

As a guideline this paper should be completed in 1 hour.

No Calculator to be used in this examination.

Section A [36 marks]

1. [Maximum mark 6]

If $\sin A = \frac{2}{3}$ and A is obtuse, find the exact values of:

- a) $\cos A$,
- b) $\sin 2A$,
- c) $\tan 2A$.

2. [Maximum mark 6]

$z_1 = \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$ and $z_2 = 3\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$, where $i = \sqrt{-1}$.

Write $z_1 \div z_2$ in the form $a(\cos x + i \sin x)$, where a and x are exact real numbers.

3. [Maximum mark 5]

Find the equation of the curve that passes through $(1, -3)$ and has the differential equation,

$$\frac{dy}{dx} = \frac{y}{x \ln x}.$$

4. [Maximum mark 5]

If $f(x) = \ln(4 - 3x)$, find $f''(x)$.

5. [Maximum mark 4]

The curve $y = 3x^4 - 4x^3$ has two turning points. Find each point and determine the nature of the turning point.

6. [Maximum mark 6]

A biased die is such that the probabilities of landing on each of the numbers from 1 to 6 is given below.

Score	1	2	3	4	5	6
Probability	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	x	x	x

- Find the value of x .
- Hence, find the mean expected score, $E(x)$.
- Find the variance of the expected score, $V(x)$.

7. [Maximum mark 4]

$$z^3 = 8 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right), \text{ where } i = \sqrt{-1}.$$

Find the solutions for z in the form $z = a(\cos \theta + i \sin \theta)$.

Section B [24 marks]

8. [Maximum mark 24]

- i) The line $\frac{x-2}{3} = \frac{2-y}{1} = \frac{z+1}{2}$ is reflected in $x + y + z = 1$. Find the equation of the line. [5 marks]
- ii) A plane, π , contains the points $A(2, 1, 4)$ and $B(3, -2, -5)$.
- a) Find the unit vector in the direction AB . [2 marks]
- b) Show that the equation of the plane that contains the points A and B is π : [4 marks]
- c) Hence, show that the point C with vertices $(1, 3, 5)$ exists on the plane π . [1 mark]
- d) Find the area of the triangle ABC . [4 marks]
- e) The point P is in space defined as $(1, 3, 5)$. Find the shortest distance from P to the plane π . [5 marks]
- f) Find the angle between the plane π and the x plane. [3 marks]