As a guideline, this paper should be completed in 1 hour.

No Calculator to be used in this examination.

Section A [38 marks]

1. *[Maximum mark 6]*

   Find the area of the triangle that has vertices (2,1,1), (4,2,1), and (-3,2,-1).

2. *[Maximum mark 6]*

   Solve the differential equation $\frac{dy}{dx} = y^2(1 + x^2)$, given that x=-3 when y=1. Give your answer in the form $y = f(x)$.

3. *[Maximum mark 6]*

   Find the Cartesian equation of the straight line that is formed at the intersection of the planes:

   $$\pi_1 : 3x - y + z = -2 \text{ and } \pi_2 : 2x + y - 2z = 1$$

4. *[Maximum mark 6]*

   The curve $y = x^3 + ax^2 + bx + 1$ at the point (2,7) has the tangent $y = 17x - 27$.

   Find the values of a and b.

5. *[Maximum mark 4]*

   The complex number z satisfies the equation,

   $iz + 4 = (2 - i)$

   where z is in the form $(a + ib)$, and $i = \sqrt{-1}$.

   Find z in the form $a + ib$, where a and b are real.

6. *[Maximum mark 5]*

Find $\int \left(x^2 \sin x\right) dx$.

7. *[Maximum mark 5]*

The function $f(x) = x^3 - ax^2 - bx + 30$ is exactly divisible by $(x - 2)$ and leaves a remainder of 16 when divided by $(x - 1)$.

Find the values of a and b.

Section B [22 marks]

8. *[Maximum mark 22]*

i) Write the complex number $r = \sqrt{3} - i$ in modulus and argument form. *[2 marks]*

ii) Use De Moivre's theorem to show the identity,

$$\tan 5\theta \equiv \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$ *[6 marks]*

iii) $z_1 = 5\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ and $z_2 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, where $i = \sqrt{1}$.

a) Find $\left(z_1\right)^2 \times \left(z_2\right)^3$. *[4 marks]*

b) Find $\dfrac{\left(z_1\right)^3}{\left(z_2\right)^2}$ *[5 marks]*

iv) If $z = \cos\theta + i\sin\theta$, prove that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$. *[5 marks]*

Answers

1. $\frac{1}{2}\sqrt{69}$

2. $y = \frac{-3}{x + x^3 - 3}$

3. $(t =)\frac{5x+1}{1} = \frac{5y+13}{8} = \frac{z}{1}$ (or equivalent)

4. $a = 3, b = -7$

5. $a = -1, b = -2$

6. $-x^2 \cos x + 2x \sin x + 2\cos x + c$

7. $a = 4, b = 11$

8. i) $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

iii) a) $200\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right)$

b) $\frac{125}{4}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$