

As a guideline this paper should be completed in 1 hour.

You will need a Graphics Display Calculator (GDC) for this examination.

Section A [30 marks]

1. [Maximum mark 6]

The angle θ satisfies the equation $3 \tan^2 \theta - 2 \sec \theta = 8$, where θ is in the first quadrant. Find the value of θ in degrees.

2. [Maximum mark 6]

$$f(x) = \sqrt{3 - \frac{1}{x^2}}.$$

a) Find the set of values of x such that $f(x)$ is real and finite.

b) Find the range of $f(x)$.

3. [Maximum mark 6]

The ACME chocolate factory runs tours. On average 80% of the people who take these tours are overweight.

a) Calculate the probability that exactly 6 people out of 8 who take the tour are overweight.

b) Calculate the probability that at least 6 people out of 8 who take the tour are overweight.

4. [Maximum mark 6]

Solve the inequality $\frac{1}{x-4} > \frac{x}{x-6}$.

5. [Maximum mark 6]

A survey was conducted to find the average number of passengers on any given flight between London and Barcelona. The results are given in the table below.

Number of passengers	31 - 60	61 - 80	81 - 120	121 - 150	151 - 180	181 - 220
Frequency	21	52	73	95	62	47

- a) Calculate an estimate of the mean number of passengers on these flights.
- b) Calculate an estimate of the standard deviation of the number of passengers on these flights.

Section B [30 marks]

6. [Maximum mark 14]

- i) A company truck is bought at the start of 2004 for \$24000. It has been calculated that each year the truck will depreciate by 15%.
 - a) Show that at the end of 2005 the truck will be worth \$17340. [1 mark]
 - b) The truck will be replaced when the value of the truck falls below \$10000. Evaluate the year in which the truck will be replaced. [2 marks]
 - c) The company will require extra premises. They have planned to buy land in 10 years time, at a value of \$120000. In order to purchase the land they will save x amount per year. The bank has offered an interest rate of 8% per annum on the savings.

Calculate the value of x in order to afford the land in 10 years time. [5 marks]

- ii) Prove by induction that,

$$\sum_{r=1}^{r=n} nr = \frac{n(n+1)(2n+1)}{6} \quad [6 \text{ marks}]$$

7. [Maximum mark 16]

- i) The function f is defined on the domain $x > 0$ by $f(x) = \frac{e^x}{3x}$.

- a) Find $f'(x)$,
- b) find $f''(x)$,
- c) hence find the exact value of the coordinates of the minimum value of $f(x)$. [7 marks]

- ii) The function g is defined by $g(x) = \frac{x}{x^2 + 5}$.

- a) Find the R , area bounded by the function g , the x -axis, and the lines $x=0$ and $x=4$.
Give your answer to 3 decimal places. [4 marks]
- b) Find the coordinates of the three turning points of $g(x)$, distinguishing between each. [5 marks]

Answers

1. $\theta = 26.05^\circ$

2. a) $x \geq \frac{1}{\sqrt{3}}, x \leq -\frac{1}{\sqrt{3}}$

b) $y \geq \sqrt{2}$

3. a) 0.294 b) 0.7973

4. $2 < x < 3$

5. a) $\bar{x} = 127.4$ b) $\sigma = 44.9$

6. i) b) 2009

c) $x \approx 7670$

7. i) a) $f'(x) = \frac{e^x(x-1)}{3x^2}$

b) $f''(x) = \frac{e^x(x^2 - 2x + 2)}{3x^3}$

c) $\left(1, \frac{e}{3}\right)$

ii) a) 0.718

b) (2.236, 0.224) maximum point

(-2.236, -0.224) minimum point

(0, 0) point of inflexion