INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

November 2001

MATHEMATICAL METHODS

Standard Level

Paper 2

22 pages

1. (a) (i) $f'(x) = -x + 2$ ***(A1)***

(ii) $f'(0) = 2$ ***(A1)***

[2 marks]

(b) Gradient of tangent $= 2 \Rightarrow$ gradient of normal $= \frac{-1}{2} = -0.5$ ***(A1)***

y-intercept is 2.5 ***(A1)***

Therefore, equation of the normal is $y = -0.5x + 2.5$ ***(AG)***

[2 marks]

(c) $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$

$\Rightarrow 0.5x^2 - 2.5x = 0$ ***(A1)***

$\Rightarrow x^2 - 5x = x(x-5) = 0$ ***(M1)***

$\Rightarrow x = 0$ or $x = 5$ ***(A1)***

OR

$0.5x^2 - 2.5x = 0$ ***(A1)***

$\Rightarrow x = 0$ or $x = 5$ ***(G2)***

[3 marks]

(d) $x = 5 \Rightarrow y = -0.5(5) + 2.5 = 0$ ***(M1)***

$(5, 0)$ ***(A1)***

OR

Coordinates $(5, 0)$ ***(G2)***

[2 marks]

(e) Area $= \int_0^5 (-0.5x^2 + 2x + 2.5)\mathrm{d}x - \frac{1}{2} \times 5 \times 2.5$ (or equivalent) ***(M2)(A1)***

Note: Award ***(A1)*** for the correct limits on the integral.

[3 marks]

(f) Area $= \left[-\frac{0.5x^3}{3} + x^2 + 2.5x \right]_0^5 - \frac{25}{4}$ ***(A1)***

$= \frac{125}{12}$ (or 10.4 (3 s.f.)) ***(A1)***

OR

Area $= \frac{125}{12} = 10.4$ (to 3 s.f.) ***(G2)***

[2 marks]

Total [14 marks]

2. (a) $\text{BC}^2 = 65^2 + 104^2 - 2(65)(104)\cos 60^\circ$ ***(M1)(A1)***

$= 4225 + 10816 - 6760 = 8281$

$\Rightarrow \text{BC} = 91\text{ m}$ ***(A1)***

OR

$\text{BC} = 91\text{ m}$ ***(G3)***

[3 marks]

(b) Area $A = \frac{1}{2}(65)(104)\sin 60^\circ$ ***(M1)***

$= 1690\sqrt{3}$ ***(A1)***

Therefore $p = 1690$

[2 marks]

(c) Smaller area $A_1 = \left(\frac{1}{2}\right)(65)(x)\sin 30^\circ = \frac{65x}{4}$ ***(M1)(AG)***

Larger area $A_2 = \left(\frac{1}{2}\right)(104)(x)\sin 30^\circ = 26x$ ***(M1)(A1)***

[3 marks]

(d) $A_1 + A_2 = A$ ***(M1)***

$\Rightarrow \frac{65x}{4} + 26x = 1690\sqrt{3}$

$\Rightarrow \frac{169x}{4} = 1690\sqrt{3}$ ***(A1)***

$\Rightarrow x = \frac{4 \times 1690\sqrt{3}}{169}$

$\Rightarrow x = 40\sqrt{3}$ ***(A1)***

Note: Award no marks for other methods.

[3 marks]

(e) (i) $\text{A}\hat{\text{D}}\text{C} = 180^\circ - \text{A}\hat{\text{D}}\text{B} \Rightarrow \sin \text{A}\hat{\text{D}}\text{C} = \sin \text{A}\hat{\text{D}}\text{B}$ ***(R1)***

(ii) ΔADB $\frac{\text{BD}}{\sin 30^\circ} = \frac{65}{\sin \text{A}\hat{\text{D}}\text{B}} \Rightarrow \frac{\text{BD}}{65} = \frac{\sin 30^\circ}{\sin \text{A}\hat{\text{D}}\text{B}}$ ***(M1)***

ΔACD $\frac{\text{DC}}{\sin 30^\circ} = \frac{104}{\sin \text{A}\hat{\text{D}}\text{C}} \Rightarrow \frac{\text{DC}}{104} = \frac{\sin 30^\circ}{\sin \text{A}\hat{\text{D}}\text{C}}$ ***(M1)***

since $\sin \text{A}\hat{\text{D}}\text{B} = \sin \text{A}\hat{\text{D}}\text{C}$

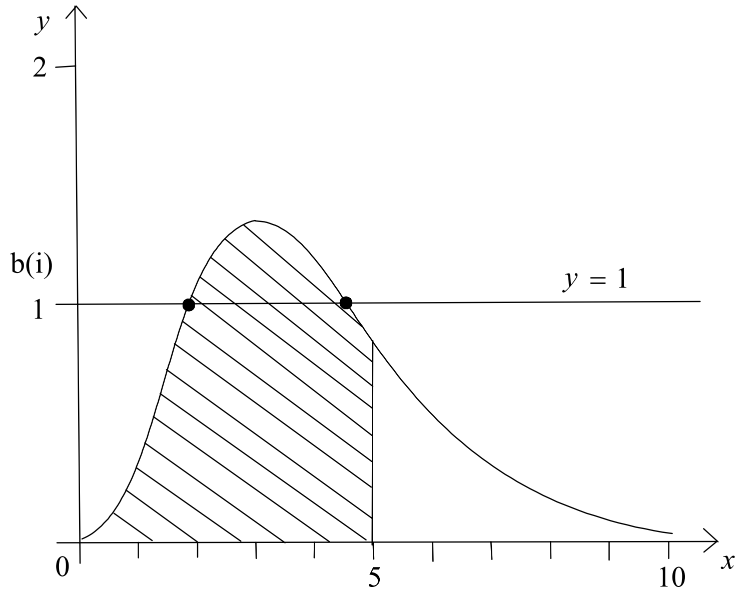
$\frac{\text{BD}}{65} = \frac{\text{DC}}{104} \Rightarrow \frac{\text{BD}}{\text{DC}} = \frac{65}{104}$ ***(M1)***

$\Rightarrow \frac{\text{BD}}{\text{DC}} = \frac{5}{8}$ ***(AG)***

[4 marks]

Total [15 marks]

3. (a) ***(A4)***



Note:	Award ***(A1)*** for labelled axes drawn with a straight edge, ***(A1)*** for maximum near (3, 1.3) (coordinates not required), ***(A1)*** for (0, 0), ***(A1)*** for the correct shape, with $y \to 0$ as $x \to 10$.

[4 marks]

(b) (i) See above diagram ***(A1)***

(ii) 4.41 ***(G1)***

[2 marks]

(c) $y = 0$ ***(A1)***

[1 mark]

(d) (i) See line $y = 1$ on above graph ***(A1)***

(ii) $x_1 = 1.86$, $x_2 = 4.54$ ***(G1)(G1)***

[3 marks]

(e) (3.00, 1.34) ***(G1)(G1)***

[2 marks]

Total [12 marks]

4. **Note:** In all parts, accept unsimplified fractions as the answers.

(a) $P(RR) = \left(\frac{4}{6}\right)\left(\frac{3}{5}\right) = \frac{2}{5}$ ***(M1)(A1)***

$P(GG) = \left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{1}{15}$ ***(A1)***

$P(\text{RG or GR}) = \left(\frac{4}{6}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right)$ ***(M1)***

$= \frac{8}{15}$ ***(A1)***

OR

$P(\text{RG or GR}) = 1 - [P(RR) + P(GG)] = 1 - \left(\frac{2}{5} + \frac{1}{15}\right)$ ***(M1)***

$= \frac{8}{15}$ ***(A1)***

[5 marks]

