## MATHEMATICAL STUDIES <br> STANDARD LEVEL <br> PAPER 1

Monday 7 May 2001 (afternoon)
1 hour

## INSTRUCTIONS TO CANDIDATES

- Write your name and candidate number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures, as appropriate.
- Write the make and model of your calculator in the box below e.g. Casio $f x-9750 G$, Sharp EL-9400, Texas Instruments TI-85.

Calculator

| Make | Model |
| :---: | :---: |
|  |  |


| EXAMINER | TEAM LEADER |  | IBCA |  |  |
| ---: | :--- | :--- | :--- | :---: | :---: |
| TOTAL | TOTAL |  | TOTAL |  |  |
|  |  |  |  |  |  |
|  | 160 |  |  |  |  |

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working. Working may be continued below the box, if necessary. (If graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.)

1. (a) Solve the inequality

$$
2 x+3 \leq-5, \quad x \in \mathbb{R} .
$$

(b) Represent your solution on the number line below.


Working:

## Answer:

(a)
2. Twenty students are asked how many detentions they received during the previous week at school. The results are summarised in the frequency distribution table below.

| Number of <br> detentions <br> $x$ | Number of <br> students <br> $f$ | $f x$ |
| :---: | :---: | :--- |
| 0 | 6 |  |
| 1 | 3 |  |
| 2 | 10 |  |
| 3 | 1 |  |
| Total | 20 |  |

(a) What is the modal number of detentions received?
(b) (i) Complete the table.
(ii) Find the mean number of detentions received.

Working:

Answers:
(a)
(b) (ii)
3. A function $f(x)$ is defined as:

$$
f(x)=\left\{\begin{array}{lll}
2 x & \text { if } & 0 \leq x \leq 3 \\
9-x & \text { if } & x \geq 3
\end{array} \quad x \in \mathbb{R}\right.
$$

(a) Complete the table below.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 0 | 2 |  |  |  |  | 3 |

(b) Complete the graph of $f(x)$ on the axes below.


Working:
4. The following diagram shows a triangle $\mathrm{ABC} . \mathrm{AB}=8 \mathrm{~m}, \mathrm{AC}=14 \mathrm{~m}, \mathrm{BC}=18 \mathrm{~m}$, and $\widehat{B A C}=110^{\circ}$.


## Diagram not to scale

Calculate
(a) the area of triangle ABC ;
(b) the size of angle $\mathrm{A} \widehat{\mathrm{C}} \mathrm{B}$.

Working:

Answers:
(a) $\qquad$
(b)
5. $[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
(a) Complete the truth table below for the compound statement above.

| $p$ | $q$ | $p \Leftrightarrow q$ | $(p \Leftrightarrow q) \wedge p$ | $[(p \Leftrightarrow q) \wedge p] \Rightarrow q$ |
| :---: | :---: | :---: | :---: | :--- |
| T | T |  |  |  |
| T | F |  |  |  |
| F | T |  |  |  |
| F | F |  |  |  |

(b) Explain the significance of your result.

Working:

Answer:
(b) $\qquad$
6. The rate of inflation from the beginning of 1995 has been $4.5 \%$ per year.
(a) A loaf of bread cost $\$ 1.70$ on January 1, 1996. What did it cost on January 1, 1999?
(b) A car cost $\$ 40000$ on January 1, 1999. What did it cost on January 1, 1997? (Give your answer to the nearest thousand dollars.)

Working:

Answers:
(a)
(b) $\qquad$
7. The graph below shows part of the function $y=2 \sin x+3$.

(a) Write the domain of the part of the function shown on the graph.
(b) Write the range of the part of the function shown on the graph.

## Working:

Answers:
(a) $\qquad$
(b) $\qquad$
8. The graph below shows the cumulative frequency for the yearly incomes of 200 people.


Use the graph to estimate
(a) the number of people who earn less than 5000 British pounds per year ;
(b) the median salary of the group of 200 people;
(c) the lowest income of the richest $20 \%$ of this group.

## Working:

Answers:
(a) $\qquad$
(b) $\qquad$
(c) $\qquad$
9. The following diagram shows a cuboid ABCDEFGH . The coordinates of C are $(4,5,3)$. M is the mid point of $[\mathrm{FC}]$.

(a) Write down the coordinates of M.
(b) Calculate the length of $[\mathrm{CM}]$.

## Working:

## Answers:

(a) $\qquad$
(b)
10. Hank has a small business making wallets and picture frames from leather. The number of each item he can produce each week is subject to certain constraints.

The shaded region on the graph below shows the possible combinations of the number of wallets ( $x$ ) and the number of picture frames ( $y$ ) that Hank can produce each week.


Hank makes a profit of $\$ 18$ for each wallet and $\$ 9$ for each picture frame.
(a) Write an expression that represents the profit that Hank could make each week.
(b) How many wallets and how many picture frames should Hank make each week to maximise his profit?

## Working:

Answers:
(a)
(b) $\qquad$
11. A is the point $(2,3)$, and $B$ is the point $(4,9)$.
(a) Find the gradient of the line segment $[\mathrm{AB}]$.
(b) Find the gradient of a line perpendicular to the line segment $[\mathrm{AB}]$.
(c) The line $2 x+b y-12=0$ is perpendicular to the line segment $[\mathrm{AB}]$. What is the value of $b$ ?

Working:

## Answers:

(a)
(b)
(c) $\qquad$
$\qquad$

12. Two propositions $p$ and $q$ are defined as follows:
$p$ : the number ends in zero
$q$ : the number is divisible by 5
(a) Write in words
(i) $p \Rightarrow q$;
(ii) the converse of $(p \Rightarrow q)$.
(b) Write in symbolic form
(i) the inverse of $(p \Rightarrow q)$;
(ii) the contrapositive of $(p \Rightarrow q)$.

Working:

Answers:
(a) (i) $\qquad$
$\qquad$
(ii) $\qquad$
$\qquad$
(b) (i)
(ii) $\qquad$
13. The diagram below shows part of the graph of $y=a x^{2}+4 x-3$. The line $x=2$ is the axis of symmetry. M and N are points on the curve, as shown.

(a) Find the value of $a$.
(b) Find the coordinates of
(i) M ;
(ii) N .

Working:

Answers:
(a)
(b) (i)
(ii) $\qquad$
14. The table below shows the frequencies and frequency densities of the different ages of people living in an apartment building.

| Age <br> Interval | Frequency | Frequency <br> Density |
| :---: | :---: | :---: |
| $0-19$ | 8 |  |
| $20-29$ |  | 2.8 |
| $30-39$ | 24 | 2.4 |
| $40-49$ | 20 | 2.0 |
| $50-89$ |  | 0.3 |

(a) What time period, in years, is represented by the interval 30-39?
(b) Complete the table above.
(c) What is the frequency density if the intervals $30-39$ and $40-49$ are combined into one interval?

## Working:

Answers:
(a)
(c) $\qquad$
15. A rectangular block of wood with face $A B C D$ leans against a vertical wall, as shown in the diagram below. $\mathrm{AB}=8 \mathrm{~cm}, \mathrm{BC}=5 \mathrm{~cm}$ and angle $\mathrm{BAE}=28^{\circ}$.


Find the vertical height of C above the ground.

## Working:

Answer:

