visiting each town only once.

- (i) Draw the sub-graph that he should use.
  - (ii) How many kilometres must he travel to collect the raw materials? [5 marks]
- (c) All the roads connecting towns C, P, G, S and W are shown in the directed graph below.



(This question continues on the following page)

(Question 6 (c) continued)

(i) Copy and complete the adjacency matrix,  $M$ , for this diagram.

$$M = \begin{matrix} & \begin{matrix} C & P & G & S & W \end{matrix} \\ \begin{matrix} C \\ P \\ G \\ S \\ W \end{matrix} & \begin{pmatrix} & 0 & & & \\ 0 & & & & \\ & & & 1 & \\ & & 1 & & \\ & & & & 0 \end{pmatrix} \end{matrix}$$

(ii) What information could you obtain from the matrix  $M^2$ ?

[6 marks]

(d) Consider the following four graphs.

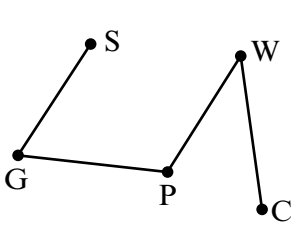


Fig. 1

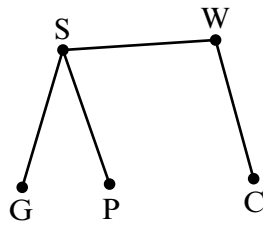


Fig. 2

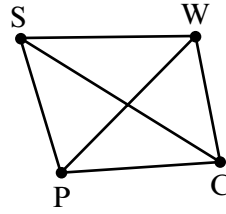


Fig. 3

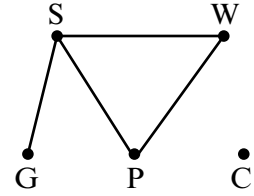


Fig. 4

Which of the graphs are

- (i) connected?
- (ii) complete?
- (iii) a tree?

[5 marks]

(e) During his lunch break, the contractor and his friend play a two-person zero-sum game. The contractor has 2 strategies C1 and C2 and his friend has 2 strategies F1 and F2.

The following matrix shows how much the contractor may win, depending on the strategies used (pay-off matrix).

$$\begin{matrix} & \begin{matrix} F1 & F2 \end{matrix} \\ \begin{matrix} C1 \\ C2 \end{matrix} & \begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} \end{matrix}$$

- (i) What is the outcome for each player, when the contractor uses C2 and his friend uses F2?
- (ii) What strategy should the contractor use to minimise his losses?
- (iii) What is the outcome for both players if they both try to minimise their losses?

[6 marks]

**Further Statistics and Probability**

7. [Maximum mark: 30]

- (i) Speed checks on a very large number of cars at a certain point on a motorway show that the speeds are normally distributed.

10% of cars have speeds more than  $132.8 \text{ km h}^{-1}$  and only 7% of cars have speeds less than  $103.2 \text{ km h}^{-1}$ .

- (a) Draw a normal distribution diagram to illustrate this information, indicating clearly both the percentages and the speeds. [2 marks]

- (b) Given that the standard deviation of the speeds is  $10.7 \text{ km h}^{-1}$ , calculate the mean speed of the cars. [3 marks]

- (c) Calculate the percentage of cars travelling at more than  $130 \text{ km h}^{-1}$ . [2 marks]

- (ii) For his Mathematical Studies Project a student gave his classmates a questionnaire to fill out. The results for the question on the gender of the student and specific subjects taken by the student are given in the table below, which is a  $2 \times 3$  contingency table of **observed** values.

	History	Biology	French	
Female	22	20	18	(60)
Male	20	11	9	(40)
	(42)	(31)	(27)	

The following is the table for the **expected** values.

	History	Biology	French
Female	$p$	18.6	16.2
Male	$q$	$r$	10.8

- (a) Calculate the values of  $p$ ,  $q$  and  $r$ . [3 marks]

*(This question continues on the following page)*

(Question 7 (ii) continued)

The chi-squared test is used to determine if the choice of subject is independent of gender, at the 5% level of significance.

- (b) (i) State a suitable null hypothesis  $H_0$ .
- (ii) Show that the number of degrees of freedom is two.
- (iii) Write down the critical value of chi-squared at the 5% level of significance. [3 marks]
- (c) The calculated value of chi-squared is 1.78. Do you accept  $H_0$ ? Explain your answer. [2 marks]

(iii) The heights and weights of 10 students selected at random are shown in the table below.

