INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
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MARKSCHEME

November 2001

MATHEMATICAL METHODS

Standard Level

Paper 2

20 pages

1. (a) $r = \frac{360}{240} = \frac{240}{160} = \frac{3}{2} = 1.5$ ***(A1)***

[1 mark]

(b) 2002 is the 13th year. ***(M1)***

$u_{13} = 160(1.5)^{13-1}$ ***(M1)***

$= 20759$ (Accept 20760 or 20800.) ***(A1)***

[3 marks]

(c) $5000 = 160(1.5)^{n-1}$

$\frac{5000}{160} = (1.5)^{n-1}$ ***(M1)***

$\log\left(\frac{5000}{160}\right) = (n-1)\log 1.5$ ***(M1)***

$n - 1 = \frac{\log\left(\frac{5000}{160}\right)}{\log 1.5} = 8.49$ ***(A1)***

$\Rightarrow \quad n = 9.49 \quad \Rightarrow \quad 10^{\text{th}}$ year

$\Rightarrow \quad 1999$ ***(A1)***

OR

Using a gdc with $u_1 = 160, u_{k+1} = \frac{3}{2}u_k, u_9 = 4100, u_{10} = 6150$ ***(M2)***

1999 ***(G2)***

[4 marks]

(d) $S_{13} = 160\left[\frac{1.5^{13} - 1}{1.5 - 1}\right]$ ***(M1)***

$= 61958$ (Accept 61960 or 62000.) ***(A1)***

[2 marks]

(e) Nearly everyone would have bought a portable telephone so there would be fewer people left wanting to buy one. ***(R1)***

OR

Sales would saturate. ***(R1)***

[1 mark]

Total [11 marks]

2. (a) (Using mid-intervals)

$$\bar{v} = \frac{65(7) + 75(25) + \ldots + 135(5)}{7 + 25 + \ldots + 5}$$ ***(M1)***

$$= \frac{29450}{300} = 98.2 \text{ km h}^{-1}$$ ***(A1)***

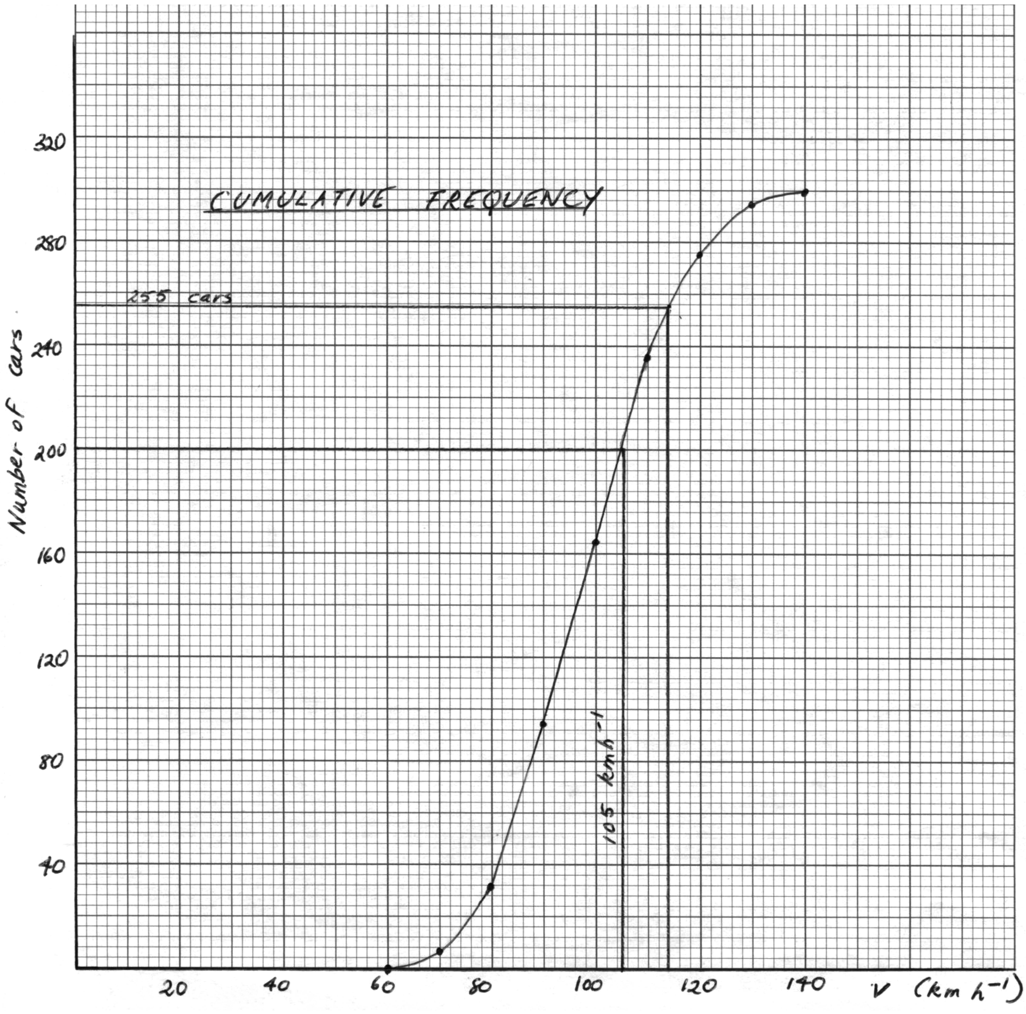
OR

$\bar{v} = 98.2$ ***(G2)***

[2 marks]

(b) (i) $a = 165, b = 275$ ***(A1)***

(ii)



(A4)

Note:	Award ***(A1)*** for properly marked scales and axes, ***(A2)*** for 9 correctly plotted points, ***(A1)*** for 7 or 8 points, ***(A1)*** for a smooth curve through the points.

[5 marks]

continued…

Question 2 continued

(c) (i) Vertical line on graph at $105\ \text{km h}^{-1}$ ***(M1)***

$\frac{300-200}{300}\times 100\ \% = 33.3\ (\pm 1.3\ \%)$ ***(A1)***

OR

$33.3\ (\pm 1.3\ \%)$ ***(A2)***

(ii) $15\ \%$ of $300 = 45$ $300 - 45 = 255$

Horizontal line on graph at 255 cars ***(M1)***

Speed $= 114\ (\pm 2\ \text{km h}^{-1})$ ***(A1)***

OR

Speed $= 114\ (\pm 2\ \text{km h}^{-1})$ ***(A2)***

[4 marks]

Total [11 marks]

3. (a) (i) $a = -3$ ***(A1)***

(ii) $b = 5$ ***(A1)***

[2 marks]

(b) (i) $f'(x) = -3x^2 + 4x + 15$ ***(A2)***

(ii) $-3x^2 + 4x + 15 = 0$

$-(3x+5)(x-3) = 0$ ***(M1)***

$x = -\frac{5}{3}$ or $x = 3$ ***(A1)(A1)***

OR

$x = -\frac{5}{3}$ or $x = 3$ ***(G3)***

(iii) $x = 3 \Rightarrow f(3) = -3^3 + 2(3^2) + 15(3)$ ***(M1)***

$= -27 + 18 + 45 = 36$ ***(A1)***

OR

$f(3) = 36$ ***(G2)***

[7 marks]

(c) (i) $f'(x) = 15$ at $x = 0$ ***(M1)***

Line through (0, 0) of gradient 15

$\Rightarrow \quad y = 15x$ ***(A1)***

OR

$y = 15x$ ***(G2)***

(ii) $-x^3 + 2x^2 + 15x = 15x$ ***(M1)***

$\Rightarrow \quad -x^3 + 2x^2 = 0$

$\Rightarrow \quad -x^2(x-2) = 0$

$\Rightarrow \quad x = 2$ ***(A1)***

OR

$x = 2$ ***(G2)***

[4 marks]

