

As a guideline this paper should be completed in 1 hour.

No Calculator to be used in this examination.

Section A [36 marks]

1. [Maximum mark 6]

Use the substitution $u = x - 1$ to find $\int \left(\frac{x}{x-1} \right)^2 dx$.

2. [Maximum mark 6]

$$x = \log_a 5 \text{ and } y = \log_a 2.$$

Find in terms of x and y ,

a) $\log_a 100$,

b) $\log_2 25$.

3. [Maximum mark 6]

A die is biased such that the probability of getting a six is $\frac{1}{4}$. The die is rolled 2000 times. Let X be the number of sixes obtained. Find,

a) the mean of X ,

b) the standard deviation of X , leaving your answer as a surd.

4. [Maximum mark 6]

Write the complex number $\frac{(1 - \sqrt{3}) + i(1 + \sqrt{3})}{(1 + i)}$ in the form $z = r(\cos \theta + i \sin \theta)$, where $i = \sqrt{-1}$, and r is a real number.

5. [Maximum mark 6]

Find the equation of the normal at the point $(2, -1)$ on the curve with the equation $y^2 + 3xy = x^2 - 9$.

Give your answer in the form $ax + by + c = 0$.

6. [Maximum mark 6]

The probability of event $A = 0.5$, and the probability of $(A \cup B) = 0.7$. Given that the probabilities of A and B are independent, find the probability of event B happening.

Section B [24 marks]

7. [Maximum mark 24]

i) Find the coordinate at the point of intersection between the two lines:

$$\frac{x-1}{3} = \frac{y}{2} = \frac{z+4}{-1} \text{ and } \frac{x-12}{-2} = \frac{y-7}{-1} = \frac{z+11}{4} \quad [6 \text{ marks}]$$

ii) A triangle has the three vertices $A(2, 0, -5)$, $B(4, 1, 1)$, $C(3, -1, -2)$.

a) Find the vector product $AB \times AC$, giving your answer in the form $ai + bj + ck$.

b) Hence, find the exact area of the triangle ABC .

c) Find the Cartesian equation of the plane that contains the triangle ABC . [13 marks]

iii) Find the values of a and b , such that the simultaneous equations,

$$x + 4y + 2z = -1$$

$$5x + 25y + z = 3$$

$$2x + 3y + az = b$$

have an infinite number of solutions.

[5 marks]

Answers

1. $(x-1) - 2\ln(x-1) + \frac{1}{x-1} + c$

2. a) $2x + 2y$ b) $\frac{2x}{y}$

3. a) 500 b) $5\sqrt{15}$

4. $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

5. $7x - 4y - 10 = 0$

6. $P(B) = 0.4$

7. i) $(10, 6, -7)$

ii) a) $9i - 3k$

b) $\frac{3}{2}\sqrt{10}$

c) $9x - 3z = 33$

iii) $a = 13, b = -10$