Student	1	2	3	4	5	6	7	8	9	10
Height $x$ cm	155	161	173	150	182	165	170	185	175	145
Weight $y$ kg	50	75	80	46	81	79	64	92	74	108

- (a) Plot this information on a scatter graph. Use a scale of 1 cm to represent 20 cm on the  $x$ -axis and 1 cm to represent 10 kg on the  $y$ -axis. [4 marks]
- (b) Calculate the mean height. [1 mark]
- (c) Calculate the mean weight. [1 mark]
- (d) It is given that  $S_{xy} = 44.31$ .
- (i) By first calculating the standard deviation of the heights, correct to two decimal places, show that the gradient of the line of regression of  $y$  on  $x$  is 0.276.
- (ii) Calculate the equation of the line of best fit.
- (iii) Draw the line of best fit on your graph. [6 marks]
- (e) Use your line to estimate
- (i) the weight of a student of height 190 cm;
- (ii) the height of a student of weight 72 kg. [2 marks]
- (f) It is decided to remove the data for student number 10 from all calculations. Explain **briefly** what effect this will have on the line of best fit. [1 mark]

**Introductory Differential Calculus**

**8.** [Maximum mark: 30]

- (i) The curve  $y = f(x)$  has its only local minimum value at  $x = a$  and its only local maximum value at  $x = b$ .
- (a) If  $a < 0$  and  $b > 0$ , **sketch** a possible curve of  $y = f(x)$  indicating clearly the points  $(a, f(a))$  and  $(b, f(b))$ . [2 marks]
- (b) Given that  $0 < h < 1$  and  $b - a > 1$ , are the following statements about the curve  $y = f(x)$  TRUE or FALSE?
- (i)  $f(a + h) < f(a)$
- (ii)  $f'(b - h)$  is positive
- (iii) The tangent to the curve at the point  $(a, f(a))$  is parallel to the vertical axis.
- (iv) The gradient of the tangent to the curve at the point  $(a, f(a))$  is equal to zero.
- (v)  $f(a - h) < f(a) < f(a + h)$
- (vi)  $f'(a) = 0 \Rightarrow (a, f(a))$  is a point of inflexion. [6 marks]

(ii) The function  $g$  is defined as follows

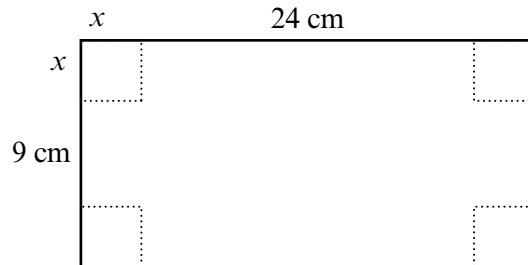
$$g : x \mapsto px^2 + qx + c, \quad p, q, c \in \mathbb{R}$$

- (a) Find  $g'(x)$  [1 mark]
- (b) If  $g'(x) = 2x + 6$ , find the values of  $p$  and  $q$ . [2 marks]
- (c)  $g(x)$  has a minimum value of  $-12$  at the point A. Find
- (i) the  $x$ -coordinate of A;
- (ii) the value of  $c$ . [4 marks]
- (iii) An object starts moving from rest towards a building which is 192 m away from the starting point. After  $t$  seconds, its velocity  $v \text{ ms}^{-1}$  is given by  $v = 9t^2$ .
- (a) Find an expression for the distance,  $s$  metres, travelled after  $t$  seconds, given that  $s = 0$  when  $t = 0$ . [2 marks]
- (b) After how many seconds does the object reach the building? [2 marks]
- (c) What does the expression  $\frac{dv}{dt}$  represent? [1 mark]
- (d) Find the acceleration of the object when  $t = 2$  seconds. [2 marks]

(This question continues on the following page)

(Question 8 continued)

- (iv) A rectangular piece of card measures 24 cm by 9 cm. Equal squares of length  $x$  cm are cut from each corner of the card as shown in the diagram below. What is left is then folded to make an **open** box, of length  $l$  cm and width  $w$  cm.



- (a) Write expressions, in terms of  $x$ , for
- (i) the length,  $l$ ;
  - (ii) the width,  $w$ . [2 marks]
- (b) Show that the volume ( $B \text{ m}^3$ ) of the box is given by  $B = 4x^3 - 66x^2 + 216x$ . [1 mark]
- (c) Find  $\frac{dB}{dx}$ . [1 mark]
- (d) (i) Find the value of  $x$  which gives the maximum volume of the box.
- (ii) Calculate the maximum volume of the box. [4 marks]
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