As a guideline this paper should be completed in 1 hour.

No Calculator to be used in this examination.

Section A [16 marks]

1. *[Maximum mark 6]*

   Find the exact value of $\int_1^4 \left(\frac{3\ln x}{x}\right)dx$.

2. *[Maximum mark 4]*

   The function $f(x) = x^3 - 5x^2 + ax + b$ has a factor of $(x+1)$ and leaves a remainder of 6 when divided by $(x-1)$.

   Find the constant values of a and b.

3. *[Maximum mark 6]*

   $f(x) = \frac{3x+1}{x+2}$ where $x \neq -2$.

   a) Find $f^{-1}(x)$.

   b) State the domain of $f^{-1}(x)$.

Section B [44 marks]

4. *[Maximum mark 22]*

   i) a) Prove by mathematical induction that,
   $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$,
   when n is a positive integer. *[4 marks]*

   b) Use De Moivre's theorem to write $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$. *[5 marks]*

ii) Consider $z^4 - 81 = 0$, where $z = a + ib$.

a) Show that $z_1 = 3$ is one of the complex roots of this equation.

b) Find z_2, z_3, z_4. Give your answer in modulus argument form.

c) Plot the points z_1, z_2, z_3, z_4 in the complex plane.

[8 marks]

iii) Consider the complex number $z = \dfrac{\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^5}{\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^6}$.

a) Simplify the complex number z.

b) State the argument and modulus of z. *[5 marks]*

5. *[Maximum mark 22]*

i) Two planes are defined as: $\pi_1 : 3x + y - z = 23$ and $\pi_2 : 2x - 3y + 2z = 5$

a) A point X is at the foot of the perpendicular from the point (2,1,6) to the plane π_1.

Find the coordinates of X. *[5 marks]*

b) Find the Cartesian equation of the line formed at the point of intersection of the two planes. *[5 marks]*

ii) The points A (3,2,1) and B(4,-5,-1) lie on a straight line, l.

a) Find an equation of *the* line l, giving your answer in parametric form. *[4 marks]*

b) Express the line l in Cartesian form. *[2 marks]*

c) Find the point, P, on l that is perpendicular to OP. *[3 marks]*

iii) The points P(1,2,3), Q(4,2,-1) and R(2,1,5) lie on the same plane.

Find the equation of the plane. *[4 marks]*

Answers

1. $\frac{3}{2}(\ln 4)^2$

2. $a = 2, b = 8$

3. a) $f(x)^{-1} = \frac{1-2x}{x-3}$

   b) $x \in \Re, x \neq 3$

4. i) b) $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 10\cos\theta\sin^4\theta$

   ii) a) $z_1 = 3(\cos 0 + i\sin 0)$

   b) $z_2 = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

   $z_3 = 3(\cos\pi + i\sin\pi)$

   $z_2 = 3\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

   iii) a) $z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

   b) modulus = 1, argument = $\frac{\pi}{2}$.

5. i) a) (8,3,4)

b) $(t =)\dfrac{11x-70}{1}=\dfrac{11y-31}{8}=\dfrac{z}{1}$ (or equivalent)

ii) a) $\begin{pmatrix}3\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-7\\-2\end{pmatrix}$

b) $\dfrac{x-3}{1}=\dfrac{y-2}{-7}=\dfrac{z-1}{-2}$

c) $P\left(3\dfrac{7}{27},\dfrac{4}{27},\dfrac{13}{27}\right)$

iii) $4x-10y-3z=-25$