(b) $P(A) = \frac{1}{3} \quad P(B) = \frac{2}{3}$ ***(A1)***

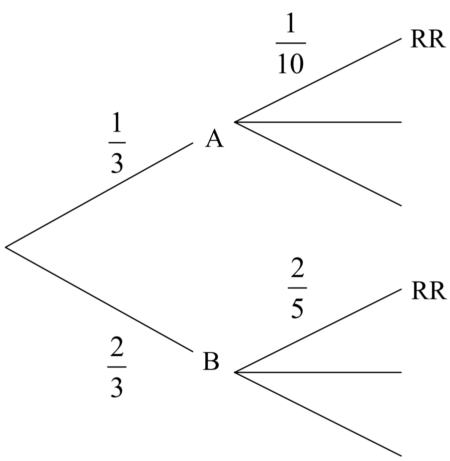
$P(RR) = P(A \cap RR) + P(B \cap RR)$ ***(M1)***

$= \left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{5}\right)$

$= \frac{1}{30} + \frac{4}{15} = \frac{1}{30} + \frac{8}{30} = \frac{9}{30}$

$\frac{9}{30}\left(= \frac{3}{10}\right)$ ***(A1)***

OR



(A1)

$P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{5}$ ***(M1)***

$= \frac{1}{30} + \frac{4}{15} = \frac{1}{30} + \frac{8}{30}$

$= \frac{9}{30} = \frac{3}{10}$ ***(A1)***

[3 marks]

continued…

Question 4 continued

(c) $\mathrm{P}(1 \text{ or } 6) = \mathrm{P}(\mathrm{A})$ ***(M1)***

$$\mathrm{P}\left(\mathrm{A} \middle| \mathrm{RR}\right) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{RR})}{\mathrm{P}(\mathrm{RR})}$$ ***(M1)***

$$= \frac{\left[\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)\right]}{\frac{3}{10}}$$ ***(M1)***

$$= \frac{1}{30} \times \frac{10}{3}$$

$$= \frac{1}{9}$$ ***(A1)***

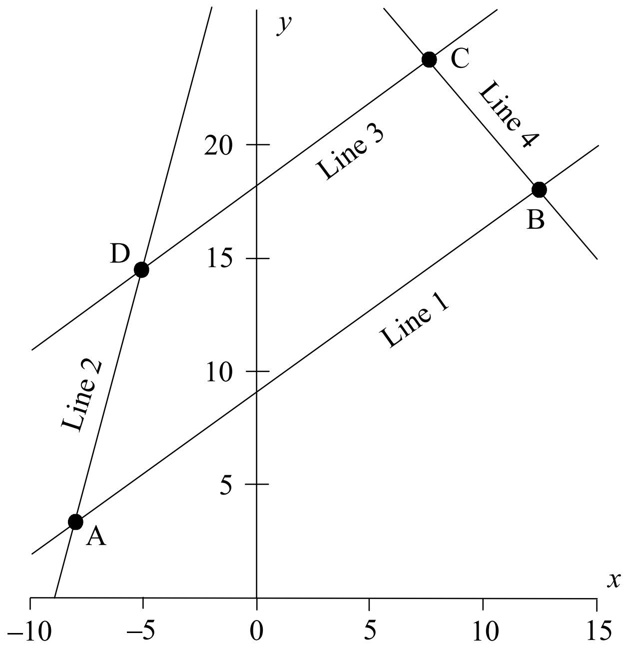
[4 marks]

Total [12 marks]

5. (a) Line 1 and Line 3 or (AB) and (DC) ***(A1)***

[1 mark]

(b)



(A4)

Note: Award ***(A1)*** for axes drawn with straight edge and labelled, ***(A2)*** for 4 lines correctly drawn, ***(A1)*** for 3 correct, ***(A0)*** for 2 or fewer correct. Award ***(A1)*** for correctly identifying the vertices.

[4 marks]

(c) (i) C(7, 23) ***(A1)***

(ii) Since D lies on Line 2 and Line 3, $x = -8 + 3q = 7 - 4s \Rightarrow 3q + 4s = 15$ ***(M1)***
$y = 3 + 11q = 23 - 3s \Rightarrow 11q + 3s = 20$ ***(M1)***
$\Rightarrow q = 1, s = 3$ ***(A1)***
$\Rightarrow x = -8 + 3(1) = -5,\ y = 3 + 11(1) = 14$ ***(A1)***
Thus D is (–5, 14) ***(AG)***

OR

Consider $x = -5$ on Line 2, $\Rightarrow -8 + 3q = -5 \Rightarrow q = 1$ ***(M1)***
$\Rightarrow y = 3 + 11 = 14, \Rightarrow (-5, 14)$ is on Line 2 ***(A1)***
Similarly for Line 3, $7 - 4s = -5$
$\Rightarrow 7 + 5 = 4s = 12$ ***(M1)***
$\Rightarrow s = 3,\ \Rightarrow y = 23 - 9 = 14$ ***(A1)***
That is (–5, 14) is also on Line 3
Thus D(–5, 14) is the intersection of Line 2 and Line 3 ***(AG)***

[5 marks]

continued…

Question 5 continued

(d) $\overrightarrow{AD} = [-5-(-8)]\boldsymbol{i} + [14-3]\boldsymbol{j} = 3\boldsymbol{i} + 11\boldsymbol{j}$ ***(A1)***

$|-3\boldsymbol{i} + 4\boldsymbol{j}| = \sqrt{(-3)^2 + 4^2} = 5$ ***(A1)***

Unit vector $\boldsymbol{u} = \frac{1}{5}(-3\boldsymbol{i} + 4\boldsymbol{j})$

$\boldsymbol{u} \cdot \overrightarrow{AD} = \frac{1}{5}[3(-3) + 11(4)]$ ***(M1)***

$= 7$ ***(A1)***

[4 marks]

(e) $|AB| = \sqrt{(12-(-8))^2 + (18-3)^2} = 25$ ***(A1)***

$|CD| = \sqrt{(-5-(7))^2 + (14-23)^2} = 15$ ***(A1)***

Area $= \left(\frac{(25+15)}{2}\right)(7) = 140$ ***(A1)***

[3 marks]

Total [17 marks]

6. **Note:** Throughout this question allow differences of ±1 in the last figure of the answer. Candidates may obtain slightly different values from tables and calculators.

(i) (a)

Observed

	A	B	C	Total
P	47	95	58	200
P′	103	145	152	400
Total	150	240	210	600

Expected

	A	B	C	Total
P	**50**	**80**	**70**	200
P′	**100**	**160**	**140**	400
Total	150	240	210	600

(G3)

Note: Award ***(G3)*** for 5 or 6 correct bold values, ***(G2)*** for 3 or 4, ***(G1)*** for 1 or 2.

OR

Assuming independence: P(take physics) $= \frac{200}{600} = \frac{1}{3}$

P(not take physics) $= \frac{400}{600} = \frac{2}{3}$ *(M1)*

Expected number taking physics at A $= \frac{1}{3}(150) = 50$

Expected number not taking physics at A $= \frac{2}{3}(150) = 100$

Expected number taking physics at B $= \frac{1}{3}(240) = 80$

Expected number not taking physics at B $= \frac{2}{3}(240) = 160$

Expected number taking physics at C $= \frac{1}{3}(210) = 70$

Expected number not taking physics at C $= \frac{2}{3}(210) = 140$ *(A2)*

Note: Award ***(A2)*** for 6 correct answers, ***(A1)*** for 4 or 5, ***(A0)*** for 3 or less.

[3 marks]

(b) $\chi^2 = \sum \frac{(f_e - f_o)^2}{f_e}$ *(M1)*

$$\chi^2 = \frac{(50-47)^2}{50} + \frac{(100-103)^2}{100} + \frac{(80-95)^2}{80} + \frac{(160-145)^2}{160} + \frac{(70-58)^2}{70} + \frac{(140-152)^2}{140}$$ *(A1)*

$$= \frac{9}{50} + \frac{9}{100} + \frac{225}{80} + \frac{225}{160} + \frac{144}{70} + \frac{144}{140}$$ *(A1)*

$= 7.574$ *(AG)*

Note: Award ***(A0)*** for just giving the value $\chi^2 = 7.574$.

[3 marks]

continued...

