



**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 2**

Friday 3 November 2000 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.
- Write the make and model of your calculator on the front cover of your answer booklets
e.g. Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

*You are advised to start each new question on a new page. A correct answer with **no** indication of the method used will usually receive **no** marks. You are therefore advised to show your working. (If graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.)*

SECTION A

*Answer all **five** questions from this section.*

1. [Maximum mark: 14]

In a club with 60 members, everyone attends either on Tuesday for Drama (D) or on Thursday for Sports (S) or on both days for Drama and Sports.

One week it is found that 48 members attend for Drama and 44 members attend for Sports and x members attend for both Drama and Sports.

- (a) (i) Draw and **label fully** a Venn diagram to illustrate this information. [3 marks]
(ii) Find the number of members who attend for both Drama and Sports. [2 marks]
(iii) Describe, in words, the set represented by $(D \cap S)'$. [2 marks]
(iv) What is the probability that a member selected at random attends for Drama only or Sports only? [3 marks]

The club has 28 female members, 8 of whom attend for both Drama and Sports.

- (b) What is the probability that a member of the club selected at random
(i) is female and attends for Drama only or Sports only? [2 marks]
(ii) is male and attends for both Drama and Sports? [2 marks]

2. [Maximum mark: 14]

(i) The propositions p , q and r are defined as follows :

- p : this is a good course
- q : the course is worth taking
- r : the grading is lenient

(a) Write a symbolic statement for each of the following sentences.

(i) *If this is a good course, then it is worth taking.*

(ii) *Either the grading is lenient, or the course is not worth taking.* [2 marks]

(b) Write the following argument using p , q , r and logic symbols or connectives only.

If this is a good course, then it is worth taking. Either the grading is lenient, or the course is not worth taking. But the grading is not lenient. Therefore, this is not a good course. [2 marks]

(c) Use a truth table to test the validity of the argument in part (b). [6 marks]

(Hint: Begin your truth table like this.)

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(This question continues on the following page)

(Question 2 continued)

(ii) **Diagram 1** shows a part of the graph of $y = x^2$.

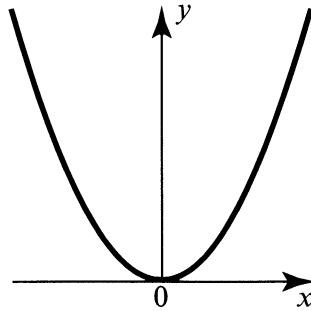


Diagram 1

Diagrams 2, 3 and 4 show a part of the graph of $y = x^2$ after it has been moved parallel to the x -axis, or parallel to the y -axis, or parallel to one axis, then the other.

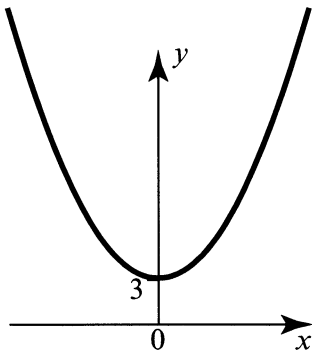


Diagram 2

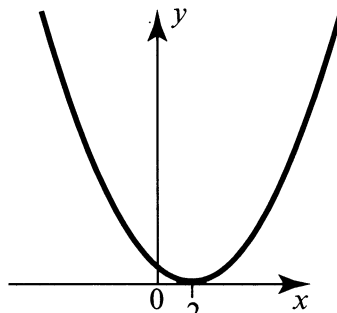


Diagram 3

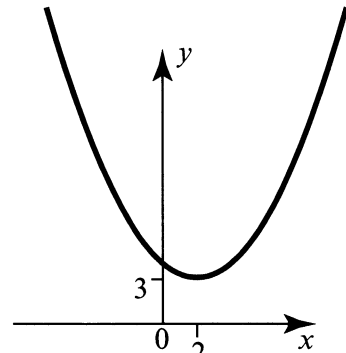


Diagram 4

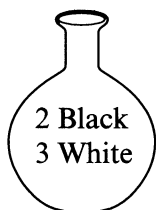
Write down the equation of the graph shown in

- (a) **Diagram 2** ;
- (b) **Diagram 3** ;
- (c) **Diagram 4** .