(d) Area $= 115$ (3 s.f.) ***(G2)***

OR

Area $= \int_0^5 (-x^3 + 2x^2 + 15x)\,\mathrm{d}x = \left[-\frac{x^4}{4} + 2\frac{x^3}{3} + 15\frac{x^2}{2}\right]_0^5$ ***(M1)***

$= \frac{1375}{12} = 115$ (3 s.f.) ***(A1)***

[2 marks]

Total [15 marks]

4. (a) (i) $\vec{OA} = \begin{pmatrix} 240 \\ 70 \end{pmatrix}$ $OA = \sqrt{240^2 + 70^2} = 250$ ***(A1)***

unit vector $= \frac{1}{250}\begin{pmatrix} 240 \\ 70 \end{pmatrix} = \begin{pmatrix} 0.96 \\ 0.28 \end{pmatrix}$ ***(M1)(AG)***

(ii) $\vec{v} = 300\begin{pmatrix} 0.96 \\ 0.28 \end{pmatrix} = \begin{pmatrix} 288 \\ 84 \end{pmatrix}$ ***(M1)(A1)***

(iii) $t = \frac{240}{288} = \frac{5}{6}\text{hr } (= 50 \text{ min})$ ***(A1)***

[5 marks]

(b) $\vec{AB} = \begin{pmatrix} 480 - 240 \\ 250 - 70 \end{pmatrix} = \begin{pmatrix} 240 \\ 180 \end{pmatrix}$ ***(A1)***

$AB = \sqrt{240^2 + 180^2} = 300$

$\cos\theta = \frac{\vec{OA} \bullet \vec{AB}}{OA \times AB} = \frac{(240)(240) + (70)(180)}{(250)(300)}$ ***(M1)***

$= 0.936$ ***(A1)***

$\Rightarrow \quad \theta = 20.6^\circ$ ***(A1)***

[4 marks]

(c) (i) $\vec{AX} = \begin{pmatrix} 339 - 240 \\ 238 - 70 \end{pmatrix} = \begin{pmatrix} 99 \\ 168 \end{pmatrix}$ ***(A1)***

(ii) $\begin{pmatrix} -3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 240 \\ 180 \end{pmatrix} = -720 + 720 = 0$ ***(M1)(A1)***

$\Rightarrow \quad \boldsymbol{n} \perp \vec{AB}$ ***(AG)***

(iii) Projection of $\vec{AX}$ in the direction of $\boldsymbol{n}$ is

$XY = \frac{1}{5}\begin{pmatrix} 99 \\ 168 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \frac{-297 + 672}{5} = 75$ ***(M1)(A1)(A1)***

[6 marks]

(d) $AX = \sqrt{99^2 + 168^2} = 195$ ***(A1)***

$AY = \sqrt{195^2 - 75^2} = 180 \text{ km}$ ***(M1)(A1)***

[3 marks]

Total [18 marks]

5. (a) (i) $v(0) = 50 - 50e^0 = 0$ ***(A1)***

(ii) $v(10) = 50 - 50e^{-2} = 43.2$ ***(A1)***

[2 marks]

(b) (i) $a = \dfrac{dv}{dt} = -50(-0.2e^{-0.2t})$ ***(M1)***

$= 10e^{-0.2t}$ ***(A1)***

(ii) $a(0) = 10e^0 = 10$ ***(A1)***

[3 marks]

(c) (i) $t \to \infty \Rightarrow v \to 50$ ***(A1)***

(ii) $t \to \infty \Rightarrow a \to 0$ ***(A1)***

(iii) when $a = 0$, v is constant at 50 ***(R1)***

[3 marks]

(d) (i) $y = \int v\,dt$ ***(M1)***

$= 50t - 50\dfrac{e^{-0.2t}}{-0.2} + k$ ***(A1)***

$= 50t + 250e^{-0.2t} + k$ ***(AG)***

(ii) $0 = 50(0) + 250e^0 + k = 250 + k$ ***(M1)***

$\Rightarrow k = -250$ ***(A1)***

(iii) Solve $250 = 50t + 250e^{-0.2t} - 250$ ***(M1)***

$\Rightarrow \quad 50t + 250e^{-0.2t} - 500 = 0$

$\Rightarrow \quad t + 5te^{-0.2t} - 10 = 0$

$\Rightarrow \quad t = 9.207s$ ***(G2)***

[7 marks]

