1.   *[Maximum mark: 13]*

At Brightsville School 80, students sat the end of year examination in maths. They received a grade from A to E and the results are broken down by gender and shown in the table below.

	A	B	C	D	E	Total
Male	8	9	15	12	6	50
Female	12	7	6	3	2	30
Total	20	16	21	15	8	80

A mathematics teacher believes that the gender of the students has some effect on the grade of the test.

a)   Write suitable hypotheses for the mathematician's claims.
[2 marks]

b)   Complete a table of expected values for the test. *[4 marks]*

c)   Calculate the chi-squared test statistic, *p*, for this test.
[3 marks]

d)   Write down the degrees of freedom, and the critical value at the 1% level of significance, from the chi-squared statistics tables.
[2 marks]

e)   Write down a conclusion for your test.   *[2 marks]*

2.   *[Maximum mark: 10]*

The mass of Trout in a river satisfies the normal distribution $X \sim N(2.5, 0.4)$ with the mass given in kg.

a)   Samples of 30 trout are taken from the river and their masses are found. If the masses are below 2.3 kg then it will be assumed that the mass of trout is in fact less than 2.5 kg. Show that the probability of a type I error under these circumstances is 0.041.   *[4 marks]*

b)   Find the probability of a type II error being made if in fact the mass of the trout has fallen to 2.15 kg.   *[6 marks]*

3.   *[Maximum mark: 11]*

Sunnydale tourist board claim they have on average a mean summer temperature of 25°C with a standard deviation of 6°C.

The tourist board in the neighbouring town disputes the claim by Sunnydale and believes that they have a lower average temperature. To test this claim 50 days temperatures were recorded throughout the summer and the mean temperature was recorded as 23°C. You may assume that the temperatures are distributed normally.

a)   Write suitable hypotheses for the test.   *[2 marks]*

b)   Carry out a suitable test to test the claim made by the neighbouring town.   *[9 marks]*

4.   *[Maximum mark: 10]*

In a local rugby league one of the teams of 15 players is chosen at random and the masses of the players is recorded. Each players masses, *x*, are recorded in kg.

$$\sum x = 1320kg \text{ and } \sum x^2 = 116998kg$$

a)   Assuming the masses of the rugby players are normally distributed, calculate a 95% confidence interval for the players of the entire league.   *[8 marks]*

b)   Justify your choice of test in a).   *[2 marks]*

5.   *[Maximum mark: 16]*

Vladimir commutes into central Moscow each day. He turns up at random at his local metro station, the *Novokuznetskaya* station.

Vladimir finds that the time spent waiting for his train follows an exponential distribution given as, $f(t) = \lambda e^{-\lambda t}$, where t is the waiting time in minutes.

a)   Show that $\lambda = \dfrac{1}{8}$ given that the median waiting time fro Vladimir is 5.545 minutes.   *[5 marks]*

b)   Find the mean and variance of Vladimir's waiting time in minutes.   *[2 marks]*

c)   Find the probability that on any random day Vladimir will wait for more than 3 minutes for a train.   *[2 marks]*

d)   Given that Vladimir waits more than 3 minutes for a train, calculate the probability that he waits no more than 6 minutes for the train.   *[4 marks]*

e)   In the evening, Vladimir travels home from the *Arbatskaya* station. Trains from his platform will go either to the *Novokuznetskaya* or to the *Prospekt Mira* station. The probability of the next train arriving being the one Vladimir wants is always 0.3. Find the probability that the third train to arrive will be the first one that Vladimir needs to go home to *Novokuznetskay*.   *[3 marks]*

Answers

1. a) H_0: The gender is independent of the grade.
         H_1: The grade is effected by the gender.

    b)

	A	B	C	D/E
Male	12.5	10	13.125	14.375
Female	7.5	6	7.875	8.625

    c) $p=7.739$

    d) $v = 3, \chi^2 = 11.345$

    e)   Do not reject the null hypothesis. At the 1% level of significance there is no evidence to suggest that grades and gender are related.

2. a) $p=0.041$ b) $p=0.097$

3. a) $H_0 : \mu = 25$ and $H_1 : \mu < 25$

    b)   standard error = 0.849

$$\frac{x - 25}{0.849} = -1.6448$$

$x = 23.6$

As 23<23.6 reject the null. There is sufficient evidence to suggest that the temperature in Sunnydale is less than 25.

4. a) $\overline{x} = 88$, $S_{n-1} = 7.737$

$$88 \pm 2.145 \times \left(\frac{7.737}{\sqrt{15}} \right)$$

[83.71, 92.29]

   b)   Use a *t*-test as we do not know the population variance and we have a small sample size.

5. b)   mean = 8   variance = 64

   c)   0.686

   d)   0.313

   e)   0.147