Question 6 (i) continued

(c) Number of degrees of freedom $= (2-1)(3-1) = 2$ ***(M1)***

For 2 degrees of freedom

$P(\chi^2 > 5.991) = 0.05$, $P(\chi^2 > 9.210) = 0.01$ ***(A1)***

(i) Reject independence hypothesis at 5% level ***(A1)***

(ii) Accept independence hypothesis at 1% level ***(A1)***

OR

$P(\chi^2 > 7.574) = 0.02266$ ***(G2)***

(i) Reject independence hypothesis at 5% level ***(A1)***

(ii) Accept independence hypothesis at 1% level ***(A1)***

[4 marks]

(ii) (a) $\mu_{\bar{x}} = 65$ ***(A1)***

[1 mark]

(b) The standard error of the mean $\sigma_E = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{25}}$ ***(M1)(A1)***

Note: Award ***(M1)*** for correct expression, ***(A1)*** for substitution of values.

Therefore $\sigma_E = 2.4$ ***(AG)***

[2 marks]

(c) For normal population 95% of population is within 1.96 standard deviations of mean. ***(M1)***

$65 - 1.96(2.4) < \bar{X} < 65 + 1.96(2.4)$ ***(A1)(A1)***

$60.3 < \bar{X} < 69.7$ ***(A1)***

OR

$60.3 < \bar{X} < 69.7$ ***(G4)***

[4 marks]

(d) We require that $1.96\sigma_E = \frac{5}{2}$ ***(M1)***

$\Rightarrow \sigma_E = 1.276$ ***(A1)***

$1.276 = \frac{12}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{12}{1.276}$ ***(M1)***

$\Rightarrow n = 88.51$

$\Rightarrow$ minimum sample size $= 89$ ***(A1)***

Note: Award ***(A1)*** for the correct smallest integer which is greater than the calculated value of *n*.

[4 marks]

continued…

Question 6 continued

(iii) (a) That y is decreasing when x is increasing ***(R1)***

[1 mark]

(b) (i) $r_u = -0.992$ ***(G2)***

OR

$\sigma_x = 8.6487 \quad \sigma_y = 0.44822 \quad \sigma_{xy} = \frac{65.8}{5} - 11(1.546) = -3.846$ ***(M1)***

$r_u = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = -0.992$ ***(A1)***

(ii) There is a stronger correlation between x^2 and y than between x and y but it is not perfect. ***(R1)***

[3 marks]

(c) (i) There is a very strong linear relationship between x^3 and y. ***(R1)***

(ii) $y = -0.00983v + 1.99$ ***(G1)(G1)***

OR

$a = \frac{Sxy}{Sx^2} = \frac{-20.426}{2078.0} = -0.00983$ ***(A1)***

$Sx^2 = (45.585)^2 = 2078.0$

$Sxy = \frac{245.72}{5} - 45(1.546) = -20.426$

$y - 1.546 = -0.00983(v - 45)$

$y = -0.00983v + 0.4423 + 1.546$

$= -0.00983v + 1.988$

$b = 1.99$ ***(A1)***

(iii) Weight touches floor $\Rightarrow y = 0$

$\Rightarrow -0.00983v + 1.99 = 0$ ***(M1)***

$\Rightarrow v = \frac{1.99}{0.00983} = 202.4$

$v = x^3 \Rightarrow x = \sqrt[3]{202.4} = 5.87$ m ***(A1)***

[5 marks]

Total [30 marks]

7. (i) (a) $a = 4 \quad b = 3$ (or -3) ***(A1)(A1)***

[2 marks]

(b) $f'(x) = \dfrac{(x^2 - 9)2 - 2x(2x)}{(x^2 - 9)^2}$ ***(M1)(A1)***

> **Note:** Award ***(M1)*** for using the quotient rule and ***(A1)*** for correct substitution into the quotient rule.