Total [15 marks]

6. (i) (a) $Z = \dfrac{25 - 25.7}{0.50} = -1.4$ ***(M1)***

$\mathrm{P}(Z < -1.4) = 1 - \mathrm{P}(Z < 1.4)$

$= 1 - 0.9192$

$= 0.0808$ ***(A1)***

OR

$\mathrm{P}(W < 25) = 0.0808$ ***(G2)***

[2 marks]

(b) $\mathrm{P}(Z < -a) = 0.025 \Rightarrow \mathrm{P}(Z < a) = 0.975$

$\Rightarrow \quad a = 1.960$ ***(A1)***

$\dfrac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96(0.50)$ ***(M1)***

$= 25 + 0.98 = 25.98$ ***(A1)***

$= 26.0$ (3 s.f.) ***(AG)***

OR

$\dfrac{25.0 - 26.0}{0.50} = -2.00$ ***(M1)***

$\mathrm{P}(Z < -2.00) = 1 - \mathrm{P}(Z < 2.00)$

$= 1 - 0.9772 = 0.0228$ ***(A1)***

≈ 0.025 ***(A1)***

OR

$\mu = 25.98$ ***(G2)***

$\Rightarrow$ mean $= 26.0$ (3 s.f.) ***(A1)(AG)***

[3 marks]

(c) Clearly, by symmetry $\mu = 25.5$ ***(A1)***

$Z = \dfrac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma$ ***(M1)***

$\Rightarrow \sigma = 0.255$ kg ***(A1)***

[3 marks]

(d) On average, $\dfrac{\text{cement saving}}{\text{bag}} = 0.5$ kg ***(A1)***

$\dfrac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = \$\,0.40$ ***(M1)***

To save \$ 5000 takes $\dfrac{5000}{0.40} = 12500$ bags ***(A1)***

[3 marks]

continued…

Question 6 continued

(ii) (a) (i) H_0 : The mean is equal to 175 cm. ***(A1)***

(ii) It is a one-tailed test. ***(A1)***

[2 marks]

(b) For a sample of size 36, standard error of mean $= \dfrac{12.0}{\sqrt{36}} = 2.0$ ***(M1)(A1)***

Critical value for 5 % significance $= 1.645$ ***(A1)***

$175 + 1.645(2.0) = 178.3$ cm ***(A1)***

(Sample mean $> 178.3 \Rightarrow$ reject H_0)

[4 marks]

(c) We accept H_1 at the 5% level of significance. However, 178.9 is only slightly larger than 178.3 so the result may not be significant at a lower level of significance. ***(R1)*** ***(R1)***

[2 marks]

(iii) (a) $\bar{x} = \dfrac{104}{8} = 13.0 \qquad \bar{y} = \dfrac{123}{8} = 15.375$

$\dfrac{1}{n}\sum x^2 = \dfrac{1548}{8} = 193.5 \qquad \dfrac{1}{n}\sum y^2 = \dfrac{2179}{8} = 272.375$

$\dfrac{1}{n}\sum xy = \overline{xy} = \dfrac{1836}{8} = 229.5$

$S_x^2 = 193.5 - (13.0)^2 = 24.5 \Rightarrow S_x = 4.950$

$S_y^2 = 272.375 - (15.375)^2 = 35.984 \Rightarrow S_y = 5.999$

$S_{xy} = 229.5 - (13.0)(15.375) = 29.625$ ***(M1)***

$\Rightarrow p = \dfrac{29.625}{24.5} = 1.2092 = 1.21$ (3 s.f.) ***(A1)***

$y - 15.375 = 1.2092(x - 13.0)$

$= 1.2092x - 15.719$

$\Rightarrow \quad y = 1.2092x - 0.344$

$\Rightarrow \quad q = -0.344$ ***(A1)***

OR

$y = 1.21x - 0.344$ ***(G3)***

[3 marks]

(b) $r = \dfrac{S_{xy}}{S_x S_y} = \dfrac{29.625}{(4.950)(5.999)} = 0.998$ ***(M1)(A1)***

OR

$r = 0.998$ (or 0.997) ***(G2)***

[2 marks]

continued...

Question 6 (iii) continued

(c) (i) $A: 1.21(16) - 0.344 = 19$ ***(A1)***

(ii) $B: 1.02(16) - 3.08 = 13$ ***(A1)***

[2 marks]

(d) (i) B : III ***(A1)***

(ii) C : II ***(A1)***

[2 marks]

(e) Teacher C is the most inconsistent. Sometimes the grades are too high, sometimes too low. The lower value of *r* indicates this. ***(R2)***

[2 marks]

Total [30 marks]

7. (i) (a) (i) $x = -\frac{5}{2}$ ***(A1)***

(ii) $y = \frac{3}{2}$ ***(A1)***

[2 marks]

(b) By quotient rule ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x+5)(3)-(3x-2)(2)}{(2x+5)^2}$ ***(A1)***

$= \frac{19}{(2x+5)^2}$ ***(A1)***

[3 marks]

(c) There are no points of inflexion. ***(A1)***

[1 mark]

(ii) (a) $u = \tan 3x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3\left(\frac{1}{\cos^2 3x}\right)$ ***(M1)***

$\Rightarrow \quad \frac{1}{\cos^2 3x} = \frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}x}$

$\Rightarrow \quad \int \tan^3 3x \left(\frac{1}{\cos^2 3x}\right)\mathrm{d}x$

$= \int u^3 \left(\frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}x}\right)\mathrm{d}x = \frac{1}{3}\int u^3 \mathrm{d}u$ ***(M1)***

$\frac{u^4}{12} + c$ ***(A1)***

[3 marks]

(b) $x = 0 \quad \Rightarrow \quad u = 0$

$x = \frac{\pi}{9} \quad \Rightarrow \quad u = \tan\left(\frac{3\pi}{9}\right) = \sqrt{3}$ ***(M1)***

$\Rightarrow \text{Integral} = \left[\frac{u^4}{12}\right]_0^{\sqrt{3}} = \frac{9}{12}$ ***(M1)***

$= \frac{3}{4} \; (= 0.75)$ ***(A1)***

[3 marks]

continued...

Question 7 continued

(iii) (a) $\frac{dy}{dx} = \sin x + x\cos x$ ***(A1)***

[1 mark]

(b) $\frac{d^2y}{dx^2} = \cos x + \cos x + x(-\sin x)$ ***(M1)(M1)***

$= -x\sin x + 2\cos x$ ***(AG)***

[2 marks]

(c) At point of inflexion, second derivative = 0

$\Rightarrow \quad -x\sin x + 2\cos x = 0$ ***(M1)***

$\Rightarrow \quad x\sin x = 2\cos x$

$\Rightarrow \quad x = \frac{2\cos x}{\sin x} = \frac{2}{\frac{\sin x}{\cos x}} = \frac{2}{\tan x}$ ***(A1)(AG)***

[2 marks]

(d) (i) 1.2842 ***(A1)***

(ii) 0.5895 ***(A1)***

[2 marks]

(e) (i) 1.1071 ***(A1)***

(ii) 1.0652 ***(A1)***

[2 marks]

(f) 1.076874 ***(G2)***

[2 marks]

(g) The derivative must equal 0 at a maximum and the derivative of $y = x\sin x$ is

$\frac{dy}{dx} = \sin x + x\cos x$ ***(R1)***

[1 mark]

continued…

Question 7 (iii) continued

(h) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f(x_n) = \sin x_n + x_n \cos x_n$ ***(M1)***

$f'(x_n) = -x_n \sin x_n + 2\cos x_n$ ***(M1)***

(i) using $x_0 = 2$ gives ***(A1)***

$x_1 = 2.0290$ ***(A1)***

OR

$x_1 = 2.0290$ ***(G4)***

(ii) $x = 2.028758$ ***(G2)***

[6 marks]