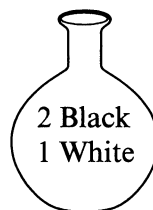
[4 marks]

3. [Maximum mark: 14]

- (i) Two jars contain a number of coloured balls as indicated in the diagrams below.



Jar One



Jar Two

Two experiments are carried out.

First Experiment: A jar is first chosen at random and then a ball is drawn from that jar.

- (a) Draw, **and label fully**, a tree diagram to show **all** possible outcomes of this experiment. [2 marks]
- (b) What is the probability that a white ball is drawn? [3 marks]

Second Experiment: The ball drawn in the first experiment is not replaced. A second ball is then drawn from the same jar.

- (c) What is the probability that both balls are white? [2 marks]

- (ii) The following table shows the times, to the nearest minute, taken by 100 students to complete a mathematics task.

Time (t) minutes	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40
Number of students	7	13	25	28	20	7

- (a) Construct a cumulative frequency table. (Use upper class boundaries 15.5, 20.5 and so on.) [2 marks]
- (b) On graph paper, draw a cumulative frequency graph, using a scale of 2 cm to represent 5 minutes on the horizontal axis and 1 cm to represent 10 students on the vertical axis. [3 marks]
- (c) Use your graph to estimate
- (i) the number of students that completed the task in less than 17.5 minutes ;
- (ii) the time it will take for $\frac{3}{4}$ of the students to complete the task. [2 marks]

4. [Maximum mark: 14]

Benny has two part-time jobs, mowing lawns and cleaning cars. He works a maximum of 15 hours per week.

He has regular clients for whom he works at least 6 hours per week mowing lawns and at least 3 hours per week cleaning cars.

Benny spends x hours a week mowing lawns and y hours a week cleaning cars.

(a) Write down **three** different inequalities to represent the above information about the time Benny spends working. [3 marks]

(b) (i) On graph paper, using a scale of 1 cm to represent 2 hours on each axis, draw the graphs of the inequalities found in part (a).

(ii) On your graph, shade and label the region R that is described by the **three** inequalities. [4 marks]

(c) Can Benny work

(i) 10 hours a week mowing lawns and 6 hours a week cleaning cars?

(ii) 3 hours a week mowing lawns and 6 hours a week cleaning cars?

(iii) 8 hours a week mowing lawns and 6 hours a week cleaning cars?

Give **one** reason for each of your answers. [3 marks]

Benny is paid £2.50 an hour for mowing lawns and £3.50 an hour for cleaning cars.

(d) (i) Write down an expression, in terms of x and y , that represents Benny's weekly income, I .

(ii) Use your graph to estimate Benny's maximum income in a week from his part-time jobs. Write down the number of hours he must work to earn this maximum income. [4 marks]

5. [Maximum mark: 14]

- (i) A gardener pegs out a rope, 19 metres long, to form a triangular flower bed as shown in this diagram.

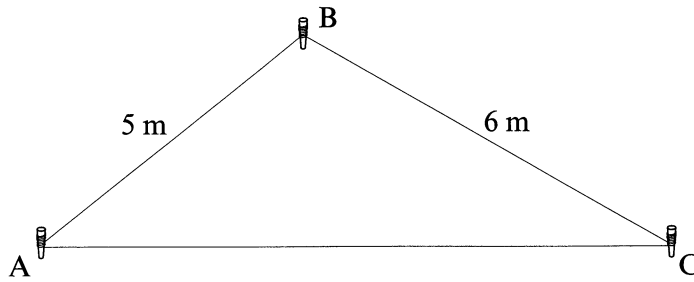


Diagram not to scale

Calculate

- (a) the size of the angle BAC ; [3 marks]
- (b) the area of the flower bed. [2 marks]
- (ii) A pyramid ABCDE has the following coordinates for the vertices of its square base ABCD:

$$A(2, 1, 3), B(2, 6, 3), C(-3, k, 3), D(-3, 1, 3).$$

- (a) Calculate
- (i) the length of [AB] ;
- (ii) the value of k ;
- (iii) the area of triangle ABC . [4 marks]
- (b) The volume of the pyramid is 40 units³.
- (i) Find the height of the pyramid.
- (ii) Write down the coordinates of E . [5 marks]

SECTION B

Answer one question from this section.