$= \dfrac{2x^2 - 18 - 4x^2}{(x^2 - 9)^2}$ (or equivalent) ***(A1)***

$= \dfrac{-2(x^2 + 9)}{(x^2 - 9)^2}$ ***(AG)***

[3 marks]

(c) Since the numerator can never equal 0, ***(R1)***
$f'(x)$ can never equal 0 $\Rightarrow$ no stationary points ***(R1)***

[2 marks]

(d) (0, 4) ***(A1)***

[1 mark]

(e) $F(x) = \int f(x)\mathrm{d}x$

$\int\left(a + \dfrac{2x}{x^2 - b^2}\right)\mathrm{d}x = ax + \ln\left|x^2 - b^2\right| + C$ ***(A3)***

> **Note:** Award ***(A1)*** for ax (or $4x$), ***(A2)*** for $\ln\left|x^2 - b^2\right|$ or $\ln\left|x^2 - 9\right|$.
> Do not penalise the omission of the absolute value sign or *C*.

[3 marks]

(f) $\left[4x + \ln(x^2 - 9\right]_4^{11} = (44 + \ln 112) - (16 + \ln 7)$ ***(M1)***

$= 28 + \ln\left(\dfrac{112}{7}\right) = 28 + \ln 16$ ***(A1)***

$= 28 + 4\ln 2$ (or $p = 28 \quad q = 4$) ***(A1)***

> **Note:** Award no marks for a calculator solution.

[3 marks]

(ii) (a) $2x^3 = 9x^2 - 11 \Rightarrow x^3 = \dfrac{9x^2 - 11}{2}$ ***(M1)***

$\Rightarrow x = \sqrt[3]{\dfrac{(9x^2 - 11)}{2}}$ ***(M1)(AG)***

[2 marks]

continued...

Question 7 (ii) continued

(b) $x_0 = 4.25$

$x_1 = 4.231755703$ ***(A1)***

$x_2 = 4.218754068$ ***(A1)***

Note:	Award ***(A1)*** in each case provided the answers are correct as far as 3 decimal places.

[2 marks]

(c) $x = 4.18614$ (6 s.f.) ***(A2)***

Note:	Award ***(A1)*** for 4.18613 or 4.18615.

[2 marks]

(iii) (a) $\int_0^2 3^x \,\mathrm{d}x = \left[\frac{3^x}{\ln 3}\right]_0^2$ ***(M1)***

$\frac{3^2 - 3^0}{\ln 3} = \frac{8}{\ln 3}$ ***(A1)*** ***(AG)***

[2 marks]

(b) $h = \frac{2-0}{2} = 1$ ***(A1)***

$\Rightarrow \frac{1}{2}\left[3^0 + 2(3^1) + 3^2\right]$ ***(M1)***

$= \frac{1}{2}(1+6+9) = \frac{16}{2}$

$= 8$ ***(A1)***

OR

Area = 8 ***(G3)***

Notes:	This may be calculated as sum of areas of 2 trapezia. If incorrect form of the trapezium rule used do not use **ft** in part (b) but allow **ft** later.

[3 marks]

(c) $8 - \frac{8}{\ln 3} = 0.718086\ldots,$ ***(M1)***

= 0.72 (to 2 s.f.) ***(A1)***

[2 marks]

continued...

Question 7 (iii) continued

(d) $\frac{0.18}{0.72} = 0.25$ ***(M1)***

Error with 2	0.72
Error with 4	0.18
Error with 8	0.045
Error with 16	0.01125
Error with 32	0.0028125
Error with 64	0.000703

(M1)

$\Rightarrow 64$ intervals ***(A1)***

Note: Award ***(M2)(A0)*** if a candidate sets up a geometric sequence but gets the answer 32 or 128.

[3 marks]

8. (i) (a) $\begin{pmatrix}-3 & -4\\ 4 & -3\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}=\begin{pmatrix}-3\\ 4\end{pmatrix}$

$(-3, 4) =$ image of $(1, 0)$ ***(A1)***

$\begin{pmatrix}-3 & -4\\ 4 & -3\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix}=\begin{pmatrix}-4\\ -3\end{pmatrix}$

$(-4, -3) =$ image of $(0, 1)$ ***(A1)***

Note: Award ***(A1)(A0)*** if both correct answers are left as a column vector, not written as coordinates.

