

As a guideline this paper should be completed in 1 hour.

No Calculator to be used in this examination.

Section A [26 marks]

1. *[Maximum mark 5]*

Given that $\mathbf{a} = i + 2j + k$ and $\mathbf{b} = 4i - 3j + 2k$ and $\mathbf{c} = 2i - j + 3k$, find $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$.

2. *[Maximum mark 5]*

The complex number z satisfies $i(z + 3) = 3 - 4z$, where $i = \sqrt{-1}$. Write z in the form $a + bi$, where a and b are real numbers.

3. *[Maximum mark 4]*

A die is biased such that the probability of it landing on a 4 is $\frac{1}{4}$.

The die is rolled 720 times. Let X be the number of 4's obtained.

- the expected value of X .
- the standard deviation of X .

4. *[Maximum mark 6]*

Find the exact value of $\tan \theta$ given the equation $\frac{2}{\sin^2 \theta} - \frac{3}{\tan \theta} = 7$ and that θ lies in the third quadrant.

5. *[Maximum mark 6]*

Find the equation of the tangent to the curve $y^2 + 2xy = 3x^2 + 9$ at the point where $x=2$ and $y=3$.

Give your answer in the form $ax + by + c = 0$.

Section B [34 marks]

6. [Maximum mark 20]

- i) a) Find the value of k in the following set of equations such that the equations have an infinite set of solutions.

$$\begin{aligned}2x + y + 4z &= k \\2x + 5y + 7z &= 3 \\6x - 21y - 6z &= 0\end{aligned}$$

[6 marks]

- b) For this value of k find the general set of solutions for these equations. [3 marks]

- ii) Show that the following set of equations have no solutions,

$$\begin{aligned}2x + y - 3z &= 4 \\4x + y + 4z &= 2 \\2x + 5y - 4z &= 30\end{aligned}$$

[4 marks]

- iii) Show that the lines $\frac{x-1}{1} = \frac{y-3}{5} = \frac{z-4}{1}$ and

$\frac{x-3}{1} = \frac{y-10}{2} = \frac{z-6}{1}$ intersect and find the coordinate at the point of intersection. [7 marks]

7. [Maximum mark 14]

- i) A sequence of numbers is such that,

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

Prove the above statement by use of mathematical induction. [5 marks]

- ii) A factory starts by producing 50 widgets a week and each week will produce 6 more widgets than the previous week.
- a) Show that it will produce 500 widgets in the 76th week.
[2 marks]
- b) The factory freezes production at 500 widgets per week after the 76th week. Find the total produced number of widgets produced in the first 100 weeks of production.
[3 marks]
- iii) Find the sum to infinity of the series $\frac{3}{4}, \frac{3}{10}, \frac{3}{25}, \frac{6}{125}, \dots$
[4 marks]

Paper C

IB HL Paper 1 Practice Papers

Answers

1. -21

2. $z = \frac{9}{17} + i\frac{15}{17}$

3. a) 180 b) $3\sqrt{15}$

4. $\tan\theta = \frac{2}{5}$

5. $3x - 5y + 9 = 0$

6. i) a) $k = 2$

b) $(t =) \frac{5 - 8x}{13} = \frac{3 - 4y}{3} = \frac{z}{1}$ (or equivalent)

iii) (2, 8, 5)

7. ii) a) 76th term b) 32900

iii) $\frac{5}{4}$