INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

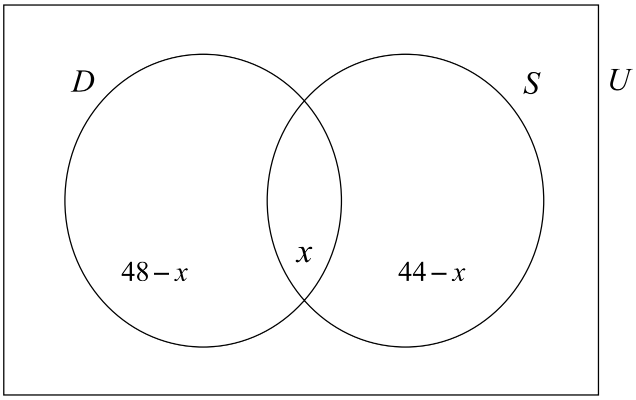
November 2000

MATHEMATICAL STUDIES

Standard Level

Paper 2

1. (a) (i)



(A3)

Note:	Award *(A1)* for a correct diagram (labelled), *(A1)* for x in the correct position, *(A1)* for either $(48-x)$ **or** $(44-x)$ correctly positioned.

[3 marks]

(ii) $48-x+x+44-x=60$ (or equivalent), allow **ft** from (i) *(M1)*

$\Rightarrow x=32$ *(A1)*

[2 marks]

(iii) The set of members who **did not** attend for **both** Drama and Sports (or equivalent) *(A2)*

[2 marks]

(iv) $\mathrm{P}(D \text{ or } S)=\left[\dfrac{48-32}{60}+\dfrac{44-32}{60}\right]$ *(M1)(M1)*

Note:	Award *(M1)* for either $\dfrac{48-32}{60}$ or $\dfrac{44-32}{60}$, *(M1)* for adding.

$=\dfrac{28}{60}$ ***or*** $\dfrac{7}{15}$ ***or*** 0.467 (3 s.f.) ***or*** 46.7% (3 s.f.) *(A1)*

[3 marks]

(b) (i) $\mathrm{P}\big(\text{Female and } (S \text{ or } D)\big)=\dfrac{20}{60}$ *(M1)*

$=\dfrac{1}{3}$ ***or*** 0.333 (3 s.f.) ***or*** 33.3% (3 s.f.) *(A1)*

[2 marks]

(ii) $\mathrm{P}(\text{Male and both } D \text{ and } S)=\left[\dfrac{32-8}{60}\right]$ *(M1)*

$=\dfrac{2}{5}$ ***or*** 0.4 ***or*** 40% *(A1)*

[2 marks]

Total [14 marks]

2. (i) (a) (i) $p \Rightarrow q$ ***(A1)***

(ii) $r \vee \neg q$ ***(A1)***

[2 marks]

(b) $p \Rightarrow q, r \vee \neg q$

$\neg r$ ***(A1)***

Therefore, $\neg p$ ***(A1)***

OR

$\{(p \Rightarrow q) \wedge (r \vee \neg q) \wedge \neg r\} \Rightarrow \neg p$ ***(A2)***

[2 marks]

(c)

			6↓		4↓	1↓	2↓	3↓	5↓	7↓
p	q	r	$\neg p$	$\neg q$	$\neg r$	$p \Rightarrow q$	$r \vee \neg q$	$1 \wedge 2$	$3 \wedge 4$	$5 \Rightarrow 6$
T	T	T	F	F	F	**T**	**T**	**T**	**F**	**T**
T	T	F	F	F	T	**T**	**F**	**F**	**F**	**T**
T	F	T	F	T	F	**F**	**T**	**F**	**F**	**T**
T	F	F	F	T	T	**F**	**T**	**F**	**F**	**T**
F	T	T	T	F	F	**T**	**T**	**T**	**F**	**T**
F	T	F	T	F	T	**T**	**F**	**F**	**F**	**T**
F	F	T	T	T	F	**T**	**T**	**T**	**F**	**T**
F	F	F	T	T	T	**T**	**T**	**T**	**T**	**T**

(A5)

Note: Award ***(A1)*** for each correct **bold** column.

From the table, the argument in part (b) is valid. ***(R1)***

[6 marks]

(ii) (a) $y = x^2 + 3$ ***(A1)***

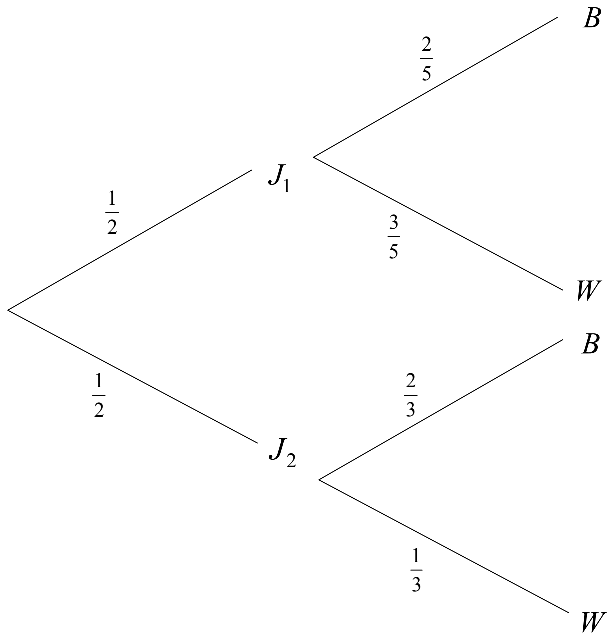
(b) $y = (x-2)^2$ ***(A1)***

(c) $y = (x-2)^2 + 3$ ***(A2)***

[4 marks]

Total [14 marks]

3. (i) (a)



(A1)

(A1)

[2 marks]

(b) $\mathrm{P}(J_1 \cap W) = \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)$, $\mathrm{P}(J_2 \cap W) = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$ ***(M1)***

Note: Award ***(M1)*** for either correct.

$\mathrm{P}(W) = \frac{3}{10} + \frac{1}{6}$ ***(M1)***

$= \frac{7}{15}$ ***or*** 0.467 (3 s.f.) ***or*** 46.7% (3 s.f.) ***(A1)***

[3 marks]

(c) $\mathrm{P}(J_1 \cap W \cap W) = \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)$, $\mathrm{P}(J_2 \cap W \cap W) = 0$ ***(M1)***

$\mathrm{P}(W \cap W) = \frac{3}{20} + 0$

$= \frac{3}{20}$ ***or*** 0.15 ***or*** 15% ***(A1)***

[2 marks]

continued…

Question 3(ii) continued

(ii) (a)

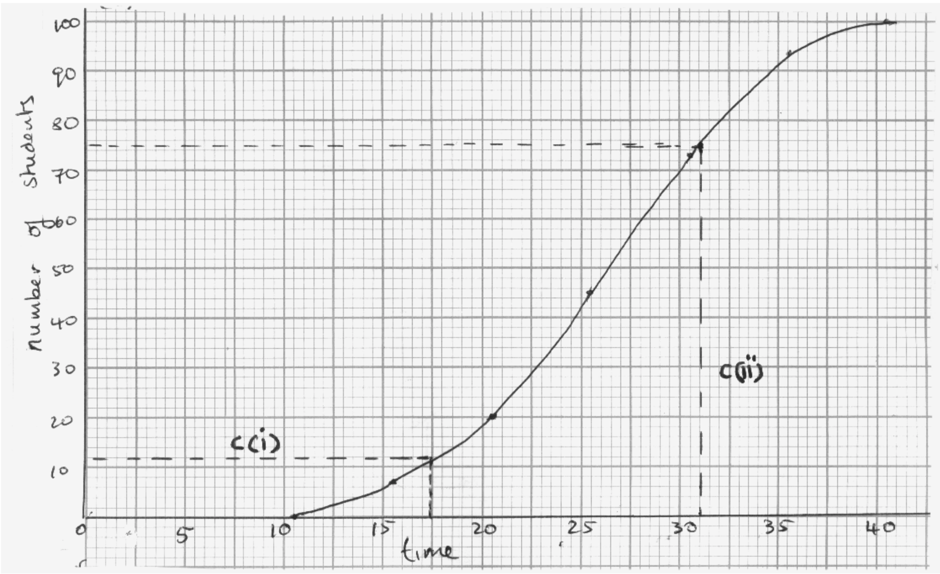
Time less than (mins)	cumulative frequency
10.5	0
15.5	7
20.5	20
25.5	45
30.5	73
35.5	93
40.5	100

(A2)

Note: Award ***(A1)*** for each correct column.

[2 marks]

(b)



(A3)

Note: Award ***(A1)*** for the correct scale and labelling.
Award ***(A2)*** for plotting 6 or 7 points correctly, ***(A1)*** for plotting 4 or 5 points correctly.

[3 marks]

(c) (i) 12 ± 1 students (allow **ft**) ***(A1)***

(ii) 31 ± 0.5 minutes (allow **ft**) ***(A1)***

[2 marks]

Total [14 marks]

4. (a) $x + y \le 15$ *(A1)*

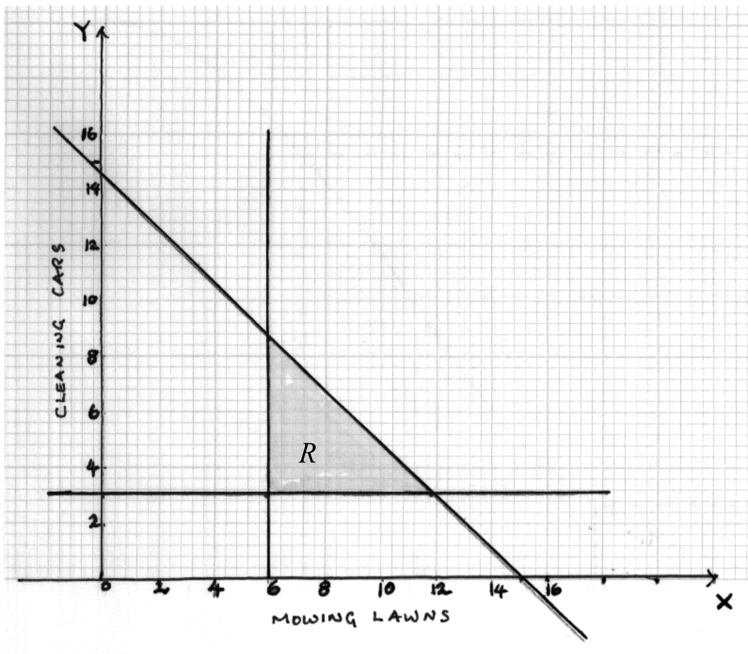
$x \ge 6$ *(A1)*

$y \ge 3$ *(A1)*

[3 marks]

(b) (i) *(A3)*

(ii) *(A1)*



Note:	Award ***(A1)*** for each correct line, and ***(A1)*** for *R*. Allow **ft** from part (a).

[4 marks]

(c) (i) **No**, because $10 + 6 > 15$ *(A1)*

(ii) **No**, because $x = 3 < 6$ *(A1)*

(iii) **Yes**, because $x = 8 > 6$, $y = 6 > 3$ and $8 + 6 < 15$ *(A1)*

[3 marks]

(d) (i) $I = 2.5x + 3.5y$ *(A1)*

(ii) Vertices $(6, 9) :\sim 6(2.5) + 9(3.5) = 46.50$

$(6, 3) :\sim 6(2.5) + 3(3.5) = 25.50$

$(12, 3) :\sim 12(2.5) + 3(3.5) = 40.50$ *(M1)*

Hence maximum income $= £46.50$ (allow **ft** from graph.) *(A1)*

6 hours mowing lawns and 9 hours cleaning cars ***or*** $(6, 9)$ *(A1)*

[4 marks]

Total [14 marks]

5. (i) (a) $AC = 19 - 11 = 8$ ***(M1)***

$6^2 = 5^2 + 8^2 - 2(5)(8)\cos BAC$ ***(M1)***

$\Rightarrow B\hat{A}C = 48.5^\circ$ (3 s.f.) ***(A1)***

[3 marks]

(b) Area $= \left(\frac{1}{2}\right)(5)(8)\sin B\hat{A}C$ ***(M1)***

$= 15.0 \text{ cm}^2$ (3 s.f.) (allow **ft** from part (a)) ***(A1)***

[2 marks]

(ii) (a) (i) $AB = 5$ ***(A1)***

(ii) $k = 6$ ***(A1)***

(iii) Area of triangle $ABC = \frac{1}{2}(5)(5)$ ***(M1)***

$= 12.5 \text{ units}^2$ ***(A1)***

[4 marks]

(b) (i) $V = \frac{1}{3}(\text{Area base}) \times \text{height} = 40 \text{ units}^3$

$\Rightarrow \frac{1}{3}(25) \times \text{height} = 40$ ***(M1)***

Height $= 3 \times 40 \div 25$ (**ft** from (b)(ii))

$= 4.8$ ***(A1)***

(ii) $x = \frac{2-3}{2} = -0.5$ ***(A1)***

$y = \frac{1+6}{2} = 3.5$ ***(A1)***

$z = 3 + 4.8 = 7.8$ ***(A1)***

OR

$E(-0.5, 3.5, 7.8)$ ***(A3)***

[5 marks]

Total [14 marks]

6. (i) (a) (i) True *(A1)*

(ii) False *(A1)*

(iii) False *(A1)*

(iv) True *(A1)*

[4 marks]

(b) (i) $A^T = \begin{pmatrix} a & 2a \\ 0 & -\frac{1}{a} \end{pmatrix}$ *(A1)*

(ii) $\det(A) = -1$ *(A1)*

(iii) $B = \begin{pmatrix} \frac{1}{a} & 0 \\ 2a & -a \end{pmatrix}$ *(M1)(A1)*

(iv) $3A = \begin{pmatrix} 3a & 0 \\ 6a & -\frac{3}{a} \end{pmatrix}$ (or equivalent) *(M1)*

$3A - A^T = \begin{pmatrix} 2a & -2a \\ 6a & -\frac{2}{a} \end{pmatrix}$ *(M1)*

$= 2a\begin{pmatrix} 1 & -1 \\ 3 & -\frac{1}{a^2} \end{pmatrix}$ *(AG)*

(v) $M = N \Rightarrow x + 2y = 10,$

$3x - y = 2,$

$xy = 8$ *(M2)*

Note: Award ***(M2)*** for any 2 correct equations.

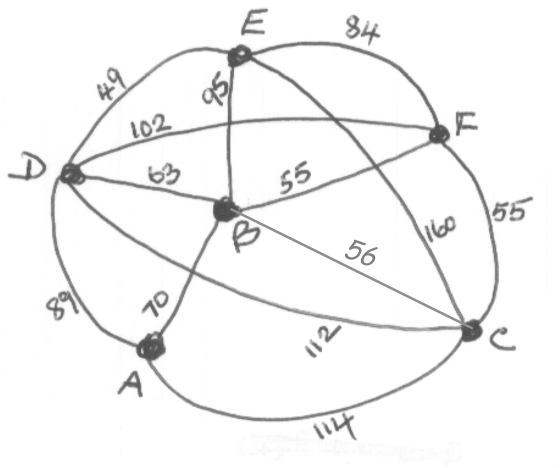
Therefore, $x = 2,\ y = 4$ *(A1)*

[10 marks]

continued…

Question 6 continued

(ii) (a)



(M3)

Note:	AC = 114 is given. Award ***(M3)*** for 11 or 12 correct distances, ***(M2)*** for 9 or 10 correct, ***(M1)*** for 7 or 8 correct, ***(M0)*** for 6 or less correct.

[3 marks]

(b) (i) $A \rightarrow B \rightarrow F$ ***(M2)***

(ii) 125 km ***(A1)***

(iii) Assuming an average speed of 40 km/h then route $A \rightarrow B \rightarrow F$ takes $\frac{125}{40} = 3.125$ hours. With delay $A \rightarrow B \rightarrow F$ takes 4.125 hours. ***(A1)***

The next shortest route is $A \rightarrow C \rightarrow F = 169$ km.

This takes $\frac{169}{40} = 4.225$ hours. ABF is still the quickest route. ***(R1)***

[5 marks]

(iii) (a)

		CHARLES (C)		
		BLACK 5	RED 3	RED 1
ROBERT (R)	BLACK 5	(R, C) (0, 0)	(R, C) (0, 2)	(R, C) (0, 4)
	RED 5	(R, C) (0, 0)	(R, C) (2, 0)	(R, C) (4, 0)

(A3)

[3 marks]

(b) (i) Play Red 5 ***(A1)***

(ii) Play Black 5 ***(A1)***

(iii) 0 (zero) ***or*** no win ***(A1)***

[3 marks]

(c) Yes ***(A1)***

Because the result of both players' optimal strategy is zero (no win) ***(R1)***

[2 marks]

7. (i) (a) $\mathrm{P}(X > 6.54) = \dfrac{1}{20} = 0.05$ ***(M1)***

$\Rightarrow \mathrm{P}\left(Z > \dfrac{0.04}{\sigma}\right) = 0.05$ ***(M1)***

$\Rightarrow 1 - \Phi\left(\dfrac{0.04}{\sigma}\right) = 0.05$ ***(M1)***

$\Rightarrow \dfrac{0.04}{\sigma} = 1.64$, therefore $\sigma = 0.0244$ (3 s.f.) ***(M1)(A1)***

(Accept $\sigma = 0.0243$ from 1.645, or $\sigma = 0.0242$ from 1.65.)

[5 marks]

(b) (i) $\mathrm{P}(X > 6.54) = \dfrac{1}{15} = 0.0667$ (3 s.f.) ***(M1)***

$\Rightarrow \mathrm{P}\left(Z > \dfrac{0.04}{\sigma}\right) = 0.0667$ ***(M1)***

$\Rightarrow 1 - \Phi\left(\dfrac{0.04}{\sigma}\right) = 0.0667$

$\Rightarrow \dfrac{0.04}{\sigma} = 1.50$, $\Rightarrow \sigma = 0.0267$ (3 s.f.) ***(M1)(A1)***

[4 marks]

(ii) $X \approx \mathrm{N}(6.50, 0.0267^2)$ (allow **ft** from part (i))

$\mathrm{P}(6.48 < X < 6.53) = \mathrm{P}\left(\dfrac{6.48 - 6.50}{0.0267} < Z < \dfrac{6.53 - 6.50}{0.0267}\right)$ ***(M1)***

$= \mathrm{P}(-0.75 < Z < 1.12)$

$= \Phi(0.75) + \Phi(1.12) - 1 \quad = 0.642$ (3 s.f.). ***(M1)(A1)***

Therefore, expected number is $(0.642 \times 1000) = 642$ ***(A1)***

OR

$\mathrm{P}(6.48 < X < 6.53) = 0.642$ ***(M0)(G1)***

Expected number is 642 ***(A1)***

[4 marks]

continued…

Question 7 continued

(ii) (a)

	Billiards	Snooker	Darts	Totals
Male Expected	**32.9**	**16.4**	**13.7**	63
Female Expected	**27.1**	**13.6**	**11.3**	52
	60	30	25	115

(A3)

Note: Award ***(A3)*** for 6 correct expected values (bold), ***(A2)*** for 4 correct, ***(A1)*** for 2 correct.

H_0 : Choice of game is independent of gender ***(A1)***

H_1 : Choice of game is not independent of gender ***(A1)***

Degree of freedom: $(3-1)(2-1)=2$ ***(A1)***

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = \frac{(39-32.9)^2}{32.9} + \ldots\ldots$$ ***(M2)***

$= 7.77$ (3 s.f.). [or 7.79 from GDC] ***(A1)***

But $\chi^2_{5\%}(2) = 5.99$ (from table) ***(M1)***

$\chi^2 = 7.77 > \chi^2_{5\%}(2)$ and we do reject H_0 ***(A1)(R1)***

Hence: Choice of game is dependent on gender. ***(A1)***

[13 marks]

(b) (i) The frequency for males choosing Billiards is less than 5 ***(R1)***

(ii) Snooker – In order to preserve the diversity of games ***(R1)***

OR

Darts – it has the next smallest number of members ***(R1)***

[2 marks]

(c) (i) $\frac{31}{122}$ ***or*** 0.254 (3 s.f.) ***(A1)***

(ii) $\frac{72}{122}$ ***or*** 0.590 (3 s.f.) ***(A1)***

[2 marks]

Total [30 marks]

8. (i) (a) $f'(x) = 3x^2 - 6x + 3$ *(A2)*

[2 marks]

(b)

