INTERNATIONAL BACCALAUREATE
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MARKSCHEME

November 2001

MATHEMATICAL METHODS

Standard Level

Paper 1

13 pages

1. **METHOD 1**

$x^2 = 3 - 2x$ *(M1)*

$\Rightarrow x^2 + 2x - 3 = 0$ *(A1)*

$\Rightarrow (x+3)(x-1) = 0$

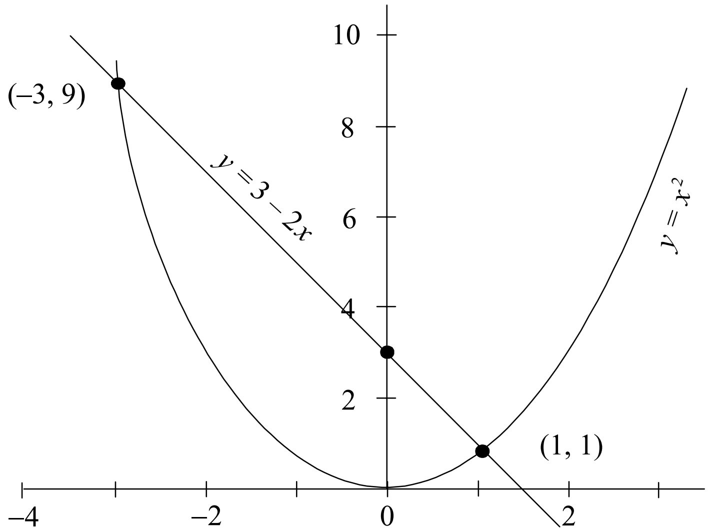
$\Rightarrow x = -3$ or $x = 1$

$\Rightarrow y = 3^2 = 9$ or $y = 1^2 = 1$

Other point is $\Rightarrow (-3, 9)$ ***(A1)(A1)*** ***(C4)***

Note: Award ***(A1)(A0)*** or ***(C3)*** if the answer is not given as coordinates.

METHOD 2



(M2)

Required point is (–3, 9) ***(G2)*** ***(C4)***

[4 marks]

2. $5000\,(1.065)^5 = \$\,6850.43\ldots$ ***(M3)***

Note: Award ***(M1)*** for multiplying by 5000, ***(M1)*** for 1.065, ***(M1)*** for raising to the 5th power.

Value $= \$\,6850$ (nearest \$) ***(A1)*** ***(C4)***

Note: Award ***(C3)*** for the answer $\$\,6851$ with no working shown.

[4 marks]

3. **METHOD 1**

Amplitude $a = 30$ *(A1)* *(C1)*

Period $\frac{2\pi}{b}$ *(M1)*

$= \frac{\pi}{2}$ *(A1)*

$\Rightarrow b = 4$ *(A1)* *(C3)*

OR

Frequency $= b$ *(M1)*

$= \frac{2\pi}{\pi/2}$ *(A1)*

$\Rightarrow b = 4$ *(A1)* *(C3)*

METHOD 2

Vertical stretch of scale factor $a = 30$ *(A1)* *(C1)*

Horizontal stretch of scale factor $\frac{1}{b} = \frac{1}{4}$ *(M1)(A1)*

$\Rightarrow b = 4$ *(A1)* *(C3)*

Note: The ***(M1)*** (in all **METHODS**) may be implied. Allow **ft** only if the ***(M1)*** is awarded.

[4 marks]

4. Recognizing an AP $a = 15$ $d = 2$ $n = 20$ (may be implied) *(M1)(A1)*

(a) $u_{20} = 15 + (20-1)2 = 53$ (that is, 53 seats in the 20th row) *(A1)* *(C2)*

(b) $S_{20} = \frac{20}{2}\left(2(15) + (20-1)2\right)$ $\left(\text{or } \frac{20}{2}(15+53)\right)$

$= 680$ (that is, 680 seats in total) *(A1)* *(C2)*

[4 marks]

5. Number of possible outcomes $= 90$. *(A1)*

(a) Set of desired outcomes $= \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$

$\Rightarrow$ number of desired outcomes $= 9$

$\Rightarrow$ P(multiple of 10) $= \frac{9}{90}\left(= \frac{1}{10}\right)$ *(A1)* *(C2)*

(b) **METHOD 1**

Outcomes giving multiple of $15 = \{15, 30, 45, 60, 75, 90\}$

$\Rightarrow$ P(multiple of 15) $= \frac{6}{90}$ *(M1)*

P(multiple of 10 and multiple of 15) $= \frac{3}{90}$

$\Rightarrow$ P(multiple of 10 or multiple of 15) $= \frac{9}{90} + \frac{6}{90} - \frac{3}{90}$

$= \frac{12}{90}\left(= \frac{2}{15}\right)$ *(A1)* *(C2)*

METHOD 2

Set of desired outcomes *(M1)*

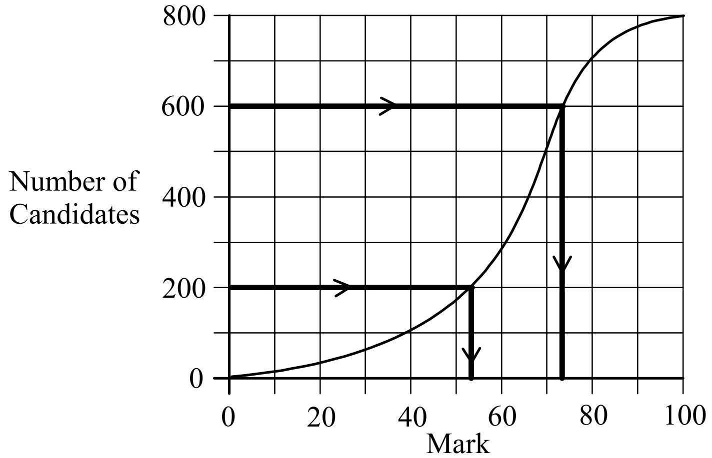
$= \{10, 15, 20, 30, 40, 45, 50, 60, 70, 75, 80, 90\}$

$\Rightarrow$ P(multiple of 10 or multiple of 15) $= \frac{12}{90}\left(= \frac{2}{15}\right)$ *(A1)* *(C2)*

Note:	Award *(M1)* for a reasonable attempt to list the desired outcomes. Allow **ft** for $\frac{12}{n}$.

[4 marks]

6.



(a) 100 students score 40 marks or fewer. ***(A1)*** ***(C1)***

(b) See graph. ***(M1)***

Note: Award ***(M1)*** for horizontal lines at 200 and 600, and/or vertical lines to points a and b.

$a = 53(\pm 1)$, $b = 75(\pm 1)$. ***(A1)(A1)*** ***(C3)***

Note: Accept statements such as middle 50% of scores between 53 and 75.

[4 marks]

7.

Note: Award ***(A1)*** for differentiating/integrating $\sin(5x-3)$, and award ***(A1)*** for multiplying/dividing by 5.

(a) $f'(x) = [2\cos(5x-3)]5 = 10\cos(5x-3)$ ***(A1)(A1)*** ***(C2)***

(b) $\int f(x)\mathrm{d}(x) = \dfrac{-2\cos(5x-3)}{5} + c = \dfrac{-2}{5}\cos(5x-3) + c$ ***(A1)(A1)*** ***(C2)***

Note: Do not penalise for the absence of c.

[4 marks]

8. (a) $a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = -10$ ***(A1)*** ***(C1)***

(b) $s = \int u\mathrm{d}t = 50t - 5t^2 + c$ ***(M1)***

$40 = 50(0) - 5(0) + c$ ***(M1)***

$s = 50t - 5t^2 + 40$ ***(A1)*** ***(C3)***

Note: Award ***(M1)*** in the first step in part (b) if c is missing, but do **not** award **ft** marks.

[4 marks]

9. (a) $f \circ g : x \mapsto 3(x+2)\ (=3x+6)$ ***(A1)*** ***(C1)***

Note: Award ***(A0)*** for $3x(x+2)$ or for $3x+2$.

(b) $f : x \mapsto 3x \Rightarrow f^{-1} : x \mapsto \dfrac{x}{3}$

$\Rightarrow f^{-1} : 18 \mapsto 6$ ***(A1)***

$g : x \mapsto x+2 \Rightarrow g^{-1} : x \mapsto x-2$

$\Rightarrow g^{-1} : 18 \mapsto 16$ ***(A1)***

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ ***(A1)*** ***(C3)***

OR

$f^{-1} : x \mapsto \dfrac{x}{3} \quad g^{-1} : x \mapsto x-2$ ***(A1)***

$f^{-1}(18) = 6 \quad g^{-1}(18) = 16$ ***(A1)***

$f^{-1}(18) + g^{-1}(18) = 6 + 16 = 22$ ***(A1)*** ***(C3)***

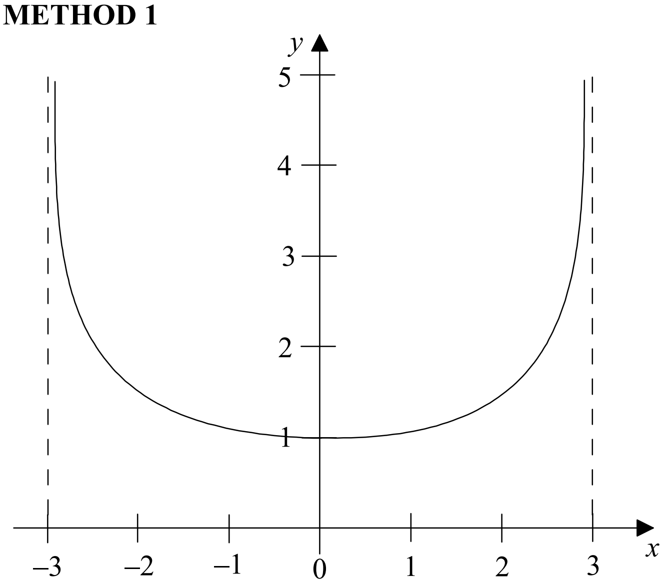
[4 marks]

10. (a) $9 - x^2 \geq 0 \Rightarrow x^2 \leq 9$

$\Rightarrow -3 \leq x \leq 3$ ***(A1)***

$\sqrt{9-x^2} \neq 0 \Rightarrow -3 < x < 3$ ***(A1)*** ***(C2)***

(b) **METHOD 1**



(M1)

$\Rightarrow y \geq 1$ ***(A1)*** ***(C2)***

METHOD 2

Maximum value of $9 - x^2$ is $9 \Rightarrow$ minimum value of y is 1 ***(A1)***

$9 - x^2$ can be as close to zero as we wish, so there is no limit on how big y can be.

$\Rightarrow y \geq 1$ ***(A1)*** ***(C2)***

Note: Award ***(C1)*** for $y > 0$ with no working shown.

[4 marks]

11. (a) $\vec{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ ***(A1)***

$\vec{AB} \cdot \vec{AC} = 4(-3) + 3(1) = -9$ ***(A1)*** ***(C2)***

(b) $|\vec{AB}| = 5$ $|\vec{AC}| = \sqrt{10}$ ***(M1)***

$\cos\theta = \dfrac{-9}{5\sqrt{10}}$ or -0.569 (3 s.f.). ***(A1)*** ***(C2)***

Note: Award ***(C1)*** for part (b) if the answer is given as 124.7° or 125°, and $\cos\theta$ not shown.

[4 marks]

12. (a) $2500 = 5000\text{e}^{-5k}$

$\Rightarrow \text{e}^{-5k} = \dfrac{1}{2}$ ***(M1)***

$\text{e}^{5k} = 2 \Rightarrow 5k = \ln 2$

$\Rightarrow k = \dfrac{\ln 2}{5} \left(= 0.139 \text{ (3 s.f.)}\right)$ ***(A1)*** ***(C2)***

(b) $50 = 5000\text{e}^{-kt}$

$\Rightarrow \dfrac{1}{100} = \text{e}^{-kt}$ ***(M1)***

$\Rightarrow \text{e}^{kt} = 100 \Rightarrow kt = \ln 100$

$\Rightarrow t = \dfrac{\ln 100}{k} = 33.2$ (Accept 33.1) ***(A1)*** ***(C2)***

[4 marks]

13. (a) $f(x) = 3x^2 - 12x + 11$

$= 3(x^2 - 4x + 4) + 11 - 12$ ***(M1)***

Note: Award ***(M1)*** for a reasonable attempt to complete the square.

$= 3(x-2)^2 - 1$ ***(A1)***

$\Rightarrow h = 2$ and $k = -1$ ***(C1)(C1)***

(b) **METHOD 1**

Vertex shifted to $(2+3, -1+5) = (5, 4)$ ***(M1)***

so the new function is $3(x-5)^2 + 4$ ***(A1)***

$\Rightarrow p = 5,\ q = 4$ ***(C1)(C1)***

METHOD 2

$g(x) = 3\left((x-3) - h\right)^2 + k + 5$ ***(M1)***

$= 3\left((x-3) - 2\right)^2 - 1 + 5$

$= 3(x-5)^2 + 4$ ***(A1)***

$\Rightarrow p = 5 \quad q = 4$ ***(C1)(C1)***

[4 marks]

14. $\frac{-3+3+a+b}{4}=0 \quad (\Rightarrow a+b=0)$ ***(M1)***

$\frac{(-3)^2+3^2+a^2+b^2}{4}=17$ ***(M1)***

$\Rightarrow a^2+b^2=68-18=50$

$a=-b \Rightarrow 2a^2=50$ ***(A1)***

$\Rightarrow a \pm 5 \quad b=\mp 5 \quad b>a$

$\Rightarrow a=-5 \quad b=5$ ***(A1)*** ***(C2)(C2)***

Note:	Award the final ***(A1)*** only if *a* and *b* are both correctly assigned. Award ***(C3)*** for the answer –5, 5 if no working is shown.

[4 marks]

15. $\binom{5}{2}(x^3)^2(-3y^2)^3$ ***(M1)***

$\binom{5}{2}=10, \ (-3y^2)^3=-27y^6$ ***(A1)(A1)***

term $=-270x^6y^6$ ***(A1)*** ***(C4)***

[4 marks]