## Paper E

## IB HL Options Stats

1. [Maximum mark: 10]

Eight students attend an Easter IB Maths revision course at Camford Revision Centre. Prior to the course they sit a Paper 1 test, and at the end of the course they sit the same Paper 1. The marks for a paper 1 test are out of 120 . The marks of the students are shown below.

|  | A | B | C | D | E | F | G | H |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Results before | 72 | 51 | 63 | 91 | 84 | 33 | 61 | 64 |
| Results after | 80 | 70 | 69 | 110 | 87 | 57 | 59 | 71 |

a) Write suitable hypotheses to test if the revision course has improved the results of the students.
b) Use a suitable test at the $1 \%$ level of significance to test if the revision course has made a significant improvement in the results of the students.
2. [Maximum mark: 10]

It is thought that the random variable $X$ follows a binomial distribution with a probability of 0.4 .

To test this claim the variable is tested by 6 groups. Each group tests the variable 40 times and records the number of successes, $x$.

| Group | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 8 | 7 | 12 | 9 | 13 | 6 |

Carry out a chi-squared goodness of fit test at the 5\% level of significance, to test the claim that the variable fits a binomial distribution with a probability of 0.4 .

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3. [Maximum mark: 13]

The first 100 runners in the London marathon times were recorded, with times given in minutes. The results were as follows:

$$
\sum x=15560 \text { and } \sum x^{2}=2506000
$$

a) Calculate the mean time of a marathon runner in minutes.
[1 mark]
b) Calculate the standard error of this sample of 100 runners.
[3 marks]
c) The New York marathon is to be run a month later. Calculate a 95\% confidence interval, based on the London results for the mean time of the first 100 runners in the marathon.
[4 marks]
d) What assumptions have you made in your calculations for c) ?
[2 marks]
4. [Maximum mark: 9]

A spinner has 5 segment coloured as follows: two blue, one green, one red, and one yellow.

A game is played whereby the spinner is spun until it lands on Blue. Two players, Arnie and Bessie take turns to spin the spinner.
a) Find the probability that Arnie wins if he spins the spinner first.
[3 marks]
b) Find the probability that Arnie wins, given that he has the second go.
[3 marks]

The players' set up a model to calculate how many blues will appear in 20 throws.
c) Calculate the probability that exactly 8 blues appear in 20 spins.
[2 marks]
d) Calculate the probability that the $8^{\text {th }}$ blue appears on the $20^{\text {th }}$ spin.
[3 marks]

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5. [Maximum mark: 16]

A shopper at Tescourt supermarket is a keen statistician, and over the course of a year has discovered that the amount of time spent queuing at the checkout follows a normal distribution with a mean of 8 minutes and a variance of 4 minutes. She has also found that the queuing time at the petrol forecourt of Tescourts is also normally distributed with a mean of 6 minutes and variance of 3 minutes.
a) In one week, she shops on 3 separate days.
i) Find the mean and variance of her total queuing time at the checkout.
ii) Hence, calculate the probability that she spends less than 15 minutes at the checkout in a queue in total for her 3 visits.
[6 marks]
b) On Saturday, she goes to the supermarket and then fills her car with petrol.
i) Calculate the mean and variance of her total queuing time at the checkout and the petrol forecourt.
ii) Hence, find the probability that she queued for more than 17 minutes in total.
[6 marks]
c) Calculate the probability that she queued for longer at the garage than in the supermarket.
[4 marks]

## Paper E

## Answers

1. a) $\mathrm{H}_{0}: \mu_{1}=\mu_{2}$ and $\mathrm{H}_{1}: \mu_{2}>\mu_{1}$
b) $\bar{d}=9.25, S_{n-1}=7.52$

$$
v=7, \chi_{(1 \%)}^{2}=18.475
$$

$$
\mathrm{p}=\frac{9.25-0}{\frac{7.52}{\sqrt{8}}}=3.479
$$

Do not reject the null hypothesis. At the 1\% level there is no evidence that the revision course provides any significant improvement in results.
2. $H_{0}$ : The distribution fits a binomial distribution with a probability of a success of 0.4.
$H_{1}$ : The distribution does not fit the binomial model.
$p=4.3$
$v=5, \chi^{2}{ }_{5 \%}=11.07$
Accept the null hypothesis, at this level of significance the distribution fits the binomial distribution with a success of 0.4.
3. a) 155.6
b) $\mathrm{s} . \mathrm{e}=2.913$
c) $[149.9,161.3]$
d) Assuming that the course and conditions are the same. The runners will be the same standard.
4.
a) $\frac{5}{8}$
b) $\frac{3}{8}$
c) 0.1797
d) 0.07188
5.
a) i) mean $=24$, variance $=36$
ii) $\quad \mathrm{p}=0.06678$
b) $\mathrm{p}=0.1284$
C) $\quad \mathrm{p}=0.224$

