N00/520/S(2)M

INTERNATIONAL BACCALAUREATE
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MARKSCHEME

November 2000

MATHEMATICAL METHODS

Standard Level

Paper 2

20 pages

1. (a)

x	15	45	75	105	135	165	195	225
f	5	15	33	21	11	7	5	3

(M1)

$\bar{x} = 97.2$ (exactly) ***(A1)***

[2 marks]

(b)

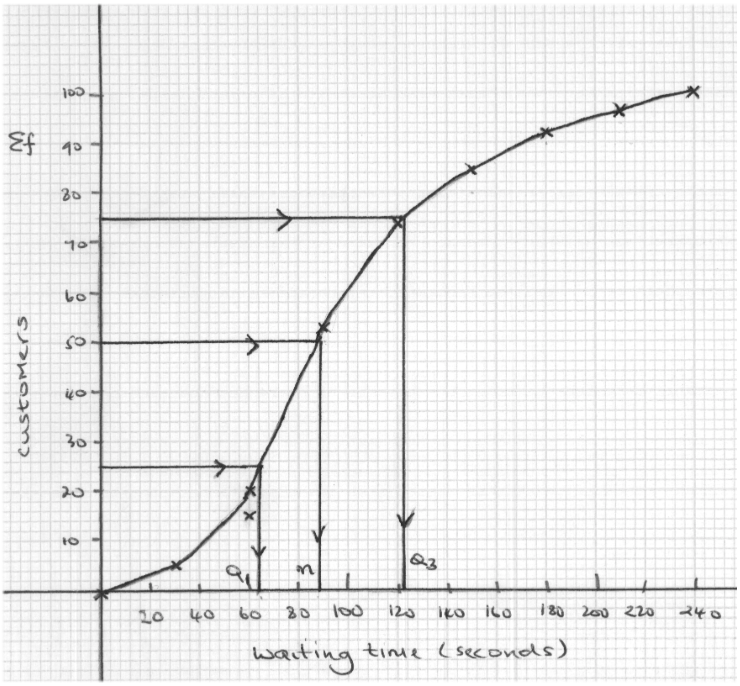
x	30	60	90	120	150	180	210	240
Σf	5	20	53	74	85	92	97	100

(A1)

Note: Award ***(A1)*** for correct values for $x, \Sigma f$.

[1 mark]

(c)



(A4)

Notes: Award ***(A2)*** for 6 or more points correct, ***(A1)*** for 4/5 points correct.
Award ***(A1)*** for a reasonable graph, ***(A1)*** for the correct axes and the given scales.

[4 marks]

(d) Median $= 87 \pm 2$ ***(A1)***

Lower quartile $= 65 \pm 2$ ***(A1)***

Upper quartile $= 123 \pm 2$ ***(A1)***

[3 marks]

Total [10 marks]

2. (a) $t = 2 \Rightarrow$ $h = 50 - 5(2^2) = 50 - 20$

$= 30$ ***(A1)***

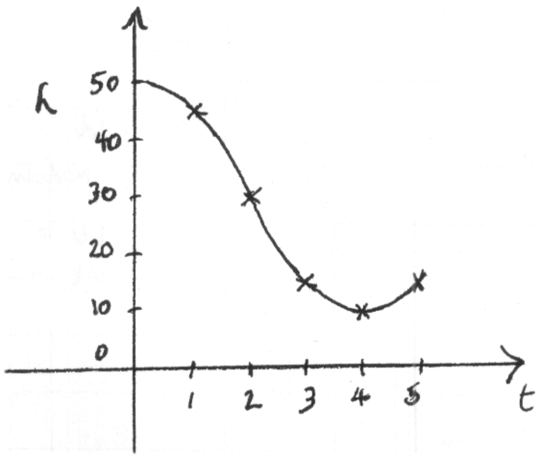
OR

$h = 90 - 40(2) + 5(2^2)$

$= 30$ ***(A1)***

[1 mark]

(b)



(A4)

Note: Award ***(A1)*** for marked scales on each axis, ***(A1)*** for each section of the curve

[4 marks]

(c) (i) $\frac{dh}{dt} = \frac{d}{dt}(50 - 5t^2)$

$= 0 - 10t = -10t$ ***(A1)***

(ii) $\frac{dh}{dt} = \frac{d}{dt}(90 - 40t + 5t^2)$

$= 0 - 40 + 10t = -40 + 10t$ ***(A1)***

[2 marks]

(d) When $t = 2$ (i) $\frac{dh}{dt} = -10(2)$ **or** $\frac{dh}{dt} = -40 + 10 \times 2$ ***(M1)***

$= -20$ $= -20$ ***(A1)***

[2 marks]

(e) $\frac{dh}{dt} = 0 \Rightarrow -10t = 0 (0 \le t \le 2)$ **or** $-40 + 10t = 0 (2 \le t \le 5)$ ***(M1)***

$t = 0$ **or** $t = 4$ ***(A1)(A1)***

[3 marks]

(f) When $t = 4$ ***(M1)***

$h = 90 - 40(4) + 5(4^2)$ ***(M1)***

$= 90 - 160 + 80$

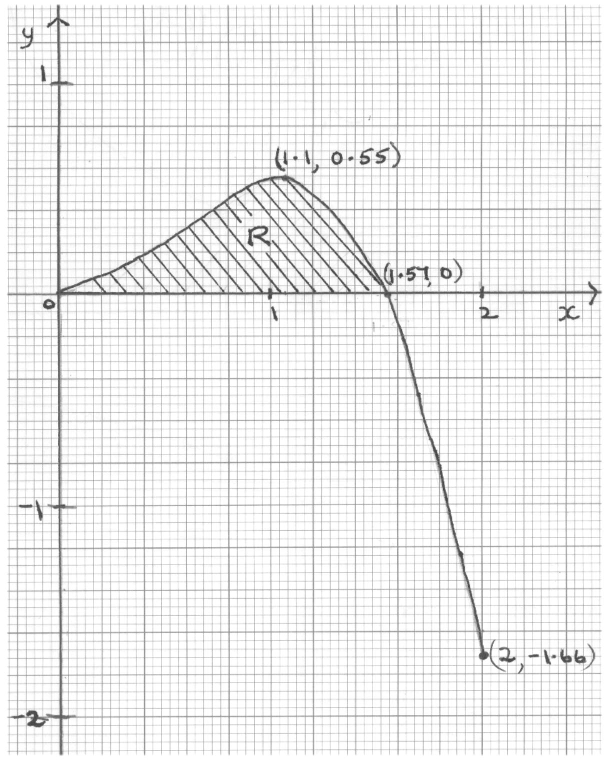
$= 10$ ***(A1)***

[3 marks]

Total [15 marks]

3. (a) (i) ***(A3)***

(c) (i)



Notes: The sketch does **not** need to be on graph paper. It should have the correct shape, and the points (0, 0), (1.1, 0.55), (1.57, 0) and 2, −1.66) should be indicated in some way.
Award ***(A1)*** for the correct shape.
Award ***(A2)*** for 3 or 4 correctly indicated points, ***(A1)*** for 1 or 2 points.

(ii) Approximate positions are
positive x-intercept (1.57, 0) ***(A1)***
maximum point (1.1, 0.55) ***(A1)***
end points (0, 0) and (2, −1.66) ***(A1)(A1)***

[7 marks]

(b) $x^2 \cos x = 0 \quad x \neq 0 \quad \Rightarrow \quad \cos x = 0$ ***(M1)***

$\Rightarrow \quad x = \dfrac{\pi}{2}$ ***(A1)***

Note: Award ***(A2)*** if answer correct.

[2 marks]

(c) (i) see graph ***(A1)***

(ii) $\int_0^{\pi/2} x^2 \cos x \, dx$ ***(A2)***

Note: Award ***(A1)*** for limits, ***(A1)*** for rest of integral correct (do not penalise missing dx).

[3 marks]

continued …

Question 3 continued

(d) Integral $= 0.467$ ***(G3)***

OR

Integral $= \left[x^2 \sin x + 2x \cos x - 2 \sin x \right]_0^{\pi/2}$ ***(M1)***

$= \left[\frac{\pi^2}{4}(1) + 2\left(\frac{\pi}{2}\right)(0) - 2(1) \right] - \left[0 + 0 - 0 \right]$ ***(M1)***

$= \frac{\pi^2}{4} - 2$ (exact) **or** 0.467 (3 s.f.) ***(A1)***

[3 marks]

Total [15 marks]

4. (a) (i) $\boldsymbol{r}_1 = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t\begin{bmatrix} 12 \\ -5 \end{bmatrix}$

$t = 0 \Rightarrow \boldsymbol{r}_1 = \begin{bmatrix} 16 \\ 12 \end{bmatrix}$ ***(M1)***

$\left|\boldsymbol{r}_1\right| = \sqrt{(16^2 + 12^2)} = 20$ ***(A1)***

(ii) Velocity vector $= \begin{bmatrix} 12 \\ -5 \end{bmatrix}$

$\Rightarrow$ speed $= \sqrt{\left(12^2 + (-5)^2\right)}$ ***(M1)***

$= 13$ ***(A1)***

[4 marks]

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t\begin{bmatrix} 12 \\ -5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 \\ 12 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix} \cdot \begin{bmatrix} 16 \\ 12 \end{bmatrix} + \begin{bmatrix} 5 \\ 12 \end{bmatrix} \cdot t\begin{bmatrix} 12 \\ -5 \end{bmatrix}$ ***(M1)***

$\Rightarrow 5x + 12y = 80 + 144$ ***(A1)***

$5x + 12y = 224$ ***(A1)(AG)***

OR

$\dfrac{x-16}{12} = \dfrac{y-12}{-5}$ ***(M1)***

$5x - 80 = 144 - 12y$ ***(A1)***

$\Leftrightarrow 5x + 12y = 224$ ***(A1)(AG)***

OR

$x = 16 + 12t,\ y = 12 - 5t \quad \Rightarrow \quad t = \dfrac{12 - y}{5}$ ***(M1)***

$\Rightarrow x = 16 + 12\left(\dfrac{12 - y}{5}\right)$ ***(A1)***

$\Rightarrow 5x = 80 + 144 - 12y$

$\Rightarrow 5x + 12y = 224$ ***(A1)(AG)***

[3 marks]

(c) $\boldsymbol{v}_1 = \begin{bmatrix} 12 \\ -5 \end{bmatrix} \quad \boldsymbol{v}_2 = \begin{bmatrix} 2.5 \\ 6 \end{bmatrix}$ ***(M1)***

$\boldsymbol{v}_1 \cdot \boldsymbol{v}_2 = \begin{bmatrix} 12 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 2.5 \\ 6 \end{bmatrix}$ ***(M1)***

$= 30 - 30$

$\Rightarrow \boldsymbol{v}_1 \cdot \boldsymbol{v}_2 = 0$ ***(A1)***

$\Rightarrow \theta = 90^\circ$ ***(A1)***

[4 marks]

continued…

Question 4 continued

(d) (i) $\begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 12 \\ -5 \end{bmatrix} = \begin{bmatrix} 23 \\ -5 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ -5 \end{bmatrix}$ ***(M1)***

$\Leftrightarrow 12x - 5y = 23 \times 12 + 25 = 301$ ***(A1)***

OR

$\frac{x-23}{2.5} = \frac{y+5}{6}$

$\Rightarrow 6x - 138 = 2.5y + 12.5$ ***(M1)***

$\Rightarrow 12x - 276 = 5y + 25$

$\Rightarrow 12x - 5y = 301$ ***(A1)***

(ii) $\left.\begin{matrix} 5x + 12y = 224 \\ 12x - 5y = 301 \end{matrix}\right\} \Leftrightarrow \left.\begin{matrix} 25x + 60y = 1120 \\ 144x - 60y = 3612 \end{matrix}\right\}$ ***(M1)***

$169x = 4732$

$x = 28, \; y = (12 \times 28 - 301) \div 5 = 7$

$(28, 7)$ ***(A1)(A1)***

Note:	Accept any correct method for solving simultaneous equations.

[5 marks]

(e) $16 + 12t = 23 + 2.5t \;\Rightarrow\; 9.5t = 7$ ***(M1)***

$12 - 5t = -5 + 6t \;\Rightarrow\; 17 = 11t$ ***(M1)***

$\frac{7}{9.5} \neq \frac{17}{11}$ ***(A1)***

$\Rightarrow$ planes cannot be at the same place at the same time ***(R1)***

OR

$\boldsymbol{r}_1 = \begin{bmatrix} 28 \\ 7 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 28 \\ 7 \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \end{bmatrix} + t \begin{bmatrix} 12 \\ -5 \end{bmatrix}$ ***(M1)***

$\Leftrightarrow \begin{cases} 12t = 12 \\ -5t = -5 \end{cases} \Leftrightarrow t = 1$ ***(A1)***

When $t = 1$ $\quad \boldsymbol{r}_2 = \begin{bmatrix} 23 \\ -5 \end{bmatrix} + \begin{bmatrix} 2.5 \\ 6 \end{bmatrix} = \begin{bmatrix} 25.5 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 28 \\ 7 \end{bmatrix}$ ***(A1)(R1)***

OR

$\boldsymbol{r}_2 = \begin{bmatrix} 28 \\ 7 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 28 \\ 7 \end{bmatrix} = \begin{bmatrix} 23 \\ -5 \end{bmatrix} + t \begin{bmatrix} 2.5 \\ 6 \end{bmatrix}$ ***(M1)***

$\Leftrightarrow t = 2$ ***(A1)***

[4 marks]

Total [20 marks]

5.

Note: A reminder that a candidate is penalised only once in this question for not giving answers to 3 s.f.

(a) $V(5) = 10000 \times (0.933^5) = 7069.8\ldots$

$= 7070$ (3 s.f.) ***(A1)***

[1 mark]

(b) We want t when $V = 5000$ ***(M1)***

$$5000 = 10000 \times (0.933)^t$$

$$0.5 = 0.933^t$$ ***(A1)***

$$\frac{\log(0.5)}{\log(0.933)} = t \quad \left(\text{or } \frac{\ln(0.5)}{\ln(0.933)}\right)$$

$$9.9949 = t$$

After 10 minutes 0 seconds, to nearest second (or 600 seconds). ***(A1)***

[3 marks]

(c) $0.05 = 0.933^t$ ***(M1)***

$\frac{\log(0.05)}{\log(0.933)} = t = 43.197$ minutes ***(M1)(A1)***

$\approx 3/4$ hour ***(AG)***

[3 marks]

(d) (i) $10000 - 10000(0.933)^{0.001} = 0.693$ ***(A1)***

(ii) Initial flow rate $= \frac{\mathrm{d}V}{\mathrm{d}t}$ where $t = 0$, ***(M1)***

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{0.693}{0.001} = 693$

$= 690$ (2 s.f.) ***(A1)***

OR

$\frac{\mathrm{d}V}{\mathrm{d}t} = 690$ ***(G2)***

[3 marks]

Total [10 marks]

6.

Note:	Candidates using tables may get slightly different answers, especially if they do not interpolate. Accept these answers.

(i) (a) $P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\dfrac{50-\mu}{10}\right)$ ***(A1)***

Hence, $\dfrac{50-\mu}{10} = \Phi^{-1}(0.7)$ ***(M1)***

$\mu = 50 - 10\Phi^{-1}(0.7)$ ***(M1)***

$= 44.75599\ldots\ldots = 44.8$ km/h (3 s.f.) (accept 44.7) ***(AG)***

[3 marks]

(b) H_1 : 'the mean speed has been reduced by the campaign'. ***(A1)***

[1 mark]

(c) One-tailed; because H_1 involves only "<". ***(A2)***

[2 marks]

(d) For a one-tailed test at 5% level, critical region is
$Z < \mu_m - 1.64\sigma_m$ (accept $-1.65\sigma_m$) ***(M1)***

Now, $\mu_m = \mu = 44.75\ldots; \sigma_m = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{25}} = 2$ (allow **ft**) ***(A1)***

So test statistic is $44.75\ldots - 1.64 \times 2 = 41.47$ ***(A1)***

Now $41.3 < 41.47$ so reject H_0, yes. ***(A1)***

[4 marks]

(ii) (a) Expected frequencies:

25.5	24.5
25.5	24.5

(A2)

[2 marks]

(b) Observed, Yates corrected.

30.5	19.5
20.5	29.5

(M1)(A1)

[2 marks]

(c) $\chi^2_{calc} = \sum \dfrac{(f_e - f_o)^2}{f_e}$ ***(M1)(M1)***

$= 4.00$ ***(A1)***

Note:	If Yates' correction is not used the answer should be 4.84. Award ***(M1)(A1)*** for 4.84, and apply **ft** to part (d).

[3 marks]

(d) 5% critical value is 3.84 ***(A1)***
$4.00 > 3.84$ ***(A1)***
So, results significant at 5% level. ***(A1)***

[3 marks]

continued…

Question 6 continued

(iii)

x	1	2	3	4
y	55	57	56	59

(a) $\bar{x} = 2.5$, $\bar{y} = 56.75$ ***(G1)***

$s_x = 1.118\ldots$, $s_y = 1.479$

$= 1.12$ (3 s.f.) $= 1.48$ (3 s.f.) ***(G1)***

[2 marks]

(b) $\overline{xy} = \frac{1}{4}(55 + 2\times 57 + 3\times 56 + 4\times 59)$ ***(M1)(A1)***

$= 143.25$ ***(AG)***

OR

$\sum xy = 573$ ***(G1)***

$\Rightarrow \overline{xy} = \frac{573}{4} = 143.25$ ***(G1)***

[2 marks]

(c) $r = \frac{s_{xy}}{s_x s_y} = \frac{(\overline{xy} - \bar{x}\,\bar{y})}{s_x \times s_y}$ ***(M1)***

$= \frac{143.25 - 2.5\times 56.75}{1.118\ldots\times 1.479\ldots}$ ***(A1)***

$= 0.8315\ldots \approx 0.83$ ***(AG)***

[2 marks]

(d) $y - \bar{y} = r\left(\frac{s_y}{s_x}\right)(x - \bar{x})$ ***(M1)***

$y - 56.75 = (0.8315\ldots)\left(\frac{1.479\ldots}{1.118\ldots}\right)(x - 2.5)$ ***(M1)***

$y = 1.1x + 54$ ***(A1)(A1)***

OR

$y = 1.1x + 54$ ***(G4)***

[4 marks]

Total [30 marks]

7. (i) (a) From graph, period $= 2\pi$ ***(A1)***

[1 mark]

(b) Range $= \{y \mid -0.4 < y < 0.4\}$ ***(A1)***

[1 mark]

(c) (i) $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left\{\cos x(\sin x)^2\right\}$

$= \cos x(2\sin x \cos x) - \sin x(\sin x)^2$ **or** $-3\sin^3 x + 2\sin x$ ***(M1)(A1)(A1)***

> **Note:** Award ***(M1)*** for using the product rule and ***(A1)*** for each part.

(ii) $f'(x) = 0$ ***(M1)***

$\Rightarrow \sin x\left\{2\cos^2 x - \sin^2 x\right\} = 0$ **or** $\sin x\left\{3\cos^2 x - 1\right\} = 0$ ***(A1)***

$\Rightarrow 3\cos^2 x - 1 = 0$

$\Rightarrow \cos x = \pm\sqrt{\left(\frac{1}{3}\right)}$ ***(A1)***

At A, $f(x) > 0$, hence $\cos x = \sqrt{\left(\frac{1}{3}\right)}$ ***(R1)(AG)***

(iii) $f(x) = \sqrt{\left(\frac{1}{3}\right)}\left(1 - \left(\sqrt{\left(\frac{1}{3}\right)}\right)^2\right)$ ***(M1)***

$= \frac{2}{3} \times \frac{1}{\sqrt{3}} = \frac{2}{9}\sqrt{3}$ ***(A1)***

[9 marks]

(d) $x = \frac{\pi}{2}$ ***(A1)***

[1 mark]

(e) (i) $\int (\cos x)(\sin x)^2\,\mathrm{d}x = \frac{1}{3}\sin^3 x + c$ ***(M1)(A1)***

(ii) Area $= \int_0^{\pi/2} (\cos x)(\sin x)^2\,\mathrm{d}x = \frac{1}{3}\left\{\left(\sin\frac{\pi}{2}\right)^3 - (\sin 0)^3\right\}$ ***(M1)***

$= \frac{1}{3}$ ***(A1)***

[4 marks]

(f) At C $f''(x) = 0$ ***(M1)***

$\Leftrightarrow 9\cos^3 x - 7\cos x = 0$

$\Leftrightarrow \cos x(9\cos^2 x - 7) = 0$ ***(M1)***

$\Rightarrow x = \frac{\pi}{2}$ (reject) ***or*** $x = \arccos\frac{\sqrt{7}}{3} = 0.491$ (3 s.f.) ***(A1)(A1)***

[4 marks]

continued…

Question 7 continued

(ii) (a)

x	e^{-x}		x^2	
0.5	0.606…	>	0.25	***(M1)***
1	0.367…	<	1	***(A1)***

> **Note:** Award ***(M1)(A1)*** for sketching the graphs of e^{-x} and x^2, or the graph of $e^{-x} - x^2$, for $0.5 \le x \le 1$.

[2 marks]

(b) (i) $x_{n+1} = \sqrt{(e^{-x_n})}$, $x_0 = 0.7$

$x_1 = 0.7046(880897)$

$x_2 = 0.7030(382037)$ ***(A2)***

$x_3 = 0.7036(184094)$

> **Note:** Award ***(A2)*** for three correct, ***(A1)*** for two correct, ***(A0)*** for one correct.

(ii) $x_{10} = 0.7034677\ldots$

$x_{11} = 0.7034673\ldots$ ***(M1)***

$x_{12} = 0.7034674\ldots$

$x = 0.703467$ (6 s.f.) ***(A1)***

> **Note:** If part (i) is correct, award ***(A2)*** for a correct answer without explanation. Award ***(A0)*** if part (i) is incorrect.

[4 marks]

(c) (i) $e^{-x} = x^2$

$\Rightarrow -x = \ln x^2$

$\Rightarrow \quad x = -\ln x^2 = -2\ln x$ ***(A1)***

$x_{n+1} = -\ln(x_n^2) = -2\ln x_n$ ***(A1)***

$x_0 = 0.7$ ***(AG)***

(ii) $x_1 = 0.7133(49887\ldots)$

$x_2 = 0.6755(66505\ldots)$

$x_3 = 0.7844(07346\ldots)$ ***(A1)***

> **Note:** Award ***(A1)*** for all three correct.

(iii) It is divergent. ***(A1)***

[4 marks]

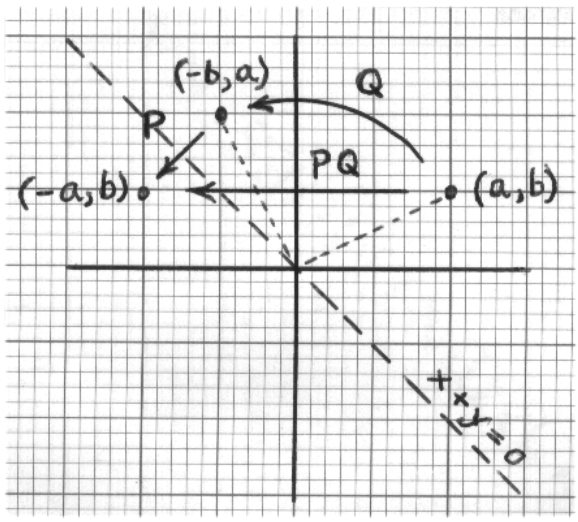
Total [30 marks]

8. (i) **Note:** Geometric or algebraic approaches may be used in parts (a) and (b).

(a) ***PQ*** is a reflection in the y-axis (or the line $x = 0$) ***(A2)***

Note: Award ***(A1)*** for reflection, ***(A1)*** for y-axis.

OR



(M1)

(A1))

PQ is a reflection in the y-axis (or the line $x = 0$)

OR

Q has matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ***P*** has matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ***(M1)***

$\Rightarrow \boldsymbol{PQ} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow$ ***PQ*** is a reflection in the y-axis (or the line $x = 0$) ***(A1)***

[2 marks]

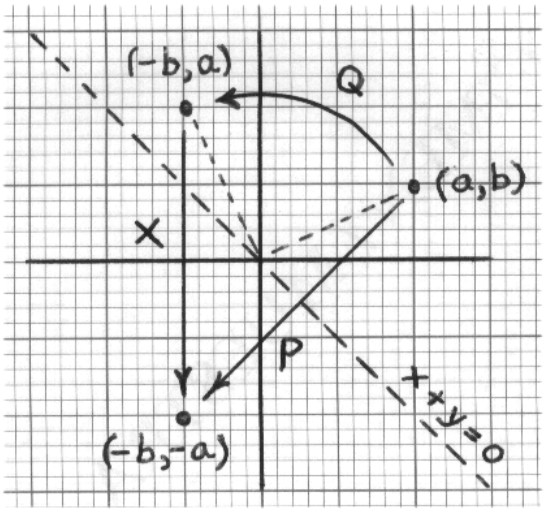
continued…

Question 8(i) continued

(b) $\boldsymbol{X}$ is a reflection in the x-axis (or the line $y = 0$). ***(A3)***

Note: Award ***(A1)*** for reflection, ***(A2)*** for x-axis.

OR



(M2)

$\boldsymbol{X}$ is a reflection in the x-axis (or the line $y = 0$). ***(A1)***

OR

$\boldsymbol{XQ} = \boldsymbol{P} \Rightarrow \boldsymbol{X} = \boldsymbol{PQ}^{-1}$ ***(M1)***

Note: If candidate gives $\boldsymbol{X} = \boldsymbol{Q}^{-1}\boldsymbol{P}$ award ***(M0)*** but use **ft**.

$\boldsymbol{Q}^{-1}$ is rotation by -90° about (0, 0) and has matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\Rightarrow \boldsymbol{X} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ***(A1)***

$\boldsymbol{X}$ is a reflection in the x-axis (or the line $y = 0$). ***(A1)***

[3 marks]

continued…

Question 8 continued

(ii) (a) $\vec{OB} = \vec{OA} + \vec{OC} = \begin{pmatrix}12\\9\end{pmatrix} + \begin{pmatrix}-10\\5\end{pmatrix} = \begin{pmatrix}2\\14\end{pmatrix}$ ***(A1)***

[1 mark]

(b) (i) (12, 9) is on the line $4y - 3x = 0$

$\Leftrightarrow 4(9) - 3(12) = 0 \Leftrightarrow 0 = 0$ ***(A1)***

(ii) $\vec{CC'} = \begin{pmatrix}-6-(-10)\\8-5\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$ ***(A1)***

Vector in direction of $4y - 3x = 0$ is $\vec{OA} = \begin{pmatrix}12\\9\end{pmatrix}$

$\vec{CC'}$ is parallel to invariant line. ***(R1)***

OR

Gradient of $(CC') = \dfrac{8-5}{-6-(-10)} = \dfrac{3}{4}$ ***(A1)***

Invariant line: $y = \dfrac{3}{4}x$ has gradient $\dfrac{3}{4}$

$\vec{CC'}$ is parallel to invariant line. ***(R1)***

[3 marks]

(c) (i) Since $\boldsymbol{M}\begin{pmatrix}12\\9\end{pmatrix} = \begin{pmatrix}12\\9\end{pmatrix}$ and $\boldsymbol{M}\begin{pmatrix}-10\\5\end{pmatrix} = \begin{pmatrix}-6\\8\end{pmatrix}$

Then $\boldsymbol{M}\begin{pmatrix}12 & -10\\9 & 5\end{pmatrix} = \begin{pmatrix}12 & -6\\9 & 8\end{pmatrix}$ ***(R1)***

(ii) $\boldsymbol{M} = \begin{pmatrix}12 & -6\\9 & 8\end{pmatrix}\begin{pmatrix}12 & -10\\9 & 5\end{pmatrix}^{-1}$ ***(M1)***

$= \begin{pmatrix}12 & -6\\9 & 8\end{pmatrix}\dfrac{1}{150}\begin{pmatrix}5 & 10\\-9 & 12\end{pmatrix}$ ***(A1)(A1)***

$= \dfrac{1}{150}\begin{pmatrix}114 & 48\\-27 & 186\end{pmatrix} = \begin{pmatrix}0.76 & 0.32\\-0.18 & 1.24\end{pmatrix}$ ***(A1)***

(iii) A shear transformation preserves area ***(R1)***

and $|\det \boldsymbol{M}|$ is the area scale-factor. Hence $\det \boldsymbol{M} = \pm 1$ ***(R1)***

In this case $\det \boldsymbol{M} = 0.76\,(1.24) - 0.32\,(-0.18) = {}^{+}1$ ***(A1)***

[8 marks]

(d) Area of parallelogram OABC = area of rectangle OAB′C′ ***(M1)***

Area $= \left|\vec{OA}\right|\left|\vec{OC'}\right| = \sqrt{9^2 + 12^2}\,\sqrt{(-6)^2 + 8^2} = 15\,(10)$ ***(M1)***

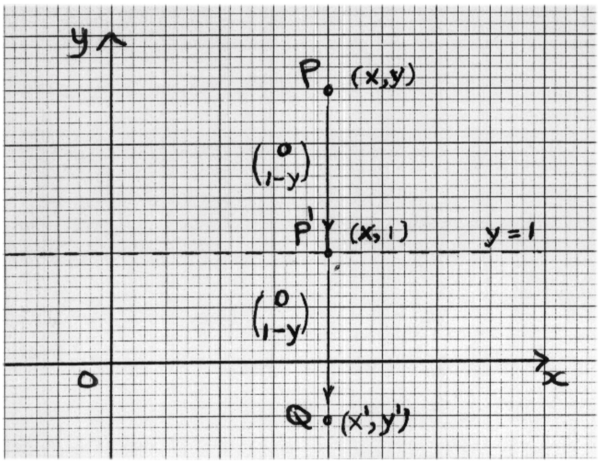
$= 150$ ***(A1)***

[3 marks]

continued…

Question 8 continued

(iii) (a)



(M1)

Now $P \mapsto Q$ where $\vec{OQ} = \vec{OP} + 2\vec{PP'}$, P′ projection of P on $y = 1$.

So $\boldsymbol{S}: \begin{bmatrix}x\\y\end{bmatrix} \mapsto \begin{bmatrix}x\\y\end{bmatrix} + 2\left\{\begin{bmatrix}x\\1\end{bmatrix} - \begin{bmatrix}x\\y\end{bmatrix}\right\}$ ***(M1)(A1)***

$= \begin{bmatrix}x\\y\end{bmatrix} + 2\begin{bmatrix}0\\1-y\end{bmatrix}$ ***(A1)***

$= \begin{bmatrix}x\\2-y\end{bmatrix}$ ***(AG)***

[4 marks]

(b) $\boldsymbol{SR}: \begin{bmatrix}x\\y\end{bmatrix} \mapsto \boldsymbol{S}\left(\boldsymbol{R}\begin{pmatrix}x\\y\end{pmatrix}\right)$

$= \boldsymbol{S}\begin{bmatrix}6-y\\6-x\end{bmatrix}$ ***(A1)***

$= \begin{bmatrix}6-y\\2-(6-x)\end{bmatrix}$ ***(A1)***

$= \begin{bmatrix}6-y\\x-4\end{bmatrix}$ ***(AG)***

[2 marks]

(c) $\begin{bmatrix}x_0\\y_0\end{bmatrix} \mapsto \begin{bmatrix}6-y_0\\x_0-4\end{bmatrix} = \begin{bmatrix}x_0\\y_0\end{bmatrix} \Leftrightarrow \begin{cases}6-y_0 = x_0\\x_0-4 = y_0\end{cases}$ ***(A1)***

$\Leftrightarrow \begin{cases}x_0+y_0 = 6\\x_0-y_0 = 4\end{cases}$

$\Leftrightarrow x_0 = 5,\ y_0 = 1$ ***(A1)***

[2 marks]

(d) Rotation of $^{+}90^{\circ}$ ***(A1)***
about (5, 1) ***(A1)***

[2 marks]

Total [30 marks]