As a guideline this paper should be completed in 1 hour.

You will need a Graphics Display Calculator (GDC) for this examination.

Section A [16 marks]

1. *[5 marks]*

   IB female students' heights are normally distributed with a mean of 162 cm and a standard devaition of 12 cm. 95% of the students lie within the range a to b, where the values of a and b are symmetrical about the mean and a is smaller than b.

   Find the values of a and b.

2. *[6 marks]*

   A sequence is described as $u_n = 2 \times 3^{n-1}$, where u_n is the next term of the sequence.

   Find,

   a) u_1

   b) the common ratio, r,

   c) the sum of the first 15 terms.

3. *[5 marks]*

   Find the angle between the y-plane and the plane $2x + 3y - z = 3$.

Section B [44 marks]

4. *[Maximum mark 27]*

i) The continuous random variable X has probability function $f(x)$, given by:

$f(x) = kx^2(3 - x)$ for $0 \leq x \leq 3$
$f(x) = 0$ otherwise.

a) Show that $k = \frac{4}{27}$, and hence sketch the graph of $f(x), 0 \leq x \leq 3$. *[3 marks]*

b) Find the expected number of X, E(X). *[3 marks]*

c) Find the variance of X, $V(X)$. *[3 marks]*

d) State the mode of X. *[1 mark]*

e) Find the exact probability that $X < 2$. *[2 marks]*

f) Use your answer to e) to state, with a reason, whether the median of X is less than 2, equal to 2, or more than 2. *[2 marks]*

ii) The proportion of people in Malawi suffering from polio is known to be 0.005. A random sample of 400 Thai people is selected.

a) Explain why the Poisson distribution can be used to approximate the number of people suffering from polio for a given sample. *[2 marks]*

b) For a sample of 400 people, state clearly the expected number, E(X), of people suffering from polio and the variance V(X) of the distribution. *[2 marks]*

c) Use the Poisson distribution to find the probability that the sample will contain 1,2 or 3 people who suffer from polio. *[5 marks]*

5. *[Maximum mark 17]*

i) $f(x) = 3\cos x - 2\sin x$ and $g(x) = \cos x + \sin x$

a) Given that $f(x) = A(\cos x + \sin x) + B(\cos x - \sin x)$, find the values of A and B. *[2 marks]*

b) Show that the exact value of $\tan x = \frac{3}{2}$ when $\frac{f(x)}{g(x)} = 0$. *[2 marks]*

c) Show that $g(x)g(3x) = \cos(2x) + \sin(4x)$. *[2 marks]*

ii) $h(x) = x^2 \ln x$.

a) Sketch a graph of $h(x)$. *[2 marks]*

b) Show that the exact value of x at the minimum point of $h(x)$ is $e^{-\frac{1}{2}}$. *[3 marks]*

c) By considering $h''(x)$ find the x-coordinate at the point of inflection of $h(x)$. *[3 marks]*

d) Find the area created below the x-axis and above the curve. *[3 marks]*

Answers

1. $a = 138.48, b = 185.52$

2. a) 2 b) $r = 3$ c) 14348906

3. $\theta = 36.7°$

4. i) b) $\frac{9}{5} = 1.8$ c) $\frac{9}{25} = 0.36$

d) $x = 2$ e) $\frac{16}{27}$

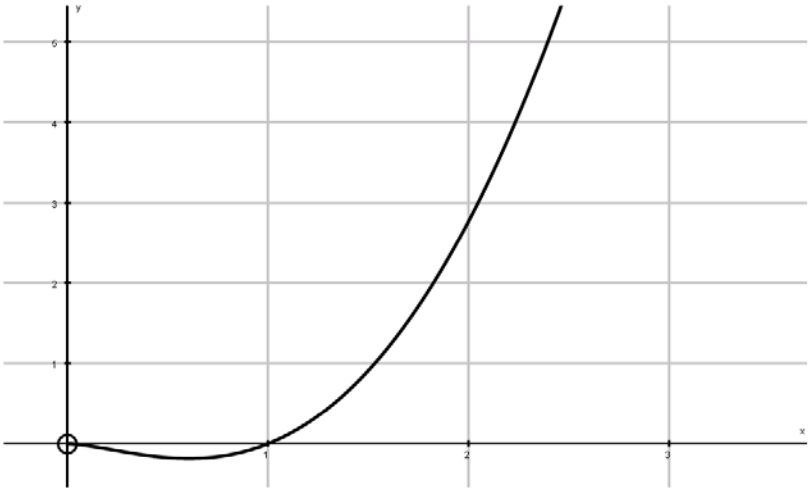
f) Median is less than 2, as probability is greater than 0.5 for e).

ii) a) Random sample within a fixed sample space.

b) $E(x) = 2, V(x) = 2$ c) 0.722

5. i) a) $A = \frac{5}{2}, B = \frac{1}{2}$

ii) a)



c) $x = e^{-\frac{3}{2}}$

d) 0.112