1. *[Maximum mark: 10]*

A normal population has an unknown mean, μ, and a variance of 49. A sample of size 64 is taken. Two hypotheses are put forward:

$H_0 : \mu = 100$ and $H_1 : \mu > 100$.

The null hypothesis will be accepted if the value of sample mean, $\overline{x}$, is less than 101.8.

a) Calculate the probability of a Type I error. *[4 marks]*

b) If the probability of a Type I error was stated as being 0.06, what would the maximum value of the sample mean $\overline{x}$ now be? *[4 marks]*

c) Explain clearly what a type I error is. *[2 marks]*

2. *[Maximum mark: 10]*

A survey of 400 male students at a college in the UK has found that 60 of the students are left-handed.

a) Calculate an unbiased estimate for the proportion of the population, *p*, that are left handed. *[2 marks]*

b) Calculate the standard error of the mean. *[3 marks]*

c) Calculate a 95% confidence interval for the proportion of the population that will be left handed. *[5 marks]*

3. *[Maximum mark: 14]*

A famous cricketer, Ricky Pointer, has scored a batting average that is normally distributed with a mean of 52 and a variance of 400.

a) Calculate that in his next three innings Ricky will score more than 75 runs,

i) in each inning,

ii) in exactly one of the innings. *[5 marks]*

b) Another famous cricketer, Adam Filchist, has a batting of 47 with a variance of 441. His average is also normally distributed. Adam and Ricky bat together against India. Find the mean and variance of the sum of Adam and Ricky's runs in the match against India. *[3 marks]*

c) Find the probability that in one match Adam and Ricky score more than 150 runs when their runs are added together. *[3 marks]*

d) Find the probability that Adam scores more runs than Ricky on any given match. *[3 marks]*

4. *[Maximum mark: 14]*

At a Casino two dice are used in a game of Craps. A visitor to the casino belives that one of the dice is biased in favour of sixes. The die is taken and rolled 100 times, and 22 sixes are recorded.

a) Test to check if the die is biased, giving your conclusion at,

i) the 10% significance level,

ii) the 5% significance level. *[12 marks]*

b) Explain clearly what is meant by the '5% level of significance'. *[2 marks]*

5. *[Maximum mark: 12]*

Fulchester United played 38 matches their local league, 19 at home and 19 away. The results are shown below.

	Won	**Drawn**	**Lost**	**Total**
Home	12	5	2	19
Away	5	8	6	19
Total	17	13	8	38

Their manager believes that there should be no difference between their home and away results. He plans a χ^2 test at the 5% level of significance.

a) Write down suitable hypotheses for this test. *[2 marks]*

b) Evaluate a table of expected results. *[3 marks]*

c) Calculate the chi-squared test statistic, *p*. *[3 marks]*

d) Write down the critical value of χ^2 from your table, clearly stating your degrees of freedom. *[2 marks]*

e) Make a conclusion for your test. *[2 marks]*

Answers

1. a) $p = 0.0198$ b) $\bar{x} = 101.36$

   c) A type I error is when the null hypothesis is rejected when in fact the null is true.

2. a) $p = 0.15$ b) standard error = 0.017854

   c) [0.115, 0.185]

3. a) i) $p = 0.00195$ ii) $p = 0.2871$

   b) mean=99 variance=841

   c) $p = 0.0393$

   d) $p = 0.4314$

4. a) $H_0 : \mu = \frac{1}{6}$ and $H_1 : \mu > \frac{1}{6}$

   $\bar{x} = 16.67, s.e = 3.727$

   i) $x = 21.44$, Reject H_0.

   ii) $x = 22.79$, Accept H_1.

   b) 5% chance that the decision to accept the null is wrong.

5. a) H_0: there is no difference between home and away results.

H_1: there is a difference between home and away results.

b)

	Won	**Drawn**	**Lost**
Home	8.5	6.5	4
Away	8.5	6.5	4

Table needs to be:-

	Won	**Drawn/Lost**
Home	8.5	10.5
Away	8.5	10.5

c) $p = 5.216$

d) $v = 1, \chi^2 = 3.841$

e) Reject the null hypothesis. There is sufficient evidence at the 5% level that the results home and away are different.