Total [30 marks]

8. (i) (a) $\det \boldsymbol{R} = (0.28)(-0.28) - (0.96)(0.96)$ ***(M1)***

$= -0.0784 - 0.9216$

$= -1$ ***(A1)***

[2 marks]

(b) $\boldsymbol{R}^{-1} = \dfrac{1}{\det \boldsymbol{R}}\begin{pmatrix} -0.28 & -0.96 \\ -0.96 & 0.28 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}$ ***(A1)***

OR

$\boldsymbol{RR} = \begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}\begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ ***(A1)***

OR

$\boldsymbol{R}^{-1} = \begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}$ ***(G2)***

[2 marks]

(c) (i) (0, 0) is its own image. ***(A1)***

(ii) $\begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ ***(A1)***

That is, (4, 3) is its own image.

[2 marks]

(d) $\boldsymbol{R}$ is a reflection in the line $y = \dfrac{3}{4}x$ ***(A1)(A1)***

[2 marks]

(e) (i) $\boldsymbol{T} = \begin{pmatrix} -0.96 & 0.28 \\ 0.28 & 0.96 \end{pmatrix}\begin{pmatrix} 0.28 & 0.96 \\ 0.96 & -0.28 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ***(A1)***

(ii) $\boldsymbol{T}$ is a positive rotation through 90° ***(A1)***

about (0, 0) ***(A1)***

[3 marks]

(ii) (a) $\boldsymbol{q} = \begin{pmatrix} x-5 \\ y-2 \end{pmatrix}$ ***(A1)***

[1 mark]

(b) $\boldsymbol{u} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x-5 \\ y-2 \end{pmatrix} = \begin{pmatrix} 2-y \\ x-5 \end{pmatrix}$ ***(A1)(A1)***

[2 marks]

continued…

Question 8 (ii) continued

(c) $\boldsymbol{v} = \begin{pmatrix} 2-y \\ x-5 \end{pmatrix} + \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 7-y \\ x-3 \end{pmatrix}$ ***(A1)***

[1 mark]

(d) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p-y \\ x+q \end{pmatrix}$ ***(M1)***

(i) $p = 7$ ***(A1)***

(ii) $q = -3$ ***(A1)***

[3 marks]

(e) $\begin{pmatrix} 7-y \\ x-3 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow x + y = 7,\, x - y = 3$ ***(M1)***

$\Rightarrow 2x = 10 \quad \Rightarrow \quad x = 5$

$2y = 4 \quad \Rightarrow \quad y = 2$ ***(M1)***

$\Rightarrow (5, 2)$ is only invariant point ***(AG)***

[2 marks]

(iii) (a) Area $\text{OA}'\text{B}'\text{C}' =$ (det of matrix) × Area OABC ***(M1)***

$= 5\left[(3 \times 2)\right] = 30$ ***(A1)***

[2 marks]

(b) $\begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} \sqrt{5} & 0 \\ 0 & \sqrt{5} \end{pmatrix}\begin{pmatrix} \cos\theta & \cos\theta - \sin\theta \\ \sin\theta & \sin\theta + \cos\theta \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} \sqrt{5}\cos\theta & \sqrt{5}(\cos\theta - \sin\theta) \\ \sqrt{5}\sin\theta & \sqrt{5}(\sin\theta + \cos\theta) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ ***(A1)***

$\Rightarrow \sin\theta = \dfrac{2}{\sqrt{5}}$ and $\cos\theta = \dfrac{1}{\sqrt{5}}$

$\Rightarrow \tan\theta = 2 \quad \Rightarrow \quad \theta = \arctan 2$ ***(A1)(AG)***

[3 marks]

(c) (i) Enlargement with scale factor $\sqrt{5}$, ***(A1)***
centre (0, 0) ***(A1)***

(ii) Shear with x-axis invariant line, ***(A1)***
scale factor 1 ***(A1)***

[4 marks]

(d) The shear is applied first. ***(A1)***

[1 mark]

Total [30 marks]