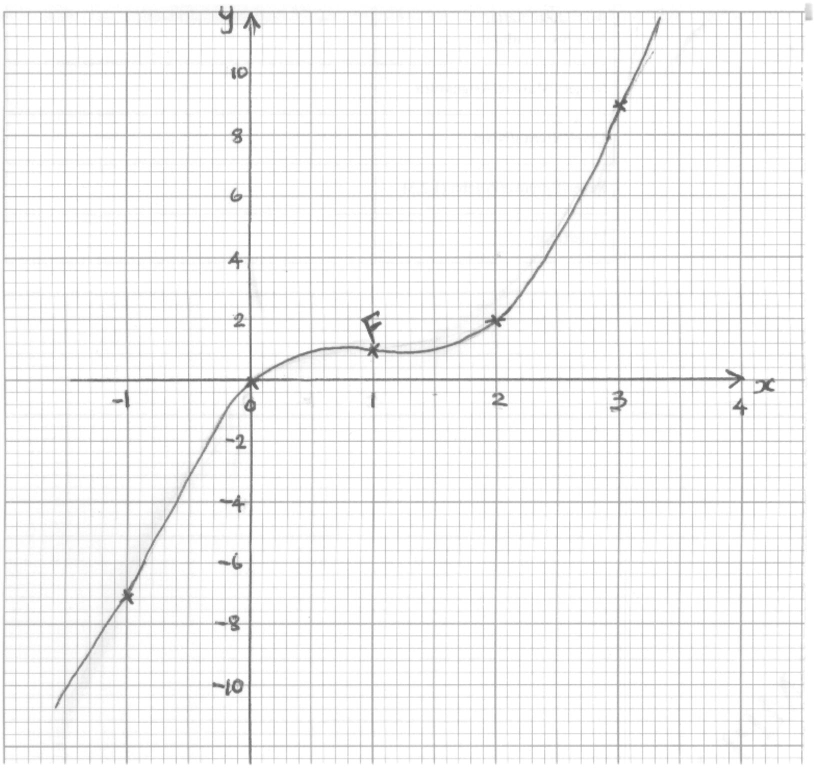
x	–1	0	1	2	3
f(x)	**–7**	0	1	2	9
$f'(x)$	12	**3**	0	**3**	12

(A3)

[3 marks]

(c)

(d)



(A2)

Note: The graph does not have to be on graph paper as long as it is reasonable.

[2 marks]

(d) See graph above *(A1)*

[1 mark]

(e) 12 *(A1)*

[1 mark]

8. (ii) (a) $a = 2, b = 20, c = 9, d = 8, e = 32$ *(A2)*

Note: Award *(A2)* for all 5 correct, *(A1)* for 3 or 4 correct, *(A0)* for 2 or less correct.

[2 marks]

(b) $A = 12x - x^2$ *(C1)*

[1 mark]

(c) $$\frac{\mathrm{d}A}{\mathrm{d}x} = 12 - 2x$$ *(A1)*

A is maximum when $12 - 2x = 0$ *(M1)*

$\Rightarrow$ length $= 6$m and width $= 6$m *(A1)*

OR

length = 6m and width = 6m *(A2)*

[3 marks]

continued…

Question 8 continued

(iii) (a) $H+h=30(T+t)-5(T+t)^2$ ***(A2)***

Note: Award ***(A1)*** for $30(T+t)$, ***(A1)*** for $-5(T+t)^2$

[2 marks]

(b) (i) $h=(30T+30t-5T^2-10Tt-5t^2)-(30T-5T^2)$ ***(M1)***

$h=30t-10Tt-5t^2$ ***(A1)***

(ii) $\lim\limits_{x\to 0}\dfrac{h}{t}=30-10T$ (allow **ft** from part (a)) ***(M1)(A1)***

[4 marks]

(c) (i) $\dfrac{\mathrm{d}H}{\mathrm{d}T}=\lim\limits_{t\to 0}\dfrac{h}{t}$

represents the velocity of the object ***(A1)***

(ii) When $T=6, \dfrac{\mathrm{d}H}{\mathrm{d}T}=30-60=-30$ ***(A1)***

(iii) The negative result means that the object is moving at 30m/s in the opposite direction to which it started, that is, downwards. ***(A1)***

[3 marks]

(d) (i) At maximum height $\dfrac{\mathrm{d}H}{\mathrm{d}T}=0$ ***(M1)***

$\Rightarrow T=3$ (**ft** from previous parts) ***(A1)***

(ii) Maximum height $=30(3)-5(3^2)=90-45$ ***(M1)***

$=$ 45 metres ***(A1)***

[4 marks]

(e) Initial velocity occurs when $T=0$

i.e. when $\dfrac{\mathrm{d}H}{\mathrm{d}T}=30-10(0)$ ***(M1)***

$=30\text{ ms}^{-1}$ ***(A1)***

[2 marks]

Total [30 marks]