M99/530/S(2)M

INTERNATIONAL BACCALAURÉAT BACHILLERATO
BACCALAUREATE INTERNATIONAL INTERNACIONAL

MARKSCHEME

May 1999

MATHEMATICAL STUDIES

Standard Level

Paper 2

20 pages

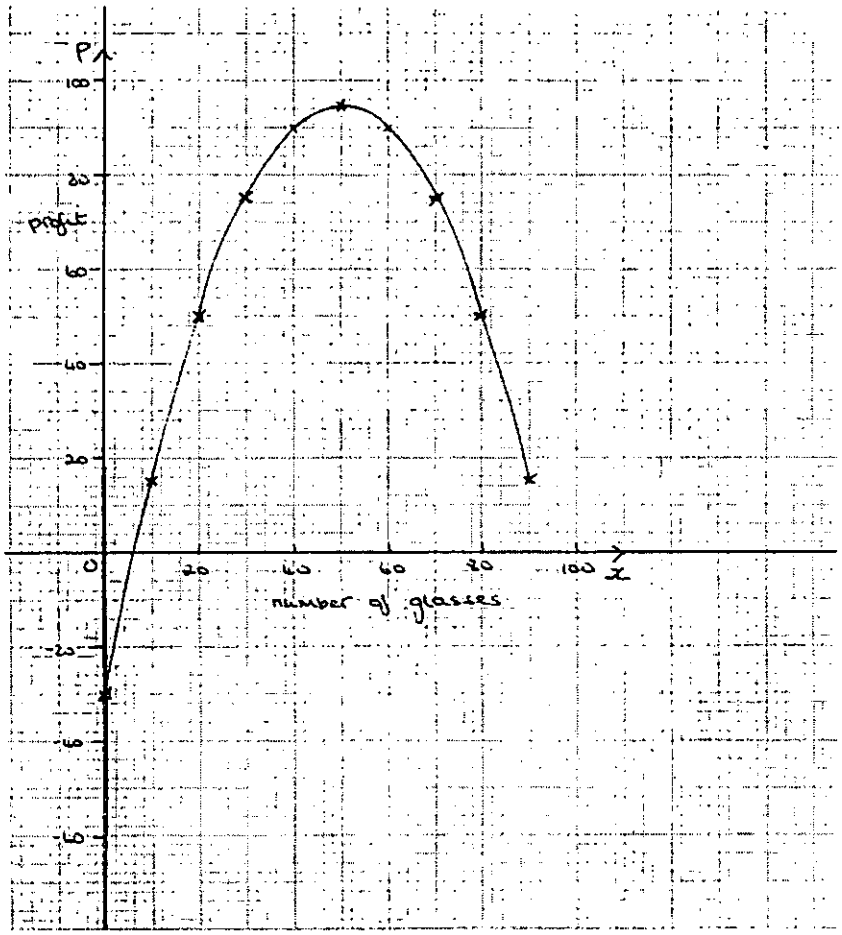
1. (a)

x	0	10	20	30	40	50	60	70	80	90
P	**–30**	15	**50**	**75**	90	**95**	**90**	75	50	**15**

(A3)

Notes: Award ***½-mark*** for each correct bold entry, and round **down**.
If a candidate obtains *(A0)* here but has clearly shown the method of substituting in the values of x into the formula award *(M1)*

(b)



(A2)(A2)(A1)

Note: For graph, follow through from candidate's table

Notes: Award *(A2)* for axes, *(A2)* for plotting points and *(A1)* for a smooth curve.

Axes: Award ***½-mark*** for each of the following and then round down:
horizontal axis labelled with 'x' or 'Numbers of glasses…'
vertical axis labelled with 'P' or 'Profit'
horizontal scale → consistent and represents values 0→90
vertical scale as for horizontal but represents their range of values for P.

Points: Award *(A2)* for 0 or 1 error
Award *(A1)* for 2 or 3 errors
Award *(A0)* otherwise

continued...

Question 1 continued

(c) (i) maximum profit = 95 swiss francs *(A1)*

(ii) 50 glasses *(A1)*

(iii) 67 ± 2 *(A1)*
33 ± 2 *(A1)*

(iv) 30 swiss francs *(A1)*

Note: Award no marks for –30 swiss francs

Note: Follow through from candidate's graph

(d) Fiona's share $= \frac{3}{6}$ *(M1)*

Profit from 40 glasses = 90 swiss francs

$$\text{Fiona's profit} = \frac{1}{2} \times 90$$

$$= 45$$ *(A1)*

[15 marks]

2. (a) (i) $\mathcal{E} = \{1, 2, 3 \ldots 16\}$ *(A1)*

Note: If they include 17, award *(A0)*

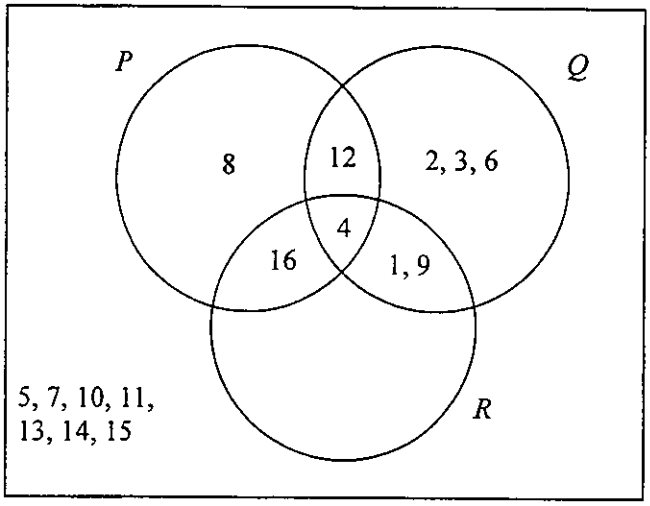
(ii) $P \cap Q \cap R = \{4\}$ *(A1)*

Note: Accept answers without brackets *e.g.* 4

(b) $P \cup Q$: the set of numbers that are either multiples of 4 or factors of 36, or everything that is in P or Q (or equivalent) *(A1)*

(c) (i) *(A2)*

(ii) *(A3)*



Notes: Follow through with candidate's list for $\mathcal{E}$, P, Q, R.

Part (i): Award *(A1)* for the rectangle, *(A1)* for 3 intersecting circles labelled appropriately

Part (ii): There are 8 regions. If a region contains an element that it shouldn't, or does not have an element that it should, then the region is incorrect. Award marks as follows:

0 or 1 region incorrect	*(A3)*
2 or 3 regions incorrect	*(A2)*
4 or 5 regions incorrect	*(A1)*
otherwise	*(A0)*

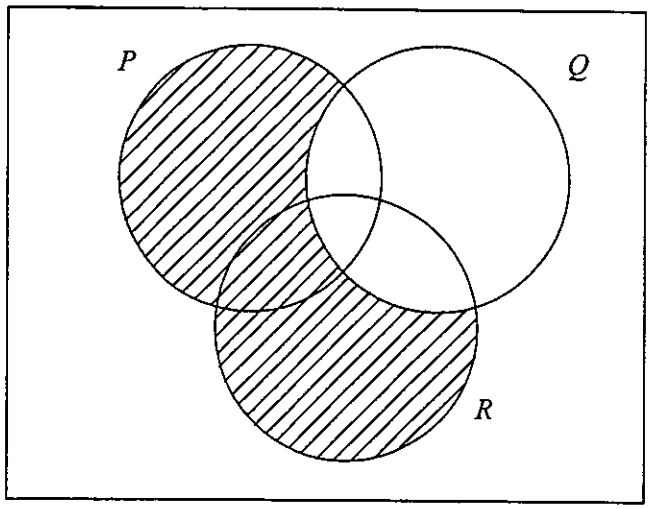
continued...

