INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

MARKSCHEME

May 2000

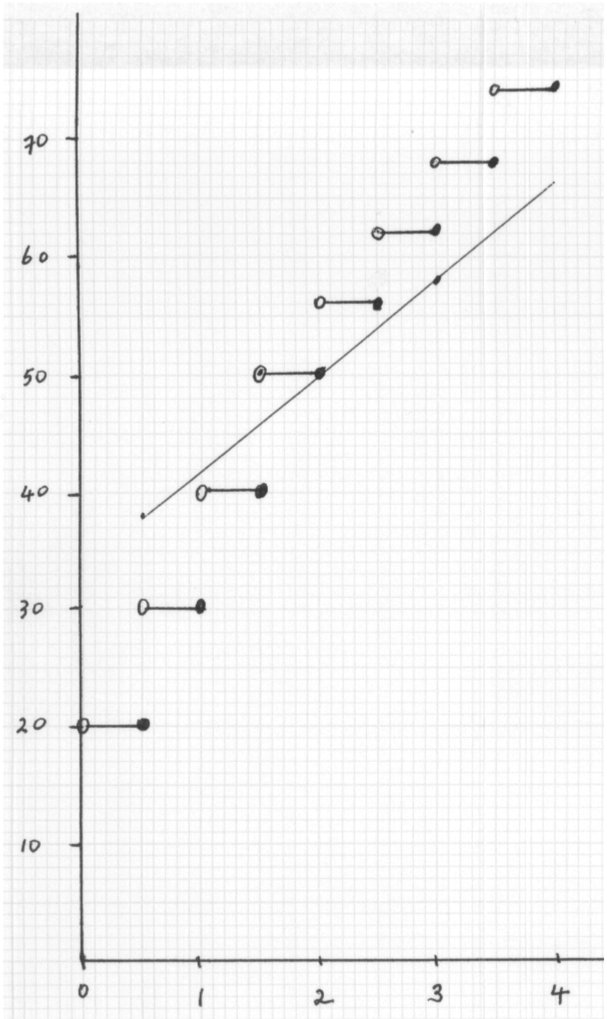
MATHEMATICAL STUDIES

Standard Level

Paper 2

1. (a) *(A5)*

(b) (ii) *(A2)*



Cost in Australian dollars

Weight (kilogrammes)

Notes: (a)	Award ***(A1)*** for both axes labelled, ***(A1)*** for correct scales, ***(A1)*** for open circles on the left of the line segments, and closed on the right (or equivalent); Award ***(A2)*** for 7 or 8 correct line segments. ***(A1)*** for 5 or 6 correct, ***(A0)*** for 4 or less correct.

[5 marks]

(b) (i)

W	0.5	1	2	3	4
C	**38**	42	**50**	**58**	**66**

(A2)

Note: Award ***[½ mark]*** for each correct bold entry and round down

(ii) See graph. *(A2)*

Note: Award ***(A1)*** for points plotted correctly, ***(A1)*** for the straight line joining the points.

[4 marks]

continued…

Question 1 continued

(c) (i) 2 kilograms. ***(A1)***

(ii) $ 68 – $ 62 ***(M1)***
= $ 6 ***(A1)***
[3 marks]

(d)

SPEEDY COURIER		*IMMEDIATE COURIER*		Total cost
W	*C*	*W*	*C*	
1.5	46	4	74	120
2	50	3.5	68	118
2.5	54	3.0	62	116
3	58	2.5	56	114
3.5	62	2.0	50	112
4	66	1.5	40	106

(M2)

Therefore he sent 4 kilograms with *SPEEDY COURIER* and 1.5 kilograms with *IMMEDIATE COURIER*. ***(A1)***
[3 marks]

Total [15 marks]

2. (a) Mean $= \dfrac{5\times 0+10\times 1+6\times 2+3\times 3+1\times 4}{25} = 1.4$ ***(M2)(AG)***

> **Note:** Award ***(M1)*** for the numerator and ***(M1)*** for the denominator.

[2 marks]

(b) $\sum f(x-\bar{x})^2 = 5(0-1.4)^2 + 10(1-1.4)^2 + 6(2-1.4)^2 + 3(3-1.4)^2 + 1(4-1.4)^2 = 28$ ***(M2)***

> **Note:** Award ***(M1)*** for $(x-\bar{x})^2$ values, and ***(M1)*** for multiplying by the appropriate frequencies.

S.D. $= \sqrt{\dfrac{28}{25}}$ ***(M1)***

$= 1.06$ ***(AG)***

[3 marks]

(c) Award ***(R1)*** for each acceptable reason, *e.g.*
Group 2 has more children in total.
Group 2 has a larger number of children per female.
Group 2 generally have larger families. ***(R2)***

[2 marks]

(d) $\mathrm{P}(>2 \text{ children}) = \dfrac{3+1}{25}$ ***(M1)***

$= \dfrac{4}{25}$ ***(A1)***

[2 marks]

(e) (i) $\mathrm{P}(\text{both females have} > 2 \text{ children}) = \dfrac{4}{25}\times\dfrac{3}{24}$ ***(M1)***

$= \dfrac{12}{600}$ or $\dfrac{1}{50}$ or 0.02 ***(A1)***

(ii) $\mathrm{P}(\text{only 1 female has} > 2 \text{ children}) = 2\times\dfrac{4}{25}\times\dfrac{21}{24}$ ***(M2)***

> **Note:** Award ***(M1)*** for $\dfrac{4}{25}\times\dfrac{21}{24}$, ***(M1)*** for multiplying by 2.

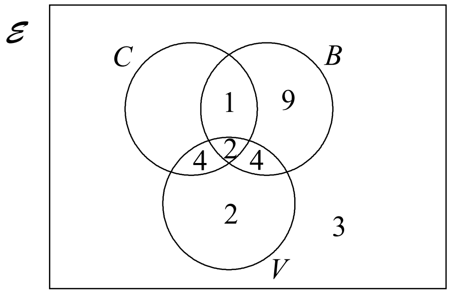
$= \dfrac{168}{600}$ or $\dfrac{21}{75}$ or 0.28 ***(A1)***

(iii) $\mathrm{P}(\text{second has 2 children} \mid \text{first has 0}) = \dfrac{6}{24}$ or $\dfrac{1}{4}$ or 0.25 ***(A1)***

[6 marks]

Total [15 marks]

3. (i) (a) **(A1)**
(b) **(A3)**



> **Notes:** (a) Award ***(A1)*** for a rectangle containing 3 intersecting circles.
> (b) Award ***(A3)*** for 6 or 7 correct numbers in the regions.
> ***(A2)*** for 4 or 5 correct, ***(A1)*** for 2 or 3 correct.

[4 marks]

(c) $1+9+4+2+4+2+3=25$ ***(M1)***

$n(C) = 30-25$

$= 5$ ***(A1)***

OR

$n(C) = 5$ ***(C2)***

[2 marks]

(ii) (a) (i) $p \Rightarrow q$ ***(A1)***

(ii) If Matthew doesn't cook dinner then Jill will not wash the dishes. ***(A1)***

[2 marks]

(b) (i)

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$\neg r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r$	$\neg p$	$[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge \neg r] \Rightarrow \neg p$
T	T	T	**T**	**T**	**F**	**F**	**F**	T
T	T	F	**T**	**F**	**T**	**F**	**F**	T
T	F	T	**F**	**T**	**F**	**F**	**F**	T
T	F	F	**F**	**T**	**T**	**F**	**F**	T
F	T	T	**T**	**T**	**F**	**F**	**T**	T
F	T	F	**T**	**F**	**T**	**F**	**T**	T
F	F	T	**T**	**T**	**F**	**F**	**T**	T
F	F	F	**T**	**T**	**T**	**T**	**T**	T

(A5)

> **Note:** Award ***(A1)*** for each correct bold column.

[5 marks]

(ii) The truth table is showing that the following argument is valid.
If Matthew arrives home before six o'clock he will cook dinner. If Matthew cooks dinner then Jill will wash the dishes. Jill did not wash the dishes. Therefore Matthew did not arrive before six o'clock. ***(R2)***

[2 marks]

Total [15 marks]

4. (a) A ; $y=0,\ 3x=24 \Rightarrow x=8$

A$(8, 0)$ ***(A1)***

B ; $x=0,\ 4y=24 \Rightarrow y=6$

B$(0, 6)$ ***(A1)***

[2 marks]

(b) M ; $x_m = \frac{8+0}{2} = 4,\ y_m = \frac{0+6}{2} = 3$

M$(4, 3)$ ***(A1)(A1)***

[2 marks]

(c) L_2: gradient $= \frac{3--2}{4-0} = \frac{5}{4}$ ***(A1)***

$y = \frac{5}{4}x - 2$ (or equivalent) ***(A1)***

[2 marks]

(d) (i) M$(4, 3)$, C$(0, -2)$

$\text{MC} = \sqrt{(4-0)^2 + (3-(-2))^2}$ ***(M1)***

$= \sqrt{41}$

$= 6.40$ ***(A1)***

(ii) A$(8, 0)$, C$(0, -2)$

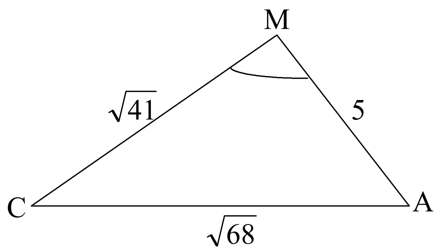
$\text{AC} = \sqrt{8^2 + (-2)^2}$ ***(M1)***

$= \sqrt{68}$

$= 8.25$ ***(A1)***

[4 marks]

(e) (i)



$\cos \text{M} = \frac{5^2 + (\sqrt{41})^2 - (\sqrt{68})^2}{2 \times 5\sqrt{41}}$ ***(M1)***

$= \frac{25+41-68}{10\sqrt{41}}$ ***(M1)***

$\text{C}\hat{\text{M}}\text{A} = 91.8^\circ$ (3 s.f.) ***(A1)***

(ii) Area of ΔCMA $= \frac{1}{2}\sqrt{41} \times 5 \sin 91.8^\circ$ ***(M1)***

$= 15.99991171\ldots$

$= 16.0$ (3 s.f.) ***(A1)***

[5 marks]

Total [15 marks]

5. (a) $A = C\left(1 + \frac{r}{100}\right)^n$

$= 1000\left(1 + \frac{5}{100}\right)^5$ ***(M1)***

$= \$\,1276.28$ ***(A1)***

[2 marks]

(b) $2000 = 1000\left(1 + \frac{5}{100}\right)^n$ ***(M1)***

$2 = 1.05^n$ ***(M1)***

n	1.05^n
10	$1.05^{10} = 1.6$
20	$1.05^{20} = 2.7$
15	$1.05^{15} = 2.07$
14	$1.05^{14} = 1.98$

(M1)

$n = 15$ years ***(A1)***

[4 marks]

(c)

Year	Deposit	Start year balance	End of year balance
1	\$ 1000	\$ 1000	\$ 1050
2	\$ 1000	\$ 2050	\$ 2152.50
3	\$ 1000	\$ 3152.50	\$ 3310.125
4	\$ 1000	\$ 4310.125	\$ 4525.63125
5	\$ 1000	\$ 5525.63125	\$ 5801.91

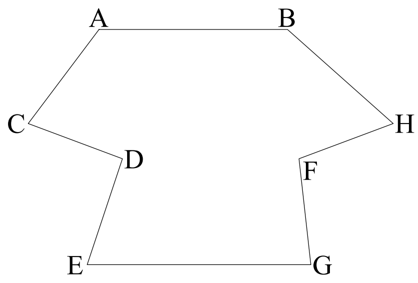
(M3)

After 5 years \$ 5801.91 ***(A1)***

[4 marks]

Total [10 marks]

6. (i) (a) (i)



or H B A C D E G F H

or $H \rightarrow F \rightarrow G \rightarrow E \rightarrow D \rightarrow C \rightarrow A \rightarrow B \rightarrow H$ ***(M3)***

> **Note:** Award ***(M1)*** if no vertex is revisited; ***(M1)*** for excluding CE; ***(M1)*** for including EG.

(ii) Minimum distance $= 10 + 55 + 30 + 25 + 30 + 20 + 10 + 15$ ***(M1)***

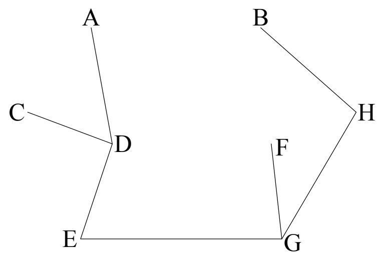
$= 195$ (follow through from candidate's diagram) ***(A1)***

OR

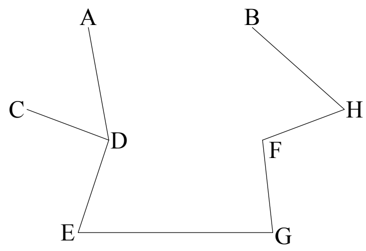
minimum distance $= 195$ ***(C2)***

[5 marks]

(b) (i)



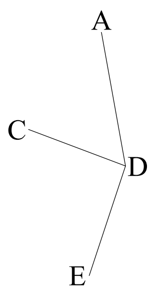
OR



(M3)

> **Note:** Award ***(M1)*** for each of the following parts of the diagram.

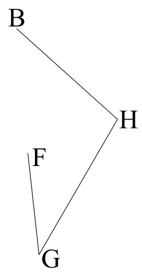
Note: Award ***(M1)*** for



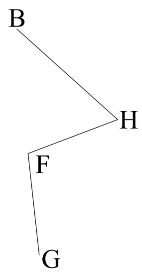
, ***(M1)*** for

,

(M1) for



or



(ii) Minimum length $= 20 + 25 + 30 + 20 + 10 + 15 + 10$ ***(M1)***

$= 130$ m ***(A1)***

OR

minimum length $= 130$ m ***(C2)***

[5 marks]

continued…

Question 6 continued

(ii) (a) No. ***(A1)***
[1 mark]

(b) 2 ***(A1)***
[1 mark]

(c) $M^2 = \begin{pmatrix} 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

(i) $a = 0 \times 0 + 2 \times 1 + 0 \times 0 + 1 \times 0$ ***(M1)***
$= 2$ ***(A1)***

OR

$a = 2$ ***(C2)***

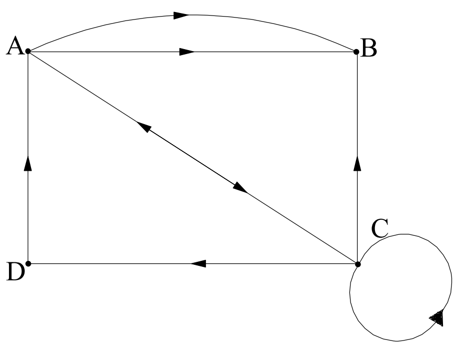
(ii) $b = 0 \times 1 + 1 \times 0 + 0 \times 0 + 0 \times 0$ ***(M1)***
$= 0$ ***(A1)***

OR

$b = 0$ ***(C2)***
[4 marks]

(d) Yes ***(A1)***
[1 mark]

(e)



(A4)

Note: Award ***[½ mark]*** for each directed segment and round down.

[4 marks]

Question 6 continued

(iii) (a) (i)

	employed	unemployed
employed	0.9	**0.15**
unemployed	**0.1**	**0.85**

(A1)
(A1)(A1)

(ii) $\begin{pmatrix} 0.9 & 0.15 \\ 0.1 & 0.85 \end{pmatrix} \begin{pmatrix} 7500 \\ 2500 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 7125 \\ 2875 \end{pmatrix}$

Therefore 2875 people will be unemployed next year. ***(A1)***

OR

2875 people will be unemployed next year. ***(C2)***

[5 marks]

(b) (i) $\begin{pmatrix} 0.825 & 0.2625 \\ 0.175 & 0.7375 \end{pmatrix} \begin{pmatrix} 7500 \\ 2500 \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}$

$m = 0.825 \times 7500 + 0.2625 \times 2500$ ***(M1)***
$= 6843.75$
$= 6844$ (4 s.f.) ***(A1)***

OR

$m = 6844$ (4 s.f.) ***(C2)***

(ii) The **following year** there will be 6844 people **employed**. ***(A2)***

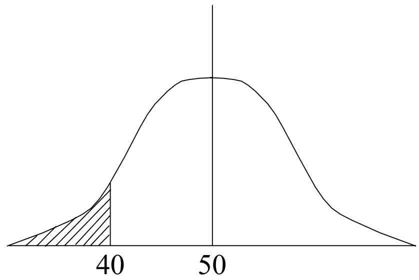
[4 marks]

Total [30 marks]

7. (i) (a)

Note: Candidates may obtain these answers directly from a graphic display calculator. In all three parts of (a), if candidates have obtained their answers in different ways, award marks as follows:
Award ***(A1)*** for correct answers with no working;
(M1)(A1) for correct answers with some indication of correct use of a GDC, *e.g.* a diagram.

(i)



$$P(D < 40) = P\left(Z < \frac{40-50}{4}\right)$$ ***(M1)***

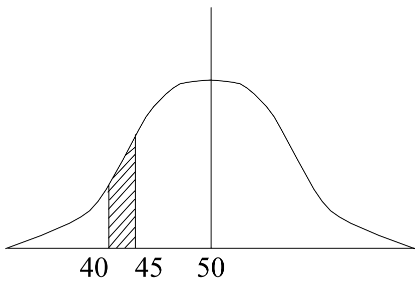
$$= P(Z < -2.5)$$

$$\Phi(2.5) = 0.9938$$ ***(M1)***

$$P(D < 40) = 1 - 0.9938$$
$$= 0.0062 \;\; (0.00621 \;\; 3 \text{ s.f.})$$ ***(A1)***

[3 marks]

(ii)



$$P(D < 45) = P\left(Z < \frac{45-50}{4}\right)$$ ***(M1)***

$$= P(Z < -1.25)$$

$$\Phi(1.25) = 0.8944$$ ***(M1)***

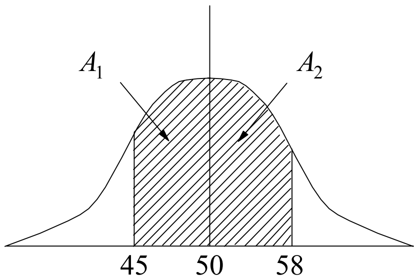
$$P(40 < D < 45) = 0.9938 - 0.8944$$
$$= 0.0994$$ ***(A1)***

[3 marks]

continued...

Question 7 (i) (a) continued

(iii)



$$P(D < 58) = P\left(Z < \frac{58-50}{4}\right)$$
$$= P(Z < 2)$$

$\Phi(2) = 0.9773$

$A_1 = 0.8944 - 0.5$
$= 0.3944$ ***(M1)***

$A_2 = 0.9773 - 0.5$
$= 0.4773$ ***(M1)***

$P(45 < D < 58) = 0.3944 + 0.4773$
$= 0.8717$ or 0.872 (3 s.f.) ***(A1)***

[3 marks]

Question 7 (i) continued

(b) (i) $H_0: \mu = 50$ and $\sigma = 4$
or
The machine is operating to its required level of performance.
or
The machine does not need adjusting. ***(A1)***

$H_1: \mu \neq 50$ and / or $\mu \neq 4$
or
The machine is not operating to its required level of performance.
or
The machine needs to be adjusted. ***(A1)***
[2 marks]

(ii) $$\chi^2_{\text{calc}} = \frac{(16-11)^2}{16} + \frac{(34-29)^2}{34} + \frac{(34-33)^2}{34} + \frac{(14-18)^2}{14} + \frac{(2-9)^2}{2}$$ ***(M2)***

Note: Award ***(M1)*** for numerators and ***(M1)*** for denominators.

$$= \frac{25}{16} + \frac{25}{34} + \frac{1}{34} + \frac{16}{14} + \frac{49}{2}$$ ***(M1)***

$= 27.97$

$= 28.0$ (3 s.f.) ***(A1)***

OR

$$\chi^2_{\text{calc}} = \frac{(16-11)^2}{16} + \frac{(34-29)^2}{34} + \frac{(34-33)^2}{34} + \frac{(16-27)^2}{16}$$ ***(M2)***

$= 9.8897$ ***(M1)***

$= 9.89$ (3 s.f.) ***(A1)***
[4 marks]

