INTERNATIONAL BACCALAUREATE
BACCALAURÉAT INTERNATIONAL
BACHILLERATO INTERNACIONAL

M00/520/S(2)

MATHEMATICAL METHODS
STANDARD LEVEL
PAPER 2

Thursday 4 May 2000 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all five questions from Section A and one question from Section B.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures as appropriate.

- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-7400G*, Sharp EL-9400, Texas Instruments TI-80.

You are advised to start each new question on a new page. A correct answer with **no** *indication of the method used will usually receive* **no** *marks. You are therefore advised to show your working. In particular, where graphs from a graphic display calculator are being used to find solutions, you should sketch these graphs as part of your answer.*

SECTION A

Answer all **five** *questions from this section.*

1. *[Maximum mark: 10]*

The *Acme* insurance company sells two savings plans, Plan A and Plan B.

For Plan A, an investor starts with an initial deposit of \$1000 and increases this by \$80 each month, so that in the second month, the deposit is \$1080, the next month it is \$1160 and so on.

For Plan B, the investor again starts with \$1000 and each month deposits 6% more than the previous month.

(a) Write down the amount of money invested under Plan B in the second and third months. *[2 marks]*

Give your answers to parts (b) and (c) correct to the nearest dollar.

(b) Find the amount of the 12th deposit for each Plan. *[4 marks]*

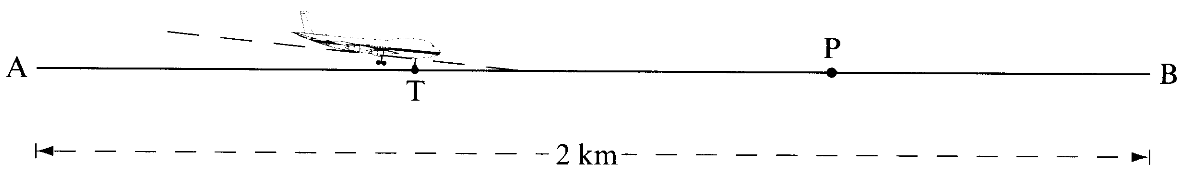
(c) Find the total amount of money invested during the first 12 months

(i) under Plan A; *[2 marks]*

(ii) under Plan B. *[2 marks]*

2. *[Maximum mark: 15]*

The main runway at *Concordville* airport is 2 km long. An aeroplane, landing at *Concordville*, touches down at point T, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point P on the runway.



Not to scale

As the aeroplane slows down, its distance, s, from A, is given by

$$s = c + 100t - 4t^2,$$

where t is the time in seconds after touchdown, and c metres is the distance of T from A.

(a) The aeroplane touches down 800 m from A, (*i.e.* $c = 800$).

(i) Find the distance travelled by the aeroplane in the first 5 seconds after touchdown. *[2 marks]*

(ii) Write down an expression for the velocity of the aeroplane at time t seconds after touchdown, and hence find the velocity after 5 seconds. *[3 marks]*

The aeroplane passes the marker at P with a velocity of 36 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker; *[2 marks]*

(iv) the distance from P to A. *[3 marks]*

(b) Show that if the aeroplane touches down before reaching the point P, it can stop before reaching the northern end, B, of the runway. *[5 marks]*

3. *[Maximum mark: 15]*

A supermarket records the amount of money d spent by customers in their store during a busy period. The results are as follows:

Money in \$ (d)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Number of customers (n)	24	16	22	40	18	10	4

(a) Find an estimate for the mean amount of money spent by the customers, giving your answer to the nearest dollar (\$). *[2 marks]*

(b) Copy and complete the following cumulative frequency table and use it to draw a cumulative frequency graph. Use a scale of 2 cm to represent \$ 20 on the horizontal axis, and 2 cm to represent 20 customers on the vertical axis. *[5 marks]*

Money in \$ (d)	< 20	< 40	< 60	< 80	< 100	< 120	< 140
Number of customers (n)	24	40					

(c) The time t (minutes), spent by customers in the store may be represented by the equation

$$t = 2d^{\frac{2}{3}} + 3.$$

(i) Use this equation and your answer to part (a) to estimate the mean time in minutes spent by customers in the store. *[3 marks]*

(ii) Use the equation and the cumulative frequency graph to estimate the number of customers who spent more than 37 minutes in the store. *[5 marks]*

4. *[Maximum mark: 10]*

(a) Sketch the graph of $y = \pi \sin x - x$, $-3 \le x \le 3$, on millimetre square paper, using a scale of 2 cm per unit on each axis.

Label and number both axes and indicate clearly the approximate positions of the x-intercepts and the local maximum and minimum points. *[5 marks]*

(b) Find the solution of the equation

$$\pi \sin x - x = 0, \qquad x > 0.$$

[1 mark]

(c) Find the indefinite integral

$$\int (\pi \sin x - x)\mathrm{d}x$$

and hence, or otherwise, calculate the area of the region enclosed by the graph, the x-axis and the line $x = 1$. *[4 marks]*

5. *[Maximum mark: 20]*

In this question, the vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ km a displacement due north.

Two crews of workers are laying an underground cable in a north-south direction across a desert. At 06:00 each crew sets out from their base camp which is situated at the origin $(0, 0)$. One crew is in a Toyundai vehicle and the other in a Chryssault vehicle.

The Toyundai has velocity vector $\begin{pmatrix} 18 \\ 24 \end{pmatrix}$ km h^{-1}, and the Chryssault has velocity vector $\begin{pmatrix} 36 \\ -16 \end{pmatrix}$ km h^{-1}.

(a) Find the speed of each vehicle. *[2 marks]*

(b) (i) Find the position vectors of each vehicle at 06:30. *[2 marks]*

(ii) Hence, or otherwise, find the distance between the vehicles at 06:30. *[3 marks]*

(c) At this time (06:30) the Chryssault stops and its crew begin their day's work, laying cable in a northerly direction. The Toyundai continues travelling in the same direction at the same speed until it is exactly north of the Chryssault. The Toyundai crew then begin their day's work, laying cable in a southerly direction. At what time does the Toyundai crew begin laying cable? *[4 marks]*

(d) Each crew lays an average of 800 m of cable in an hour. If they work non-stop until their lunch break at 11:30, what is the distance between them at this time? *[4 marks]*

(e) How long would the Toyundai take to return to base camp from its lunch-time position, assuming it travelled in a straight line and with the same average speed as on the morning journey? (Give your answer to the nearest minute.) *[5 marks]*

SECTION B

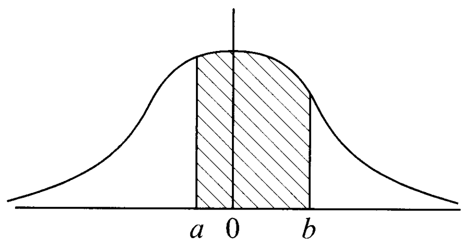
Answer **one** *question from this section.*

Statistical Methods

6. *[Maximum mark: 30]*

(i) The lifespan of a particular species of insect is normally distributed with a mean of 57 hours and a standard deviation of 4.4 hours.

(a) The probability that the lifespan of an insect of this species lies between 55 and 60 hours is represented by the shaded area in the following diagram. This diagram represents the standard normal curve.



(i) Write down the values of a and b. *[2 marks]*

(ii) Find the probability that the lifespan of an insect of this species is

(a) more than 55 hours; *[1 mark]*

(b) between 55 and 60 hours. *[2 marks]*

(b) 90% of the insects die after t hours.

(i) Represent this information on a standard normal curve diagram, similar to the one given in part (a), indicating clearly the area representing 90%. *[2 marks]*

(ii) Find the value of t. *[3 marks]*

(This question continues on the following page)

(Question 6 continued)

(ii) In a certain country, annual salaries for nurses are normally distributed with a mean of \$ 40 000 , and 95% of the nurses earn between \$ 33 000 and \$ 47 000 per year.

(a) Show that the standard deviation of nurses' salaries is \$ 3570 , correct to three significant figures. *[3 marks]*

(b) Calculate the standard error of the annual salaries of random samples of 50 nurses. *[2 marks]*

(c) Find the 95% confidence interval for the mean annual salary of random samples of 50 nurses. *[3 marks]*

(d) In a particular province of this country, 50 nurses were selected at random and their mean annual salary was found to be \$ 40 900 . Does this sample provide evidence at the 95% confidence level, that nurses' salaries are on average higher in this province? Explain your answer. *[2 marks]*

(iii) A student's final mark in mathematics is made up of two parts, an internal assessment component, which contributes 20% to the final mark, and an external examination, which contributes 80%.

A school entered 6 students for mathematics. Their marks in each of the two components were as follows:

	Internal assessment mark x	External examination mark y
Ivan	14	61
Paul	15	65
Maria	15	69
Peter	12	48
Sophie	11	35
Jane	18	70

(a) Find the regression line, $y = ax + b$ of y on x for this data. *[5 marks]*

(b) Stephen completed all the internal assessment tasks and was given a mark of 13 out of 20 , but was unable to sit the examination. Based on the data above, what mark might he have expected in the external examination? *[3 marks]*

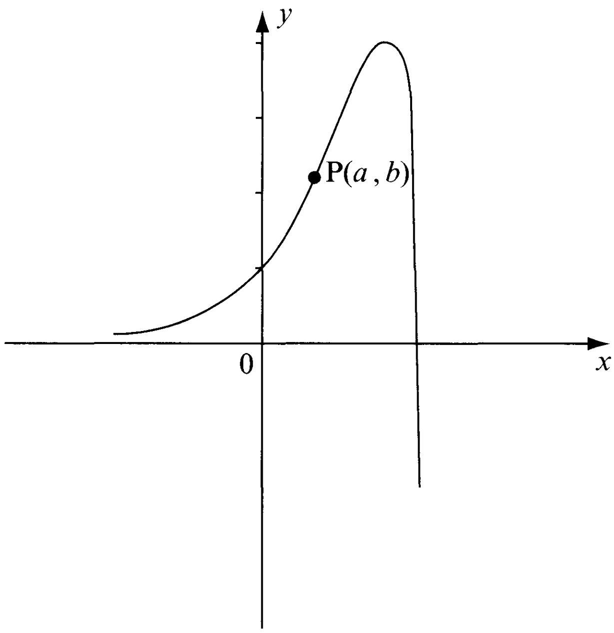
(c) Find the product-moment correlation coefficient. *[2 marks]*

Further Calculus

7. *[Maximum mark: 30]*

(i) The diagram shows part of the graph of the curve with equation

$$y = e^{2x} \cos x .$$



(a) Show that $\dfrac{dy}{dx} = e^{2x}(2\cos x - \sin x)$. *[2 marks]*

(b) Find $\dfrac{d^2y}{dx^2}$. *[4 marks]*

There is an inflexion point at $P(a, b)$.

(c) Use the results from parts (a) and (b) to prove that:

(i) $\tan a = \dfrac{3}{4}$; *[3 marks]*

(ii) the gradient of the curve at P is e^{2a} . *[5 marks]*

(This question continues on the following page)

(Question 7 continued)

(ii) A function f is defined by

$$f(x) = 2x^3 - 9x + 3.$$

The table gives values of $f(x)$ for some values of x.

x	–4	–3	–2	–1	0	1	2	3	4
$f(x)$	–89	–24	5	10	3	–4	1	30	95

(a) The equation $f(x) = 0$ has a solution between 0 and 1. Between what other pairs of successive integers can solutions of $f(x) = 0$ be found? *[2 marks]*

(b) These solutions are to be found using fixed-point iteration. Show that two possible re-arrangements of the equation $f(x) = 0$ are

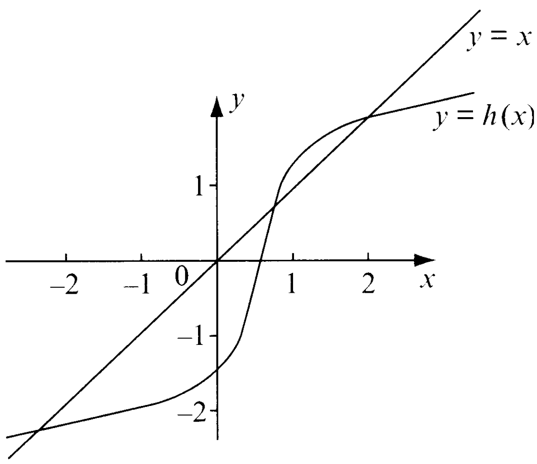
$$x = \frac{2x^3 + 3}{9} \quad \text{and} \quad x = \sqrt[3]{\frac{9x - 3}{2}}.$$ *[2 marks]*

(c) Let $g(x) = \dfrac{2x^3 + 3}{9}$ and $h(x) = \sqrt[3]{\dfrac{9x - 3}{2}}$.

(i) Use $g(x)$ with starting value $x = 0$ to find the solution between 0 and 1. Give your answer correct to six significant figures and state the number of iterations required to give this degree of accuracy. *[2 marks]*

(ii) By finding $g'(x)$ or otherwise, show that $g(x)$ will not give the other solutions. *[5 marks]*

The following diagram shows the graph of $y = h(x)$ and $y = x$.



(iii) State the features of this graph which show that $x_{n+1} = h(x_n)$ will give the other two solutions of $f(x) = 0$, but cannot be used to find the answer in part (i). *[3 marks]*

(iv) Find either one of these solutions, correct to **six** significant figures. *[2 marks]*

Further Geometry

8. *[Maximum mark: 30]*

(i) The linear transformations $\boldsymbol{R}$, $\boldsymbol{S}$, $\boldsymbol{T}$ are defined as follows:

$\boldsymbol{R}$: Rotation of 45° about the origin.

$\boldsymbol{S}$: Enlargement, centre the origin, with a scale factor of $\sqrt{2}$.

$\boldsymbol{T}$: Rotation of 45° about the origin after an enlargement, centre the origin, with a scale factor of $\sqrt{2}$.

(a) Write down the matrices $\boldsymbol{R}$, $\boldsymbol{S}$, $\boldsymbol{T}$. *[4 marks]*

(b) Find the equation of the image of the line $y = 2x$ under the transformation $\boldsymbol{T}$. *[3 marks]*

(ii) Find the matrix $\boldsymbol{Q}$ such that for all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$,

$$\boldsymbol{Q}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x + y \end{pmatrix}.$$ *[3 marks]*

(iii) $\boldsymbol{M}$ and $\boldsymbol{N}$ are transformations of the plane represented respectively by the matrices $\boldsymbol{M} = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$ and $\boldsymbol{N} = \begin{pmatrix} 0 & 3 \\ 1 & -1 \end{pmatrix}$.

(a) Find the matrix for the composite transformation $\boldsymbol{N}$ followed by $\boldsymbol{M}$. *[2 marks]*

(b) $\begin{pmatrix} -1 & 7 \\ 1 & 8 \end{pmatrix}$ is the matrix for a linear transformation $\boldsymbol{P}$.

Under this transformation, the parallelogram OABC is mapped onto O′A′B′C′, where O is the origin.

(i) Show that O′ is the same point as O. *[1 mark]*

(ii) If A has coordinates (6, –4), find the coordinates of A′. *[2 marks]*

(iii) If B′ is the point (–15, 0), find B. *[3 marks]*

(iv) Using the fact that OABC is a parallelogram, find the coordinates of C and hence of its image C′. *[3 marks]*

(v) Show that OABC is a rectangle and investigate whether O′A′B′C′ is also a rectangle. *[4 marks]*

(vi) Find the area of OABC and hence calculate the area of O′A′B′C′. *[5 marks]*