Question 2 continued

(d) (i) x is a number that is a multiple of 4 or a square number but is not a factor of 36 *(A2)*

Note: Award *(A1)* for the explanation of $p \vee r$, *(A1)* for the explanation of $\wedge \neg q$

(ii) $\mathcal{E}$



(A1)

Note: This shading should appear on the Venn diagram in part (i)(c)

(iii) (a)

p	q	r	$p \vee r$	$\neg q$	$(p \vee r) \wedge \neg q$
T	T	T	**T**	**F**	**F**
T	T	F	**T**	**F**	**F**
T	F	T	**T**	**T**	**T**
T	F	F	**T**	**T**	**T**
F	T	T	**T**	**F**	**F**
F	T	F	**F**	**F**	**F**
F	F	T	**T**	**T**	**T**
F	F	F	**F**	**T**	**F**

(A3)

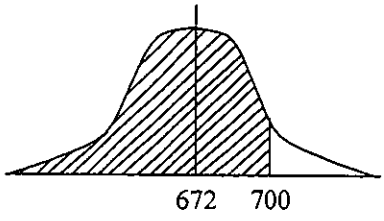
Notes: Award *(A1)* for $p \vee r$, *(A1)* for $\neg q$ and *(A1)* for $(p \vee r) \wedge \neg q$ (follow through from candidate's previous columns).
Award *(A3)* for last column only, written correctly.

(b) Either 8 or 16 *(A1)*

Note: Refer to candidate's Venn diagram, and award *(A1)* for any number that appears in the shaded region

[15 marks]

3. (a) (i)

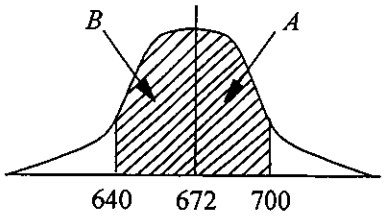


$$z = \frac{700 - 672}{50}$$ *(M1)*

$$= 0.56$$

$$\Phi(0.56) = 0.7123$$
$$= 0.712 \text{ (3 s.f.)}$$ *(A1)*

(ii)



Area $A = 0.7123 - 0.5$ *(M1)*

$$= 0.2123$$

Area B

$$z = \frac{640 - 672}{50} = -0.64$$

$$\Phi(-0.64) = 1 - 0.7389 = 0.2611$$ *(M1)*

Area $B = 0.5 - 0.2611$ *(M1)*

$$= 0.2389$$

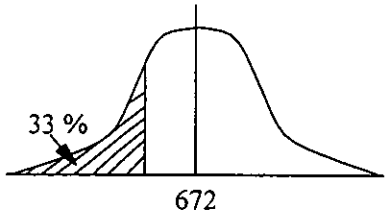
Therefore $p(640 < X < 672) = 0.2123 + 0.2389$ *(M1)*

$$= 0.4512$$
$$= 0.451 \text{ (3 s.f.)}$$ *(A1)*

continued...

Question 3 continued

(b) (i)



(M1)