Matrices and Graph Theory

6. [Maximum mark: 30]

(i) (a) Are these statements about matrices TRUE or FALSE?

(i) The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is singular if $ad - bc = 0$.

(ii) The matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is a diagonal matrix.

(iii) For all 2×2 matrices A and B , $AB = BA$.

(iv) A^T is the transpose of $A \Rightarrow (A^T)^T = A$.

[4 marks]

(b) Let $A = \begin{pmatrix} a & 0 \\ 2a & -\frac{1}{a} \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $M = \begin{pmatrix} 3 & x + 2y \\ 3x - y & xy \end{pmatrix}$,

$$N = \begin{pmatrix} 3 & 10 \\ 2 & 8 \end{pmatrix}.$$

(i) Write down the transpose of A .

[1 mark]

(ii) Find $\det(A)$.

[1 mark]

(iii) Find a matrix B such that $BA = I$.

[3 marks]

(iv) Show that $3A - A^T = 2a \begin{pmatrix} 1 & -1 \\ 3 & -\frac{1}{a^2} \end{pmatrix}$.

[2 marks]

(v) If $M = N$, find x and y .

[3 marks]

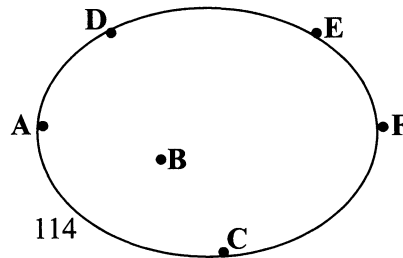
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(Question 6 continued)

- (ii) The table below shows the distances in kilometres along major routes between cities A, B, C, D, E and F.

	A	B	C	D	E	F
A		70	114	89		
B	70		56	63	95	55
C	114	56		112	160	55
D	89	63	112		49	102
E		95	160	49		84
F		55	55	102	84	

- (a) Copy, complete and **label fully** the diagram below to show its associated network.



[3 marks]

- (b) A van driver in town A is to deliver books urgently to a school in town F.

- (i) Find the shortest path for the van to travel between A and F.
- (ii) Write down this shortest distance.

- (iii) Just before leaving A, the driver is warned of a one-hour delay on the road between B and F. Assuming an average speed of 40 km h^{-1} does the driver need to change the route? Give a reason for your answer.

[5 marks]

(This question continues on the following page)

(Question 6 continued)

- (iii) In a two-person-game, players Robert and Charles have cards of different colours, numbered as shown in this diagram.

BLACK	RED
5	5

ROBERT

BLACK	RED	RED
5	3	1

CHARLES

The game they are to play is as follows:

At a given signal the players **simultaneously** turn over one of their cards. If the cards **match in colour**, Robert wins the difference between the numbers on the cards; if the cards **do not match in colour**, Charles wins the difference between the numbers on the cards played.

- (a) Copy and complete this table to show the payoffs of the game, i.e. to show the wins made by the players.

		CHARLES (C)		
		BLACK 5	RED 3	RED 1
ROBERT (R)	BLACK 5		(R , C) (0 , 2)	
	RED 5			

[3 marks]

- (b) A player's best or optimal strategy is the one he uses to ensure that he does not lose.
- What is Robert's best or optimal strategy?
 - What is Charles' best or optimal strategy?
 - If both players use their optimal strategies, who wins, and how much does he win?

[3 marks]

- (c) Is the game fair? Give a reason for your answer.