(iii) Degrees of freedom $= 4$ ***(M1)***
Critical value $= 9.488$ ***(M1)***

Therefore since $\chi^2_{\text{calc}} >$ critical value, the machine needs adjusting. ***(R1)***
[3 marks]

continued…

Question 7 continued

(ii) (a) (i) $S_x = 11.2$ ***(A1)***

$$r = \frac{36.7}{11.2 \times 3.5}$$ ***(M2)***

$= 0.936$ (3 s.f.) ***(A1)***

OR

$S_x = 11.6$ ***(A1)***

$$r = \frac{36.7}{11.6 \times 3.5}$$ ***(M2)***

$= 0.904$ (3 s.f.) ***(A1)***

(ii) The correlation coefficient suggests a strong positive correlation between the two variables. ***(R1)***

[5 marks]

(b) $$y - \bar{y} = \frac{Sxy}{(Sx)^2}(x - \bar{x})$$

$$y - 10.6 = \frac{36.7}{11.2^2}(x - 30.4)$$ ***(M1)***

$y = 0.293x + 1.69$ (or $y = 0.293x + 1.71$) (allow ft from (a) (i)) ***(A2)***

[3 marks]

(c) (i) $y = 0.293 \times 33 + 1.69$ ***(M1)***
$= 11.359$
$= 11$ hours ***(A1)***

(ii) $8 = 0.293x + 1.69$ ***(M1)***
$x = 21.54$
$= 22$ years ***(A1)***

[4 marks]

Total [30 marks]

8. (i) (a) (i) B: local maximum. ***(A1)***

(ii) D: point of inflexion. ***(A1)***

[2 marks]

(b) At B, the gradient is zero.
From B to C, the gradient is negative.
At C, the gradient is zero.
From C to D, the gradient is positive.
At D, the gradient is zero. ***(A3)***

Note: Award ***[½ mark]*** for each correct statement and round up.

[3 marks]

(c) $\text{Gradient} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{f(a+4) - f(a)}{(a+4) - (a)}$ ***(M2)***

Note: Award ***(M1)*** for $f(a+4)$

$= \dfrac{f(a+4) - f(a)}{4}$ ***(A1)***

[3 marks]

(ii) (a) $k = 4^3 - 3 \times 4^2 + 3 \times 4 + 4$ ***(M1)***
$= 32$ kilometres. ***(A1)***

[2 marks]

(b) (i) $k = t^3 - 3t^2 + 3t + 4$
$\dfrac{\mathrm{d}k}{\mathrm{d}t} = 3t^2 - 6t + 3$ ***(A2)***

Note: Award ***(A2)*** for all terms correct, ***(A1)*** for 2 correct, ***(A0)*** for 1 or none correct.

[2 marks]

(ii) When $t = 3$, $\dfrac{\mathrm{d}k}{\mathrm{d}t} = 3 \times 3^2 - 6 \times 3 + 3$ ***(M1)***
$= 12\ \mathrm{km\,h^{-1}}$ ***(A1)***

[2 marks]

continued…

Question 8 (ii) (b) continued

(iii) $\frac{dk}{dt} = 0$

$0 = 3t^2 - 6t + 3$ ***(M1)***

$0 = t^2 - 2t + 1$

$0 = (t-1)^2$ ***(M1)***

Therefore $t = 1$ hour ***(A1)***

[3 marks]

(iv) When $t = 1$, $k = 1^3 - 3 \times 1^2 + 3 \times 1 + 4$ ***(M1)***

$= 5$ kilometres. ***(A1)***

[2 marks]

(iii) (a) $2x + y$ ***(A1)***

[1 mark]

(b) $2500 = 2x + y$ ***(M1)***

$2500 - 2x = y$ ***(AG)***

[1 mark]

(c) (i) Area $A(x) = xy$ ***(M1)***

$= x(2500 - 2x)$ ***(M1)***

$= 2500x - 2x^2$ ***(AG)***

[2 marks]

(ii) $A'(x) = 2500 - 4x$ ***(A1)***

[1 mark]

(iii) $A'(x) = 0$

$0 = 2500 - 4x$ ***(M1)***

$4x = 2500$ ***(M1)***

$x = 625$ ***(A1)***

[3 marks]

(iv) $A(x) = 2500x - 2x^2$

$A(625) = 2500 \times 625 - 2(625)^2$ ***(M2)***

$= 781250$

$= 781000 \text{ m}^2$ ***(A1)***

[3 marks]

Total [30 marks]