1. *[Maximum mark: 13]*

In a series of maths tests is calculated that Ahmed makes an average of four mistakes per paper. On the same series of maths tests Bernie makes five mistakes per paper.

a) These distributions are determined to follow a Poisson model. Give justification for the use of the Poisson model. *[2 marks]*

b) Calculate the probability that Ahmed makes exactly 3 mistakes on one paper. *[2 marks]*

c) Calculate the probability that Bernie makes 3 or fewer mistakes on one paper. *[4 marks]*

d) On one paper, the mistakes of both Ahmed and Bernie are added together to give 6 mistakes. Calculate the probability that Ahmed made fewer mistakes than Bernie did. *[5 marks]*

2. *[Maximum mark: 16]*

At a county fair, forty pumpkins are entered for a competition to find the best one. It is thought that the pumpkins at the competition have a greater mass than the national average. Nationally the pumpkins follow a normal distribution with a mean mass of 1.6 kg and a variance of 0.5 kg. The total weights of all the pumpkins at the fair are 72 kg.

a) Calculate the sample mean. *[1 mark]*

b) Calculate the standard error of the sample means. *[2 marks]*

c) Conduct a hypothesis test at the 5% level of significance to test if the masses of the pumpkins at the fair are indeed greater than the national average. *[8 marks]*

d) Find the probability of a type I error in d) *[1 mark]*

e) A statistician calculates that in fact the national average has a mean mass of 1.9kg, with the variance remaining unchanged. Calculate the probability of a type II error being made in c). *[4 marks]*

3. [Maximum mark: 12]

The IBO sets two Maths Paper 1 in the May examination session, a Time Zone One and a Time Zone Two test. In order to check that the neither examination is more difficult than the other 10 students sit the examination and their results are analyzed. The percentage scores are shown below.

Student	A	B	C	D	E	F	G	H	I	J
Zone 1 test	72	54	65	71	80	48	37	59	63	50
Zone 2 test	68	43	62	65	72	30	34	60	60	45

The students sit the examination on a Monday morning one week apart.

Conduct a paired t-test at the 5% level significance to test if the two papers are of similar standard. *[12 marks]*

4. [Maximum mark: 7]

An exponential density function has a mean probability of 3.5.

a) Write down the variance of the distribution. *[2 marks]*

b) Write down the full continuous probability density function and hence calculate the median probability. *[5 marks]*

5. *[Maximum mark: 12]*

In the mathematics IB examination previous results have found that the percentage of students receiving grades 1 to 7 globally are as follows:

IB Grade	1	2	3	4	5	6	7
Percentage	5%	10%	20%	30%	20%	10%	5%

200 students in a country sit the paper and their results are recorded as follows:

IB Grade	1	2	3	4	5	6	7
Frequency	7	16	32	46	59	28	12

Conduct a chi-squared test at the 5% level of significance to test if the country's IB results follow the global model. *[12 marks]*

Answers

1. a) A Poisson model can be used because the distribution is random and in a fixed space.

   b) 0.1934

   c) 0.265

   d) 0.067

2. a) $\bar{x} = 1.8kg$

   b) s.e = 0.1118

   c) $H_0 : \mu = 1.6$ and $H_1 : \mu > 1.6$

   $$\frac{x - 1.6}{0.1118} = 1.6448$$

   $$x = 1.78$$

   As $\bar{x} > x$ then we will reject H_0 in favour of H_1. The mass of the pumpkins at the fair are greater than the nationa average.

   d) $p = 0.05$

   e) $p = 0.1546$

3. $H_0 : \mu_1 = \mu_2$ and $H_1 : \mu_1 \neq \mu_2$

   $\bar{d} = 5.8, S_{n-1} = 5.138$

   $$p = \frac{5.8 - 0}{\frac{5.138}{\sqrt{10}}} = 3.5697$$

   $v = 9$, critical value (t) = 2.262 (two tailed).

   As $p > t$ we will reject H_0 in favour of H_1. There is a difference in the two tests at the 5% level of significance.

4. a) variance = 12.25

b) p.d.f = $\frac{2}{7}e^{-\frac{2}{7}x}$

median = 2.43

5. H_0: the grades in the school fit the global model.

H_1: the grades in the school do not fit the global model.

P=19.192

$v = 6, \chi^2 = 12.592$

As $p > \chi^2$ then reject H_0 in favour of H_1. That is the school's results do not fit the global model when tested at the 5% level of significance.