[2 marks]

Further Statistics and Probability

7. [Maximum mark: 30]

(i) A cutting machine produces steel rods, which must not be more than 6.54 cm in length. The lengths of the rods are normally distributed with a mean μ cm and standard deviation, σ cm.

(a) On Monday morning, when the machine is correctly set, 1 in 20 rods are rejected as exceeding 6.54 cm. It is known that the mean length is 6.50 cm. Find the value of σ , the standard deviation, correct to three significant figures.

[5 marks]

On Tuesday, it was found that 1 in 15 rods were rejected for exceeding 6.54 cm.

(b) (i) Assuming that the mean has not changed, calculate the new standard deviation.

[4 marks]

(ii) If 1000 rods are produced on Tuesday, how many would be expected to have lengths between 6.48 cm and 6.53 cm?

[4 marks]

(This question continues on the following page)

(Question 7 continued)

- (ii) Members of a certain club are required to register for one of three games, billiards, snooker or darts.

The number of club members of each gender choosing each game in a particular year is shown in the table below.

	Billiards	Snooker	Darts
Male	39	16	8
Female	21	14	17

- (a) Use a χ^2 (Chi-squared) test at the 5% significance level to test whether choice of games is independent of gender. State clearly the null and alternative hypotheses tested, the expected values, and the number of degrees of freedom used.

[13 marks]

The following year the choice of games was widened and the figures for that year are as follows:

	Billiards	Snooker	Darts	Fencing
Male	4	15	8	10
Female	10	21	17	37

- (b) If the χ^2 test were applied to this new set of data,
- (i) why would it be necessary to combine billiards with another game?
 - (ii) which other game would you combine with billiards and why?

[2 marks]

A club member is to be selected at random.

- (c) What is the probability that the club member selected is a
- (i) female who chose billiards or snooker?
 - (ii) male or female who chose darts or fencing?

[2 marks]

Introductory Differential Calculus

8. [Maximum mark: 30]

(i) The function $f(x)$ is given by $f(x) = x^3 - 3x^2 + 3x$, for $-1 \leq x \leq 3$.

(a) Differentiate $f(x)$ with respect to x . [2 marks]

(b) Copy and complete the table below.

x	-1	0	1	2	3
$f(x)$		0	1	2	9
$f'(x)$	12		0		12

[3 marks]

(c) Use the information in your table to sketch the graph of $f(x)$. [2 marks]

(d) Mark, with an F, a point of inflexion on your graph. [1 mark]

(e) Write down the gradient of the tangent to the curve at the point (3, 9). [1 mark]

(ii) The perimeter of a rectangle is 24 metres.

(a) The table shows some of the possible dimensions of the rectangle. Find the values of a , b , c , d and e .

Length (m)	Width (m)	Area (m ²)
1	11	11
a	10	b
3	c	27
4	d	e

[2 marks]

(b) If the length of the rectangle is x m, and the area is A m², express A in terms of x only. [1 mark]

(c) What are the length and width of the rectangle if the area is to be a maximum? [3 marks]

(This question continues on the following page)

(Question 8 continued)

- (iii) An object thrown upwards reaches a height of H metres after T seconds, where $H = 30T - 5T^2$.

If T increases by an amount t seconds, the corresponding increase in H is h metres.

- (a) Write down an expression for $(H + h)$ in terms of T and t . [2 marks]

- (b) Find

(i) an expression for h in terms of T and t **only** ;

(ii) the limit of $\frac{h}{t}$ as $t \rightarrow 0$. [4 marks]

- (c) (i) What does $\frac{dH}{dT}$ represent?

(ii) What is the value of $\frac{dH}{dT}$ when $T = 6$?

(iii) What does the value of $\frac{dH}{dT}$ you obtained in part (ii) tell you about the direction of movement of the object? [3 marks]

- (d) (i) After how many seconds does the object reach its maximum height?

(ii) What is the maximum height reached by the object? [4 marks]

- (e) Calculate the initial velocity of the object. [2 marks]