



**MATHEMATICAL METHODS  
 STANDARD LEVEL  
 PAPER 1**

Friday 9 November 2001 (afternoon)

1 hour

Name

--

Number

--	--	--	--	--	--	--	--

**INSTRUCTIONS TO CANDIDATES**

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator in the box below *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Calculator

Make	Model

EXAMINER	TEAM LEADER	IBCA
TOTAL /60	TOTAL /60	TOTAL /60

*Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. Where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

1. The first three terms of an arithmetic sequence are 7, 9.5, 12.

(a) What is the 41<sup>st</sup> term of the sequence?

(b) What is the sum of the first 101 terms of the sequence?

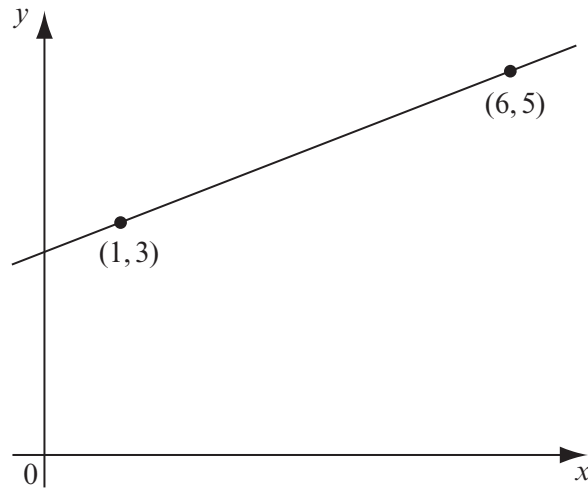
*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

2. The diagram below shows a line passing through the points (1, 3) and (6, 5).



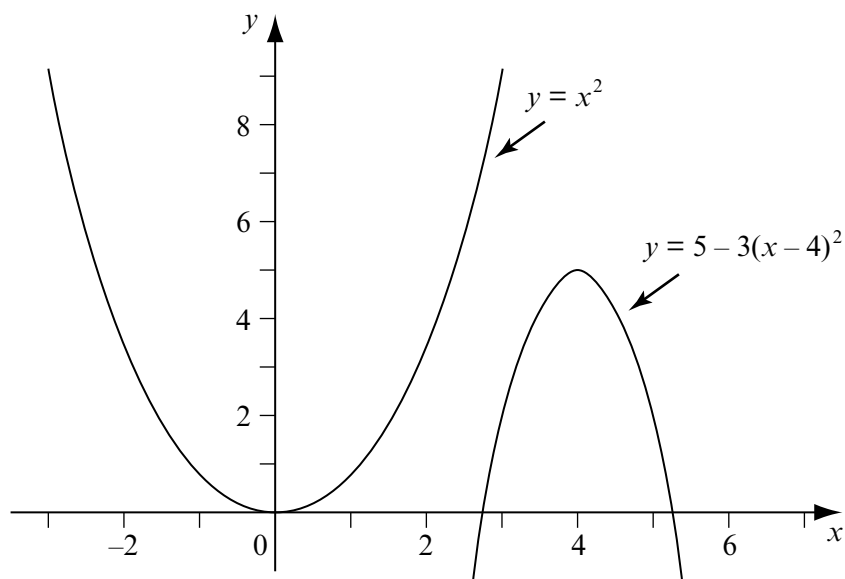
Find a vector equation for the line, giving your answer in the form

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + t \begin{pmatrix} c \\ d \end{pmatrix}, \text{ where } t \text{ is any real number.}$$

*Working:*

*Answer:*

3. The diagram shows parts of the graphs of  $y = x^2$  and  $y = 5 - 3(x - 4)^2$ .



The graph of  $y = x^2$  may be transformed into the graph of  $y = 5 - 3(x - 4)^2$  by these transformations.

A reflection in the line  $y = 0$                       **followed by**  
a vertical stretch with scale factor  $k$         **followed by**  
a horizontal translation of  $p$  units        **followed by**  
a vertical translation of  $q$  units.

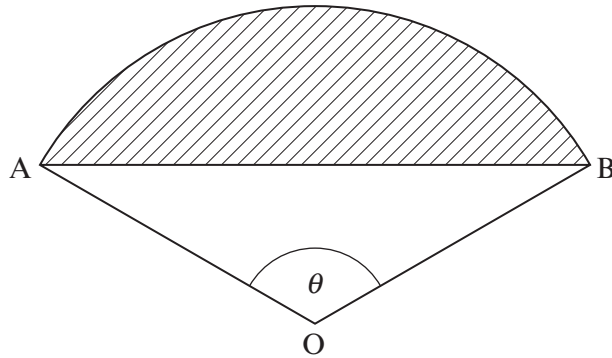
Write down the value of

- (a)  $k$  ;
- (b)  $p$  ;
- (c)  $q$  .

<p><i>Working:</i></p>	<p><i>Answers:</i></p>
	(a) _____
	(b) _____
	(c) _____

4. The diagram below shows a sector AOB of a circle of radius 15 cm and centre O . The angle  $\theta$  at the centre of the circle is 2 radians.

diagram not  
to scale



- (a) Calculate the area of the sector AOB .  
(b) Calculate the area of the shaded region.

*Working:*

*Answers:*

(a) \_\_\_\_\_

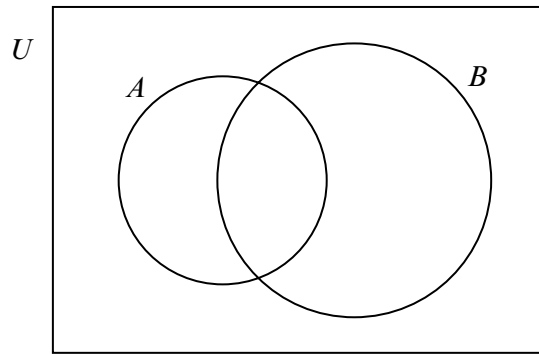
(b) \_\_\_\_\_

5. Solve the equation  $\log_9 81 + \log_9 \left(\frac{1}{9}\right) + \log_9 3 = \log_9 x$ .

*Working:*

*Answer:*

6. The following Venn diagram shows the universal set  $U$  and the sets  $A$  and  $B$ .



(a) Shade the area in the diagram which represents the set  $B \cap A'$ .

$n(U) = 100$  ,  $n(A) = 30$  ,  $n(B) = 50$  ,  $n(A \cup B) = 65$  .

(b) Find  $n(B \cap A')$  .

(c) An element is selected at random from  $U$  . What is the probability that this element is in  $B \cap A'$  ?

*Working:*

*Answers:*

(b) \_\_\_\_\_

(c) \_\_\_\_\_

7. Consider the function  $f(x) = k \sin x + 3x$ , where  $k$  is a constant.

(a) Find  $f'(x)$ .

(b) When  $x = \frac{\pi}{3}$ , the gradient of the curve of  $f(x)$  is 8. Find the value of  $k$ .

*Working:*

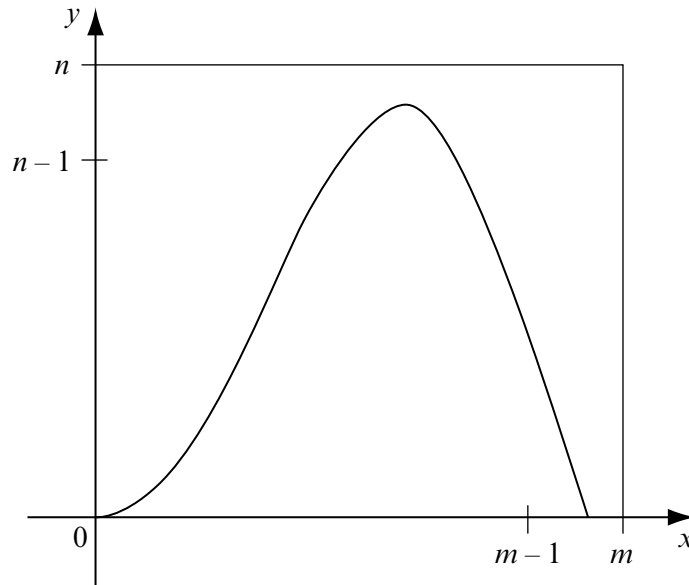
*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_



8. The diagram below shows the graph of  $y = x \sin\left(\frac{x}{3}\right)$ , for  $0 \leq x < m$ , and  $0 \leq y < n$ , where  $x$  is in radians and  $m$  and  $n$  are integers.



Find the value of

- (a)  $m$ ;
- (b)  $n$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_

9. Given that  $f(x) = 2e^{3x}$ , find the inverse function  $f^{-1}(x)$ .

*Working:*

*Answer:*

10. Consider the binomial expansion  $(1 + x)^4 = 1 + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + x^4$ .

(a) By substituting  $x = 1$  into both sides, or otherwise, evaluate  $\binom{4}{1} + \binom{4}{2} + \binom{4}{3}$ .

(b) Evaluate  $\binom{9}{1} + \binom{9}{2} + \binom{9}{3} + \binom{9}{4} + \binom{9}{5} + \binom{9}{6} + \binom{9}{7} + \binom{9}{8}$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

11. The vectors  $\begin{pmatrix} 2x \\ x-3 \end{pmatrix}$  and  $\begin{pmatrix} x+1 \\ 5 \end{pmatrix}$  are perpendicular for two values of  $x$ .

(a) Write down the quadratic equation which the two values of  $x$  must satisfy.

(b) Find the two values of  $x$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_





14. The depth,  $y$  metres, of sea water in a bay  $t$  hours after midnight may be represented by the function

$$y = a + b \cos\left(\frac{2\pi}{k}t\right), \text{ where } a, b \text{ and } k \text{ are constants.}$$

The water is at a maximum depth of 14.3 m at midnight and noon, and is at a minimum depth of 10.3 m at 06:00 and at 18:00.

Write down the value of

- (a)  $a$ ;
- (b)  $b$ ;
- (c)  $k$ .

*Working:*

*Answers:*

- (a) \_\_\_\_\_
- (b) \_\_\_\_\_
- (c) \_\_\_\_\_

15. Consider the function  $f: x \mapsto \arcsin(3x)$ .

(a) For this function to be defined,  $-a \leq 3x \leq a$ . Find the maximum value of  $a$ .

Use this value of  $a$  in parts (b) and (c).

(b) Find the domain of  $f$ .

(c) The range of  $f$  may be written as  $-b \leq y \leq b$ . Find the value of  $b$ .

*Working:*

*Answers:*

(a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

