Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – sets, relations and groups

Monday 8 May 2017 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 10]

The set A contains all positive integers less than 20 that are congruent to 3 modulo 4.
The set B contains all the prime numbers less than 20.

(a) (i) Write down all the elements of A and all the elements of B.

(ii) Determine the symmetric difference, $A \Delta B$, of the sets A and B. [4]

(b) The set C is defined as $C = \{7, 9, 13, 19\}$.

(i) Write down all the elements of $A \cap B$, $A \cap C$ and $B \cup C$.

(ii) Hence by considering $A \cap (B \cup C)$, verify that in this case the operation $\cap$ is distributive over the operation $\cup$. [6]

2. [Maximum mark: 11]

The relation R is defined such that aRb if and only if $4^a - 4^b$ is divisible by 7, where $a, b \in \mathbb{Z}^+$.

(a) (i) Show that R is an equivalence relation.

(ii) Determine the equivalence classes of R. [9]

The equivalence relation S is defined such that cSd if and only if $4^c - 4^d$ is divisible by 6, where $c, d \in \mathbb{Z}^+$.

(b) Determine the number of equivalence classes of S. [2]

3. [Maximum mark: 13]

The function $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ is defined by $f(x, y) = \left(2x^3 + y^3, x^3 + 2y^3\right)$.

(a) Show that f is a bijection. [12]

(b) Hence write down the inverse function $f^{-1}(x, y)$. [1]

4. [Maximum mark: 16]

The binary operation $*$ is defined by

$$a * b = a + b - 3 \text{ for } a, b \in \mathbb{Z}.$$

(a) Show that $\{\mathbb{Z}, *\}$ is an Abelian group. [9]

(b) Show that there is no element of order 2. [2]

(c) Find a proper subgroup of $\{\mathbb{Z}, *\}$. [2]

The binary operation $\circ$ is defined by

$$a \circ b = a + b + 3 \text{ for } a, b \in \mathbb{Z}.$$

Consider the group $\{\mathbb{Z}, \circ\}$ and the bijection $f: \mathbb{Z} \to \mathbb{Z}$ given by $f(a) = a - 6$.

(d) Show that the groups $\{\mathbb{Z}, *\}$ and $\{\mathbb{Z}, \circ\}$ are isomorphic. [3]