INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

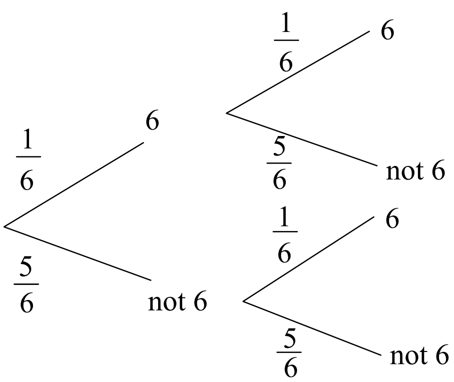
November 2000

MATHEMATICAL METHODS

Standard Level

Paper 1

1. (a)



6, 6	$\frac{1}{36}$	
6, not 6	$\frac{5}{36}$	
not 6, 6	$\frac{5}{36}$	
not 6, not 6	$\frac{25}{36}$	***(M2)*** ***(C2)***

Notes: Award ***(M1)*** for probabilities $\frac{1}{6}, \frac{5}{6}$ correctly entered on diagram.
Award ***(M1)*** for correctly listing the outcomes $6, 6$; $6, \text{not } 6$; $\text{not } 6, 6$; $\text{not } 6, \text{not } 6$, or the corresponding probabilities.

(b) $\mathrm{P}(\text{one or more sixes}) = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{5}{6}+\frac{5}{6}\times\frac{1}{6}$ **or** $\left(1-\frac{5}{6}\times\frac{5}{6}\right)$ ***(M1)***

$= \frac{11}{36}$ ***(A1)*** ***(C2)***

[4 marks]

2. $\frac{(10\times 1)+(20\times 2)+(30\times 5)+(40\times k)+(50\times 3)}{k+11} = 34$ ***(M1)(A1)***

$\frac{40k+350}{k+11} = 34$ ***(A1)***

$\Rightarrow\ k = 4$ ***(A1)*** ***(C4)***

[4 marks]

3. $f'(x) = -2x+3$

$f(x) = \frac{-2x^2}{2} + 3x + c$ ***(M1)***

Notes: Award ***(M1)*** for an attempt to integrate. Do not penalise the omission of c here.

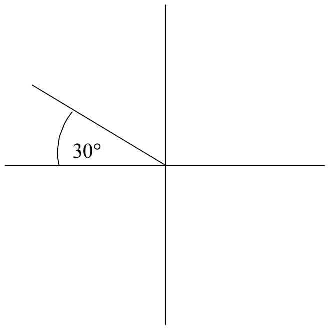
$1 = -1+3+c$ ***(A1)***

$c = -1$ ***(A1)***

$f(x) = -x^2+3x-1$ ***(A1)*** ***(C4)***

[4 marks]

4. (a)



Acute angle 30° *(M1)*

Note: Award the ***(M1)*** for 30° and/or quadrant diagram/graph seen

2nd quadrant since sine positive and cosine negative
$\Rightarrow \theta = 150^\circ$ *(A1)* *(C2)*

(b) $\tan 150^\circ = -\tan 30^\circ$ **or** $\tan 150^\circ = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$ *(M1)*

$\tan 150^\circ = -\dfrac{1}{\sqrt{3}}$ *(A1)* *(C2)*

[4 marks]

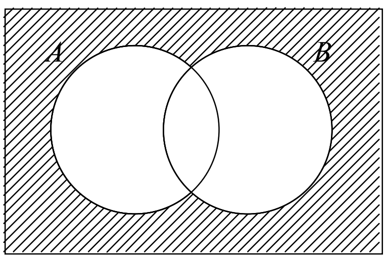
5. Vector equation of a line $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{t}$ *(M1)*

$\boldsymbol{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\boldsymbol{t} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ *(M1)(M1)*

$\Rightarrow$ $\boldsymbol{r} = \lambda(2\boldsymbol{i} + 3\boldsymbol{j})$ *(A1)* *(C4)*

[4 marks]

6. (a)



(A1) *(C1)*

(b) (i) $n(A \cap B) = 2$ *(A1)* *(C1)*

(ii) $\mathrm{P}(A \cap B) = \dfrac{2}{36}\left(\text{or } \dfrac{1}{18}\right)$ (allow **ft** from (b)(i)) *(A1)* *(C1)*

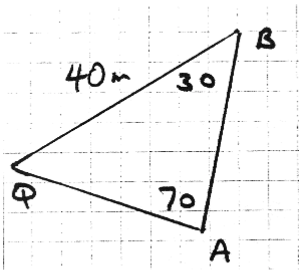
(c) $n(A \cap B) \neq 0$ (or equivalent) *(R1)* *(C1)*

[4 marks]

7. (a) $\frac{PQ}{40} = \tan 36^\circ$

$\Rightarrow PQ \approx 29.1$ m (3 s.f.) ***(A1)*** ***(C1)***

(b)



$A\hat{Q}B = 80^\circ$ ***(A1)***

$\frac{AB}{\sin 80^\circ} = \frac{40}{\sin 70^\circ}$ ***(M1)***

Note: Award ***(M1)*** for correctly substituting.

$\Rightarrow AB = 41.9$ m (3 s.f.) ***(A1)*** ***(C3)***

[4 marks]

8. (a) $f'(x) = 3(2x+5)^2 \times 2$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use the chain rule.

$= 6(2x+5)^2$ ***(C2)***

(b) $\int f(x)\,dx = \frac{(2x+5)^4}{4 \times 2} + c$ ***(A2)*** ***(C2)***

Note: Award ***(A1)*** for $(2x+5)^4$ and ***(A1)*** for /8.

[4 marks]

9. (a) $y = (x-1)^2$ ***(A2)*** ***(C2)***

(b) $y = 4(x-1)^2$ ***(A1)*** ***(C1)***

(c) $y = 4(x-1)^2 + 3$ ***(A1)*** ***(C1)***

Note: Do not penalise if these are correctly expanded.

[4 marks]

10. Perimeter $= 5(2\pi - 1) + 10$ ***(M1)(A1)(A1)***

Note: Award ***(M1)*** for working in radians; ***(A1)*** for $2\pi - 1$; ***(A1)*** for +10

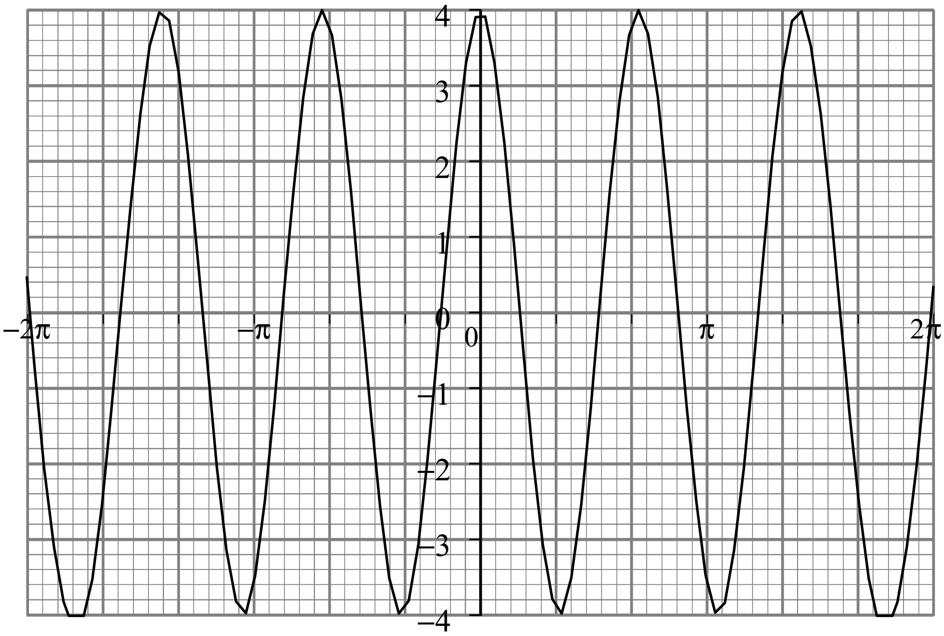
$= (10\pi + 5)$ cm ($= 36.4$, to 3 s.f.) ***(A1)*** ***(C4)***

[4 marks]

11. From sketch of graph $y = 4\sin\left(3x + \frac{\pi}{2}\right)$ ***(M2)***

or by observing $|\sin\theta| \le 1$.

$k > 4, k < -4$ ***(A1)(A1)*** ***(C2)(C2)***



[4 marks]

12. (a) $\$1000 \times 1.075^{10} = \$\,2061$ (nearest dollar) ***(A1)*** ***(C1)***

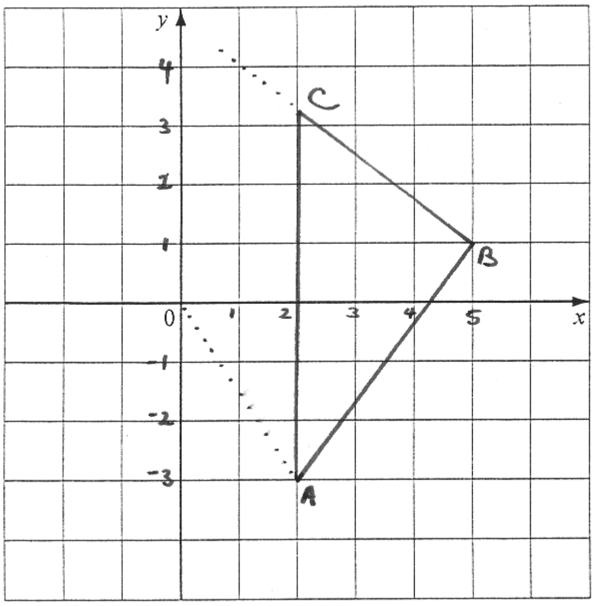
(b) $1000(1.075^{10} + 1.075^{9} + \ldots + 1.075)$ ***(M1)***

$= \dfrac{1000(1.075)(1.075^{10} - 1)}{1.075 - 1}$ ***(M1)***

$= \$\,15\,208$ (nearest dollar) ***(A1)*** ***(C3)***

[4 marks]

13. (a)



(A3) ***(C3)***

Notes:	Award ***(A1)*** for B at $(5, 1)$; ***(A1)*** for BC perpendicular to AB; ***(A1)*** for AC parallel to the *y*-axis.

(b) $\vec{\text{OC}} = \begin{pmatrix} 2 \\ 3.25 \end{pmatrix}$ ***(A1)*** ***(C1)***

Note:	Accept correct readings from diagram (allow ± 0.1).

[4 marks]

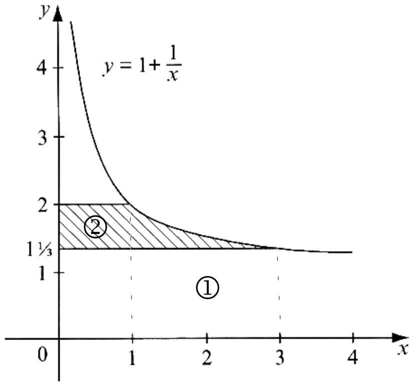
14. Graph of quadratic function.

Expression	+	–	0
a		✓	
c		✓	
$b^2 - 4a$c			✓
b	✓		

(A1) ***(C1)***
(A1) ***(C1)***
(A1) ***(C1)***
(A1) ***(C1)***

[4 marks]

15.



Area $= \int_{1\frac{1}{3}}^{2} x \, \mathrm{d}y = \int_{1\frac{1}{3}}^{2} \frac{1}{(y-1)} \mathrm{d}y$ ***(M1)(A1)***

$= \left[\ln(y-1)\right]_{1\frac{1}{3}}^{2}$

$= \ln 1 - \ln\frac{1}{3}$ ***(A1)***

$= \ln 3$ ***(A1)*** ***(C4)***

OR

Area from $x = 1$ to $x = 3$, $A = \int_{1}^{3}\left(1 + \frac{1}{x}\right)\mathrm{d}x = [x + \ln x]_{1}^{3} = (3 + \ln 3) - (1 + \ln 1)$ ***(M1)***

$= 2 + \ln 3$ ***(A1)***

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

Shaded area $= 2 + \ln 3 - 2\frac{2}{3} + \frac{2}{3}$ ***(M1)***

$= \ln 3$ ***(A1)*** ***(C4)***

OR

Area from $x = 1$ to $x = 3$, $A = \int_{1}^{3}\left(1 + \frac{1}{x}\right)\mathrm{d}x$ ***(M1)***

$A = 3.0986\ldots$ ***(G0)***

Area rectangle ① $= 2 \times 1\frac{1}{3} = 2\frac{2}{3}$, area rectangle ② $= 1 \times \frac{2}{3} = \frac{2}{3}$

Shaded area $= 3.0986 - 2\frac{2}{3} + \frac{2}{3}$ ***(M1)***

$= 1.10$ (3 s.f.) ***(A1)*** ***(C4)***

Note:	An exact value is required. If candidates have obtained the answer 1.10, and shown their working, award marks as above. However, if they do not show their working, award ***(G2)*** for the correct answer of 1.10. Award no marks for the giving of 3.10 as the final answer.

[4 marks]