Specimen/510/II(1)

INTERNATIONAL BACCALAUREATE

MATHEMATICS

Higher Level

Specimen Paper
(corresponding to examinations from May 1995 onwards)

Paper 1

2 hours

This examination paper consists of twenty questions.
The maximum mark for each question is 4.
The maximum mark for this paper is 80.
This examination paper consists of 15 pages.

BOX 1

CANDIDATE NUMBER							
CANDIDATE NAME							

INSTRUCTIONS TO CANDIDATES

DO NOT open this examination paper until instructed to do so.

Complete BOX 1 before starting the examination.

Answer ALL questions in the spaces provided.

Unless otherwise stated in the question, all numerical answers which are not exact should be given correct to three significant figures.

EXAMINER USE ONLY		

EXAMINATION MATERIALS

Required/Essential:

IB Statistical Tables
Electronic calculator
Ruler and compasses

Allowed/Optional:

A simple translating dictionary for candidates not working in their own language
Millimetre square graph paper

FORMULAE

Trigonometrical identities :

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \cos^2\theta - \sin^2\theta$$

$$\text{If } \tan\frac{\theta}{2} = t \text{ then } \sin\theta = \frac{2t}{1+t^2} \text{ and } \cos\theta = \frac{1-t^2}{1+t^2}$$

Integration by parts :

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Standard integrals :

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + c \quad (|x| < a)$$

Statistics : If $(x_1, x_2, \ldots, x_n)$ occur with frequencies $(f_1, f_2, \ldots, f_n)$ then the mean m and standard derivation s are given by

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} \qquad s = \sqrt{\frac{\Sigma f_i (x_i - m)^2}{\Sigma f_i}}, \quad i = 1, 2, \ldots n$$

Binomial distribution :

$$p_x = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots n$$

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for a correct method provided this is shown by written working in the space provided.

1. Given the function $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$, and assuming $x \in \mathbb{R}$, find

(a) the domain of f;

(b) the range of f.

Working:

Answer:

2. A box contains 50 screws of which 4 are defective. If 10 screws are chosen at random, what is the probability that

(a) none of them is defective;

(b) exactly one of them is defective?

Working:

Answer:

3. A triangle ABC is such that angle $A = 58°$, $AB = 10$ cm and $AC = 5.1$ cm.

(a) Find BC, correct to three significant figures.

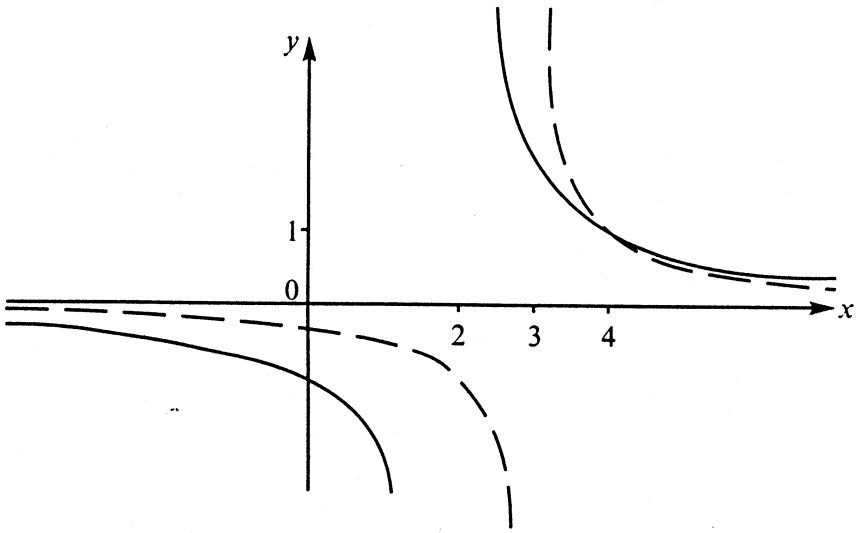
(b) Find the size of $\widehat{C}$, correct to the nearest tenth of a degree.

Working:

Answer: _____

_____

4. The graphs of $f(x) = \dfrac{2}{x-2}$ and $g(x) = \dfrac{1}{x-3}$ are shown as a solid line and a dotted line respectively.



(a) State the value of x for which $g(x)$ is undefined.

(b) Find $f(4)$ and $g(4)$.

(c) State the values of x for which $f(x) \leqslant g(x)$.

Working:

Answer:

5. (i) Vehicle licence plates are composed of two letters from a 24-letter alphabet, followed by a three-digit number whose first digit cannot be zero. How many different licence plates are possible?

(ii) How many different permutations are there of the letters LILLEHAMMER?

Working:

Answer: _____

_____

6. The second term of a geometric sequence is $-\frac{40}{3}$. The sum to infinity of the corresponding geometric series is 12.

Find the first term and the common ratio.

Working:

Answer: _____

7. Find the values of x and y, given that

$$\begin{pmatrix} x & y \\ x & 0 \end{pmatrix}\begin{pmatrix} 4 & x \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} -2 & 5 \\ 4x & 9 \end{pmatrix}.$$

Working:

Answer: _____

8. The roots of the equation

$$x^2 + kx + 2 = 0$$

are α and β. Find, in terms of k,

(a) $\alpha^2 + \beta^2$;

(b) a quadratic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.

Working:

Answer: _____

_____

12. Given $|\vec{u}| = 2$, $|\vec{v}| = 3$, and $\vec{u} \times \vec{v} = \vec{i} - 3\vec{j} - 4\vec{k}$, find the possible values of $\vec{u} \cdot \vec{v}$.

Working:

Answer:

13. Correctly rewrite each of the following erroneous mathematical statements.

(a)
$$\int_0^{\sqrt{3}} \frac{1}{1+x^2}\,dx = [\arctan x]_0^{\sqrt{3}}$$
$$= \arctan\sqrt{3} - \arctan 0$$
$$= 60 - 0 = 60$$

(b) $\cos 180° + i\sin 180° = e^{180i}$

Working:

Answer:

14. In a certain university where 65% of the students are female, it is found that 7% of the male students, and 2% of the female students, are taller than 1.8 m .

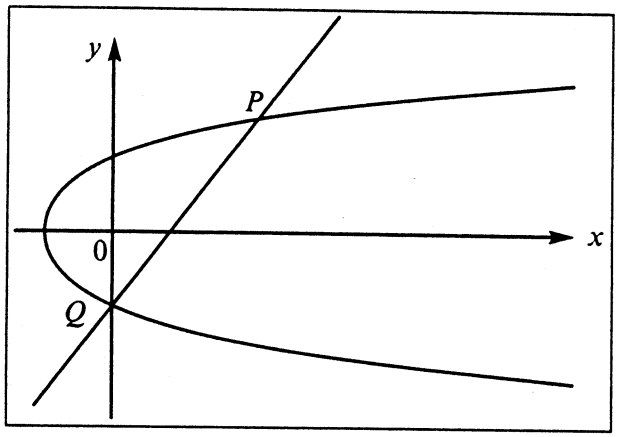
(a) Find the probability that a student selected at random is taller than 1.8 m .

(b) If a student is selected at random and is taller than 1.8 m, what is the probability that the student is male?

Working:

Answer:

15. The parabola has equation $y^2 = x + 1$ and the straight line has equation $y = x - 1$.



The real solution(s) of the equation $\sqrt{x+1} = x - 1$ can be found by observing the intersections at P only, P and Q, or Q only.

Determine which **one** of these three possibilities is correct, and give a short explanation for your choice.

Working:

Answer:

16. Given $f(x) = \cos x$, find, as an **exact value**, the volume generated when the region bounded by the curve, the x axis and the line $x = \frac{\pi}{3}$ is rotated through a full turn around the x-axis.

Working:

Answer:

17. Find, **in terms of e**, the coordinates of the point of inflection of the curve

$$y = x^3 \ln x, \qquad x > 0.$$

Working:

Answer:

18. A couple is told that the probability that they will have blue-eyed children is $\frac{1}{4}$. The couple would like to have 6 children.

(a) What is the expected number of blue-eyed children?

(b) What is the probability that 3 of the 6 children will be blue-eyed?

(c) What is the probability that all 6 children will be blue-eyed?

Working:

Answer:

19. State the values, or the range of values, of c so that the following system of equations

$$2x + y + z = 1$$
$$x - y - z = 2$$
$$4x - y - z = c$$

(a) has an infinity of solutions;

(b) has no solution.

Working:

Answer:

20. Find the possible values of $i^{\frac{3}{4}}$ in the form $a + bi$, where a and b are real numbers, and $i^2 = -1$.

Working:

Answer: