1. *[Maximum mark: 11]*

A recent opinion poll in Holland found that only 350 out of 1400 people supported a new European Union resolution.

a) Calculate the unbiased estimate of the proportion, p, of the whole population that will support the resolution. *[2 marks]*

b) Calculate the standard error of your estimate. *[3 marks]*

c) Calculate a 99% confidence interval for p. *[4 marks]*

d) State an assumption required to find this interval. *[2 marks]*

2. *[Maximum mark: 13]*

In a certain game 2 dice are thrown and their scores are added. If the score is five then the player wins a prize.

a) Show that the probability of getting a 5 is $\frac{1}{9}$. *[1 marks]*

b) Calculate the probability that the player,

(i) gets at least 1 five in his first 8 throws,

(ii) gets his five on his 7^{th} throw,

(iii) throws his second five on the 12^{th} throw. *[10 marks]*

c) On which throw he is most likely to get his first five? *[2 marks]*

3. *[Maximum mark: 9]*

Mick has been told, by the doctor, to monitor the amount of beer he consumes. Mick believes that he drinks a mean average of 4 cans per day. To test this claim he kept a record for 10 days as follows:

3 5 6 8 4 4 3 3 4 4

You may assume that the amount of cans he drinks each day are normally distributed.

a) State a suitable hypotheses. *[2 marks]*

b) Test Mick's belief, at the 5% level of significance. *[5 marks]*

c) Justify your choice of test. *[2 marks]*

4. *[Maximum mark: 12]*

English A1 IB results are known to follow a normal distribution with a mean of 62% and a variance of 100% for the entire cohort.
At Oldfield School 30 girls sat the A1 paper and their mean was 66%. The teachers believe that the girls at Oldfield school have a higher mean average than the cohort.

Carry out a hypothesis test to test this claim giving your conclusion at,

a) the 5% level of significance,

b) the 1% level of significance. *[12 marks]*

5. *[Maximum mark: 15]*

The time it takes to complete a HL maths examination is thought to be modeled by the probability density function that is defined as $\{k(t^2-1), 1 \le t \le 3$. t is the time take to complete the examination in hours.

a) Show that $k = \frac{3}{20}$. *[2 marks]*

b) A keen maths teacher studies the time taken for 50 students to complete the examination. He records his results into a table with 5 intervals as follows:

Time taken	$1 < t < 1.5$	$1.5 < t \le 2$	$2 < t \le 2.4$	$2.4 < t \le 2.8$	$2.8 < t \le 3$
Students observed	4	10	18	10	8

Use the model above to complete a table of expected values for the 50 students using the time intervals above. *[5 marks]*

c) Perform a χ^2 goodness of fit test at the 5% level of significance to test if the observed data fits the model. *[8 marks]*

Answers

1. a) $p = \frac{1}{4}$

   b) standard error = 0.01157

   c) [0.2202, 0.2798]

   d) Sample was random and unbiased

2. b) i) 0.6103 ii) 0.0548 iii) 0.0418

   c) 1st throw.

3. a) $H_0 : \mu = 4$ and $H_1 : \mu \neq 4$

   b) $\bar{x} = 4.4, S_{n-1} = 1.5776$

   $p = \frac{4.4 - 4}{1.5776} = 0.253$

   $v = 9, t = 2.262$

   Do not reject H_0. The mean is 4 at the 95% level of significance.

   c) A t-test is used as the sample is small and the variance of the population is unknown.

4. a) $H_0 : \mu = 62$ and $H_1 : \mu > 62$

   standard error = $\frac{10}{\sqrt{30}} = 1.826$

   $\frac{x - 62}{1.826} = 1.6448$ $x = 65$

   Reject H_0. At the 5% level of significance the results at Oldfield are better than the entire cohort.

   b) Accept H_0. At the 1% level of significance the results at Oldfield are better not better than the entire cohort.

5. b)

Time taken	$1 < t < 1.5$	$1.5 < t \leq 2$	$2 < t \leq 2.4$	$2.4 < t \leq 2.8$	$2.8 < t \leq 3$
Students expected	2.19	7.84	11.54	17.32	11.1

c) $p = 9.666$

$v = 4$, $\chi^2 = 9.488$

Reject H_0, in favour of H_1. The distribution at the 5% level of significance does not fit the model put forward.