Note: Award *(M1)* for indicating an area of 33 % or 0.33

(ii) $\Phi(z) = 0.67$ *(M1)*

$z = -0.44$ *(A1)*

Note: If candidate has not identified 0.67 but has $z = 0.44$ then award *(A1)*

(iii) $-0.44 = \dfrac{x - 672}{50}$ *(M1)*

$x = 650$ *(A1)*

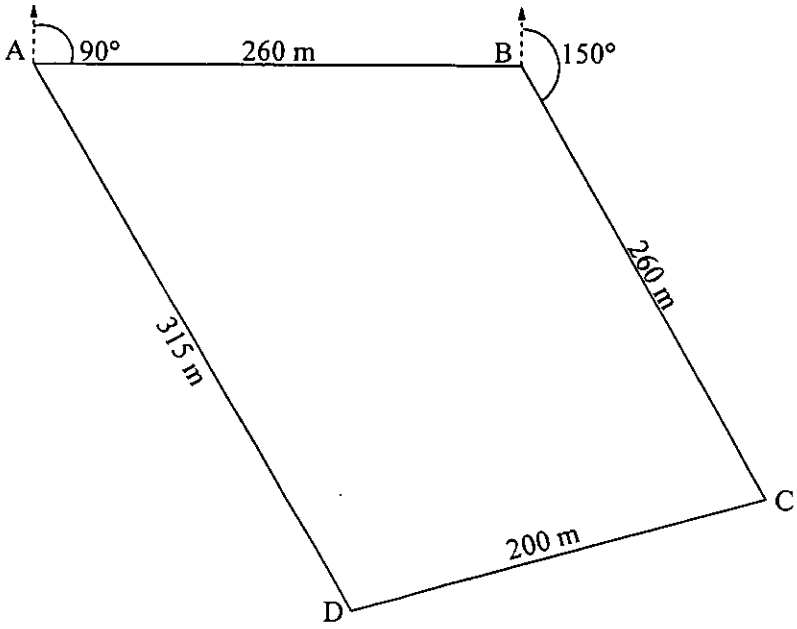
(c) p(exactly one container contains more than the printed volume)

$= 0.33 \times 0.67 \times 2$ *(M1)(M1)*

Note: Award *(M1)* for 0.33×0.67, *(M1)* for $\times 2$

$= 0.4422$
$= 0.442$ (3 s.f.) *(A1)*

4. (a)



(A3)

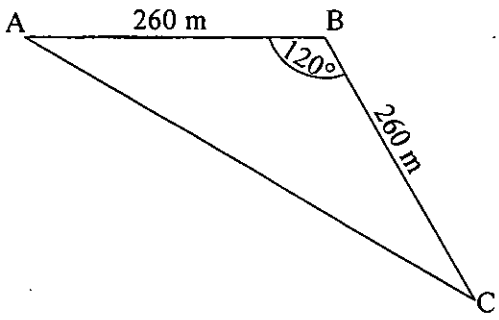
Notes:	Award *½-mark* for each correct entry. Round **down.**
	If 90° is not shown at A, accept B as east of A if the north direction is correctly indicated at B with 90° shown.

(b) (i) Angle $A\hat{B}C = 360^\circ - (150^\circ + 90^\circ)$ *(M1)*

$= 120^\circ$ *(A1)*

Notes:	Follow through with candidate's diagram.
	If $A\hat{B}C$ is shown as 150° in diagram, and no working is shown, award *(M0)(A0)* for $A\hat{B}C = 150^\circ$.

(ii)



$AC^2 = 260^2 + 260^2 - 2 \times 260 \times 260 \cos 120$ *(M1)*

$AC^2 = 202800$

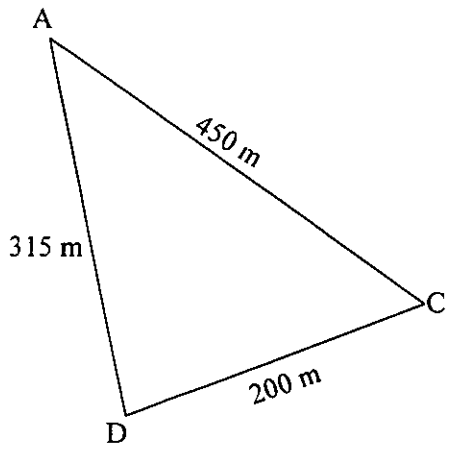
$AC = 450.33321$ m

$AC = 450$ m (3 s.f.) *(A1)*

continued...

Question 4(b) continued

(iii)



$$\cos D = \frac{315^2 + 200^2 - 202800}{2 \times 315 \times 200} \text{ (working with } AC^2 = 202800) \qquad \textit{(M1)}$$

$$\cos D = -0.5045634921\ldots$$

$$D = 120.3023\ldots°$$

$$D = 120° \text{ (3 s.f.)} \qquad \textit{(A1)}$$

OR $$\cos D = \frac{315^2 + 200^2 - 450^2}{2 \times 315 \times 200} \text{ (working with } AC = 450) \qquad \textit{(M1)}$$

$$\cos D = -0.5021825397\ldots$$

$$D = 120.144501\ldots°$$

$$D = 120° \text{ (3 s.f.)} \qquad \textit{(A1)}$$

(c) $$\text{Area } \Delta ABC = \frac{1}{2} \times 260 \times 260 \sin 120 \text{ m}^2 \qquad \textit{(M1)}$$

$$= 29271.65865\ldots \text{m}^2$$

Note: Follow through with candidate's results

$$\text{Area } ACD = \frac{1}{2} \times 315 \times 200 \sin 120 \text{ m}^2 \qquad \textit{(M1)}$$

$$= 27279.80022\ldots \text{m}^2$$

$$\text{Area } ABCD = 56551.45887\ldots \text{m}^2$$

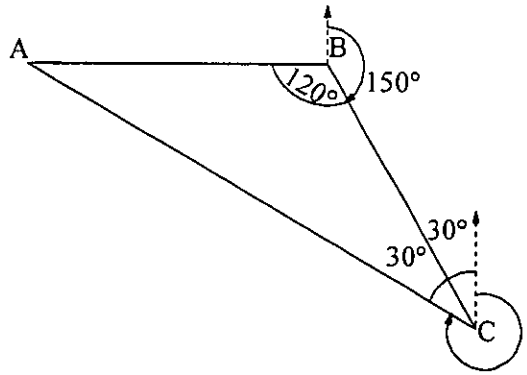
$$= 56600 \text{ m}^2 \text{ (3 s.f.)} \qquad \textit{(A1)}$$

Note: Accept other answers if candidates use unrounded values for $\hat{ADC}$

continued...

Question 4 continued

(d)



(M2)

Note:	Award ***(M1)*** for 30°, ***(M1)*** for other 30° (angle $A\hat{C}B$)

Bearing of A from C $= 360° - (2 \times 30°)$

$= 300°$

(A1)

[15 marks]

5. (i) (a) (i) $p(\text{green}) = \frac{5}{10}$ *(A1)*

(ii) $p(\text{not green}) = \frac{5}{10}$ *(A1)*

Note: Accept $\frac{1}{2}$, 0.5 or 50 % for either answer

(b) (i) $p(\text{G}|\text{G}) = \frac{4}{9}$ or 0.444 (3 s.f.) *(A1)*

(ii) p(not green then not green)

$= \frac{5}{10} \times \frac{4}{9}$ *(M1)*

$= \frac{20}{90}$ or $\frac{2}{9}$ or 0.222 (3 s.f.) *(A1)*

(iii) p(one green and one not green)

$= \frac{5}{10} \times \frac{5}{9} + \frac{5}{10} \times \frac{5}{9}$ *(M2)*

Note: Award *(M1)* for $\frac{5}{10} \times \frac{5}{9}$, *(M1)* for $(\times 2)$

$= \frac{50}{90}$ or $\frac{5}{9}$ or 0.556 (3 s.f.) *(A1)*

(c) (i) $p(3 \text{ green}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}$ *(M1)*

$= \frac{60}{720}$ or $\frac{1}{12}$ or 0.0833 (3 s.f.) *(A1)*

continued...

Question 5(i)(c) continued

(ii) $p(\text{only one green}) = 3 \times \frac{5}{10} \times \frac{5}{9} \times \frac{4}{8}$ ***(M2)***

Note: Award ***(M1)*** for $\frac{5}{10} \times \frac{5}{9} \times \frac{4}{8}$, ***(M1)*** for $(\times 3)$

$= \frac{300}{720}$ or 0.417 (3 s.f.) or $\frac{10}{24}$ or $\frac{5}{12}$ ***(A1)***

(iii) $p(\text{at least one green}) = 1 - p(\text{no green})$

$= 1 - \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8}$ ***(M1)***

$= 1 - \frac{60}{720}$

$= \frac{660}{720}$ or $\frac{11}{12}$ or 0.917 (3 s.f.) ***(A1)***

(ii) (a) $p(\text{two defective}) = 6 \times 0.05^2 \times 0.95^2$ ***(M2)***

Note: Award ***(M1)*** for $0.05^2 \times 0.95^2$, ***(M1)*** for $(\times 6)$

$= 0.0135375$

$= 0.0135$ (3 s.f.) ***(A1)***

(b) $p(\text{at least one defective}) = 1 - 0.95^4$ ***(M1)***

$= 0.18549375\ldots$

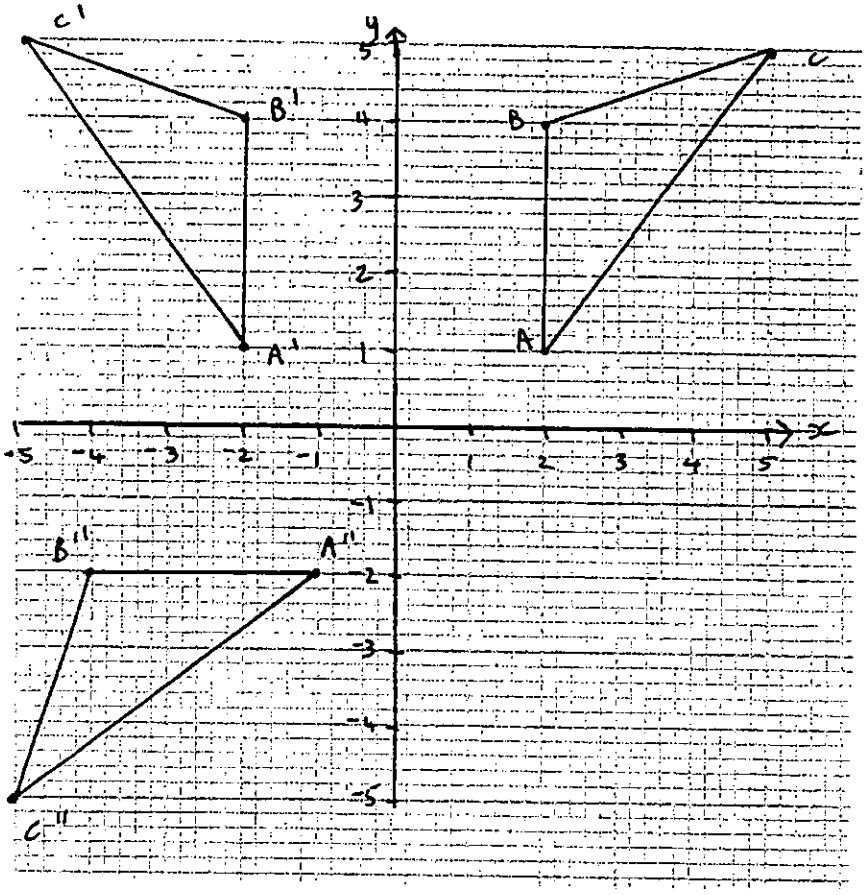
$= 0.185$ (3 s.f.) ***(A1)***

[20 marks]

6. (i) (a) (i) *(A1)*

(ii) *(A2)*

(b) (i) *(A2)*



(c) (ii) *(A1)*

Notes:	(a)(i) Award *(A1)* for axes
	(a)(ii) Award *(A1)* for the points A, B and C correctly plotted and labelled *(A1)* for A, B, C joined to give triangle
	(b)(i) Award *(A2)* for all points correctly transformed and triangle $A'B'C'$ drawn *(A1)* for triangle not drawn or 1 point incorrect *(A0)* otherwise

(ii) $A'(-2, 1)$
$B'(-2, 4)$
$C'(-5, 5)$ *(A1)*

Note:	Follow through from candidate's diagram, all correct for *(A1)*

(c) (i) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$ *(A1)*

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ *(A1)*

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$ *(A1)*

continued...

Question 6(i)(c) continued

(ii) See graph *(A1)*

Note: Award *(A1)* for the points A'', B'' and C'' correctly plotted and joined to form a triangle

(iii) Reflection in the line $y = -x$ *(A1)*

(iv) Rotation of 90° anti-clockwise about the origin *(A2)*

Notes: Award *(A1)* for 90° rotation, *(A1)* for anti-clockwise about origin.
Award *(A2)* for the transformation matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(ii) (a) B(6, 6, 0) *(A1)*

(b) Mid-point

$$x = \frac{0+6}{2} = 3, y = \frac{0+6}{2} = 3, z = \frac{0+0}{2} = 0$$ *(M1)*

M(3, 3, 0) *(A1)*

Note: If candidate has shown no working but has correct coordinates for M, award *(A2)*

(c) E(3, 3, 7) *(A1)*

Note: Follow through with candidate's M

(d) $DB = \sqrt{6^2 + 6^2}$ *(M1)*

$= \sqrt{72}$

$MB = \frac{\sqrt{72}}{2} = 4.24$ (3 s.f.) *(M1)*

Note: Award *(M1)* for dividing DB by 2

$EB = \sqrt{7^2 + 18}$

Note: $\left(\frac{\sqrt{72}}{2}\right)^2 = 18$

so $EB = \sqrt{67}$ or 8.19 (3 s.f.) *(A1)*

[20 marks]

7. (a) (i) Tulips: x hectares at £ 20 per hectare $\Rightarrow$ £ $20x$.
Carnations: y hectares at £ 12 per hectare $\Rightarrow$ £ 12y.
Therefore total cost of planting is £ $(20x+12y)$ *(R1)*

They cannot spend more than £ 240 so $20x+12y \leq 240$ *(R1)*

(ii) $5x+3y \leq 60$, or $y \leq \frac{-5}{3}x+20$ or equivalent *(A1)*

Note: Must be completely simplified for *(A1)*

(iii) $x \geq 0$: It is not possible to plant a negative number hectares of tulips (or equivalent *e.g.* must plant zero or more hectares of tulips) *(R1)*

$y \geq 6$: At least 6 hectares of carnations must be planted *(R1)*

(iv) $x+y \leq 16$ *(A1)*

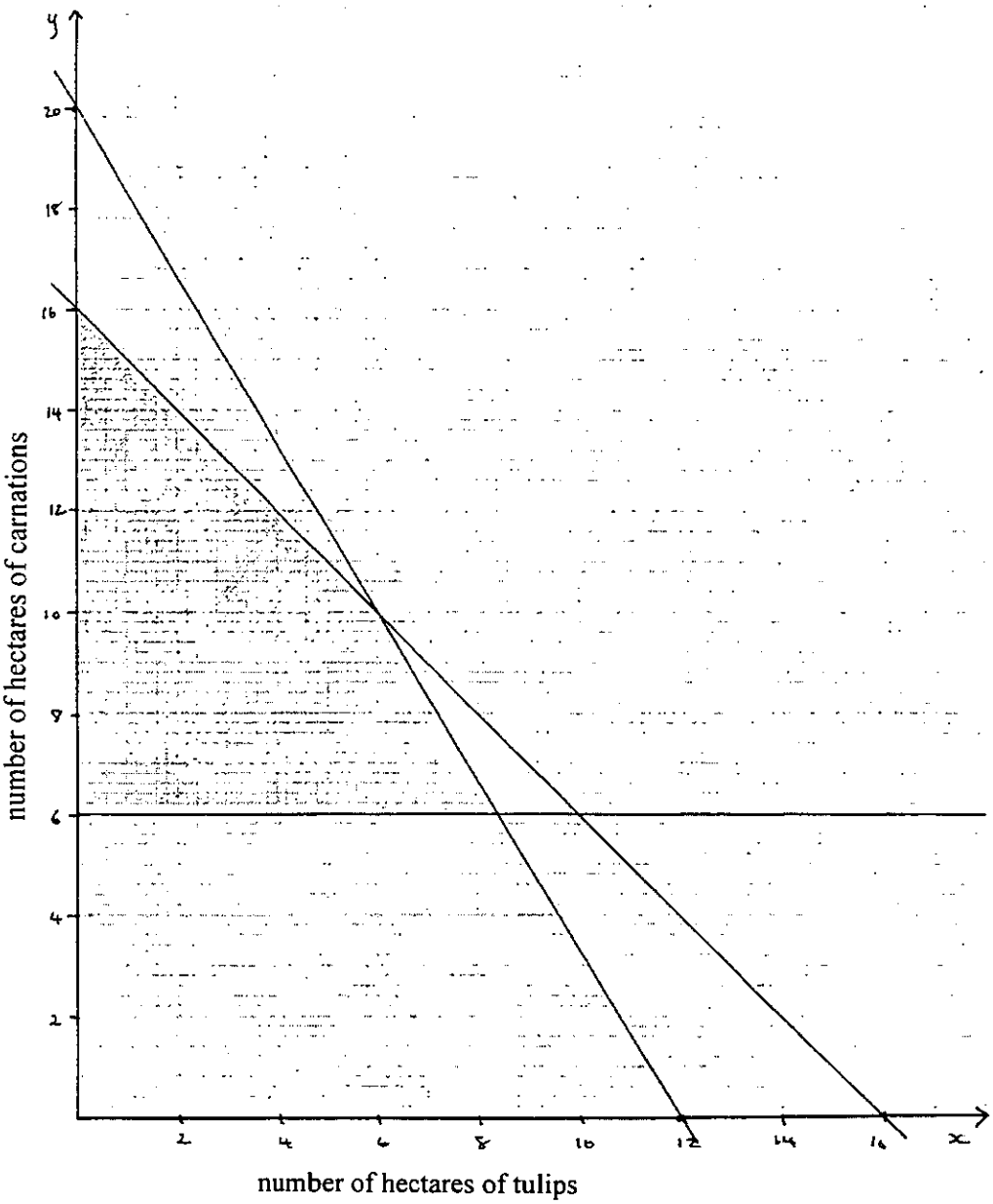
Note: Accept alternative forms

Question 7 continued

(b) (i) ***(A2)***

(A4)

(ii) ***(A1)***



Notes: (b)(i) Award ***(A2)*** for axes and label, as follows:
Award ***½-mark*** for each of the following then round up:
each axis labelled with at least x and y
consistent scale on each axis

Award ***(A4)*** for lines as follows:
Award ***(A1)*** for $y = 6$, ***(A1)*** for $x + y = 16$
Award ***(A2)*** for $20x + 12y = 240$, *i.e.* ***(A1)*** for x intercept, ***(A1)*** for y intercept

(b)(ii) Award ***(A1)*** for shaded region or appropriate region clearly indicated

continued...

Question 7(b) continued

(iii) (0, 16) (0, 6) (6, 10) *(A1)*

Note: Award *½-mark* for each correct point and round down

(8.4, 6) (accept 8.4 ± 0.1) *(A1)*

Note: Follow through from candidate's graph

(c) (i) (a) profit/hectare for tulips

$= 7000 \times £\ 0.85 = £\ 5950$ *(A1)*

(b) profit/hectare for carnations

$= 10\,000 \times £\ 0.45 = £\ 4500$ *(A1)*

(ii) $P = 5950x + 4500y$ *(A1)*

(iii)

$5950x$	$4500y$	P
0	16	72 000
0	6	27 000
8.4	6	76 980
6	10	80 700

(M1)

Note: For *(M1)* here candidate should have at least substituted bold values into the profit formula

Therefore, maximum profit = £ 80 700 *(A1)*

[20 marks]