[2 marks]

(b) (i) $\vec{OA'}=\begin{pmatrix}-3\\ 4\end{pmatrix}\Rightarrow|\vec{OA'}|=\sqrt{(-3)^2+4^2}$

$=5$ ***(A1)***

(ii) $\vec{OC'}=\begin{pmatrix}-4\\ -3\end{pmatrix}\Rightarrow|\vec{OC'}|=\sqrt{(-4)^2+(-3)^2}$

$=5$ ***(A1)***

(iii) $\vec{OA'}\cdot\vec{OC'}=(-3)(-4)+(4)(-3)$ ***(M1)***

$=0$ ***(A1)***

[4 marks]

(c) $OA'B'C'$ is a square of side 5 units ***(A1)(A1)***

[2 marks]

(d) $\boldsymbol{P}=\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$ ***(A1)***

[1 mark]

(e) $\boldsymbol{R}\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}=\begin{pmatrix}-3 & -4\\ 4 & -3\end{pmatrix}$ ***(M1)***

$\Rightarrow \boldsymbol{R}=\frac{1}{5}\begin{pmatrix}-3 & -4\\ 4 & -3\end{pmatrix}$ or $\begin{pmatrix}-\frac{3}{5} & -\frac{4}{5}\\ \frac{4}{5} & -\frac{3}{5}\end{pmatrix}$ ***(A1)***

[2 marks]

continued…

Question 8 (i) continued

(f) The matrix representing a rotation through θ is:

$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, where $\cos\theta = -\frac{3}{5}$ ***(M1)***

$\Rightarrow \theta = 126.9^\circ$ ***(A1)***

> **Note:** A diagram may also be used together with basic trigonometry to achieve $\theta = 126.9^\circ$. Award full marks.

[2 marks]

(ii) (a) $\det = 6-(-5) = 11$ ***(A1)***

$\text{inv} = \frac{1}{11}\begin{pmatrix} 3 & 1 \\ -5 & 2 \end{pmatrix}$ or $\begin{pmatrix} \frac{3}{11} & \frac{1}{11} \\ -\frac{5}{11} & \frac{2}{11} \end{pmatrix}$ ***(A1)***

OR

$\text{inv} = \begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{3}{11} & \frac{1}{11} \\ -\frac{5}{11} & \frac{2}{11} \end{pmatrix}$ ***(G2)***

[2 marks]

(b) $\boldsymbol{X} = \begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 4 & -4 \\ -1 & 12 \end{pmatrix}$ ***(M2)***

> **Notes:** Award ***(M1)*** for using the inverse, ***(M1)*** for the inverse in front. Allow **ft** from part (a).

[2 marks]

(c) $\begin{pmatrix} \frac{3}{11} & \frac{1}{11} \\ -\frac{5}{11} & \frac{2}{11} \end{pmatrix}\begin{pmatrix} 4 & -4 \\ -1 & 12 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 4 \end{pmatrix}$ ***(A2)***

> **Notes:** Award ***(A1)*** if one or two matrix coefficients incorrect, award ***(A0)*** if more than 2 incorrect.
>
> Allow **ft** from part (b), award ***(A2)***, for $\frac{1}{11}\begin{pmatrix} 32 & -4 \\ -63 & 23 \end{pmatrix}$ if the order was reversed.

[2 marks]

continued…

Question 8 continued

(iii) (a) (i) $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 5 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ ***(A1)***

(ii) $\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ***(A1)***

OR

(i) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ ***(G2)***

(ii) $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ***(G2)***

[4 marks]

(b) (3, 4) and (–1, 2) are invariant points and therefore must lie on the line of reflection. ***(M1)***

Gradient $= \dfrac{4-2}{3-(-1)} = \dfrac{1}{2}$ ***(M1)***

$\Rightarrow y - 4 = \dfrac{1}{2}(x-3)$

$\Rightarrow x - 2y + 5 = 0$ or $-x + 2y - 5 = 0$ ***(A1)***

[3 marks]

(iv) (a) $\boldsymbol{N} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ***(A1)***

[1 mark]

(b) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} h \\ k \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ***(M1)***

$\Rightarrow -2 + h = 3 \qquad 3 + k = 2$

$\Rightarrow h = 5 \qquad k = -1$ ***(A1)(A1)***

[3 marks]

Total [30 marks]