Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Friday 5 May 2017 (morning)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2217 – 7204

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider two events A and B such that $\mathrm{P}(A) = k$, $\mathrm{P}(B) = 3k$, $\mathrm{P}(A \cap B) = k^2$ and $\mathrm{P}(A \cup B) = 0.5$.

(a) Calculate k; [3]

(b) Find $\mathrm{P}(A' \cap B)$. [3]

2. [Maximum mark: 7]

The curve C is defined by equation $xy - \ln y = 1$, $y > 0$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y . [4]

(b) Determine the equation of the tangent to C at the point $\left(\frac{2}{\mathrm{e}}, \mathrm{e}\right)$. [3]

3. [Maximum mark: 6]

The coefficient of x^2 in the expansion of $\left(\frac{1}{x} + 5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a + 5x)^7$, $a \in \mathbb{R}$. Find the value of a.

4. [Maximum mark: 6]

The region A is enclosed by the graph of $y = 2\arcsin(x - 1) - \frac{\pi}{4}$, the y-axis and the line $y = \frac{\pi}{4}$.

(a) Write down a definite integral to represent the area of A. [4]

(b) Calculate the area of A. [2]

5. [Maximum mark: 6]

When carpet is manufactured, small faults occur at random. The number of faults in Premium carpets can be modelled by a Poisson distribution with mean 0.5 faults per $20\,\mathrm{m}^2$.

Mr Jones chooses Premium carpets to replace the carpets in his office building. The office building has 10 rooms, each with the area of $80\,\mathrm{m}^2$.

(a) Find the probability that the carpet laid in the first room has fewer than three faults. [3]

(b) Find the probability that exactly seven rooms will have fewer than three faults in the carpet. [3]

6. [Maximum mark: 6]

Consider the graph of $y = \tan\left(x + \frac{\pi}{4}\right)$, where $-\pi \le x \le 2\pi$.

(a) Write down the equations of the vertical asymptotes of the graph. [2]

The graph is reflected in the y-axis, then stretched parallel to the y-axis by a factor $\frac{1}{2}$, then translated by $\begin{pmatrix} \frac{\pi}{4} \\ -3 \end{pmatrix}$.

(b) Give the equation of the transformed graph. [4]

7. [Maximum mark: 6]

Find the Cartesian equation of plane Π containing the points $\mathrm{A}(6, 2, 1)$ and $\mathrm{B}(3, -1, 1)$ and perpendicular to the plane $x + 2y - z - 6 = 0$.

8. [Maximum mark: 7]

A water trough which is 10 metres long has a uniform cross-section in the shape of a semicircle with radius 0.5 metres. It is partly filled with water as shown in the following diagram of the cross-section. The centre of the circle is O and the angle KOL is θ radians.

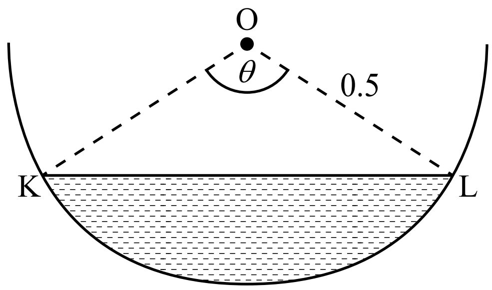

diagram not to scale

(a) Find an expression for the volume of water $V(m^3)$ in the trough in terms of θ. [3]

The volume of water is increasing at a constant rate of $0.0008\,\mathrm{m^3\,s^{-1}}$.

(b) Calculate $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ when $\theta = \frac{\pi}{3}$. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 8]

The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

(a) Find the probability that a runner selected at random will complete the marathon in less than 3 hours. [2]

It is found that 5% of the male runners complete the marathon in less than T_1 minutes.

(b) Calculate T_1. [2]

The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

(c) Find the standard deviation of the times taken by female runners. [4]

10. [Maximum mark: 15]

In triangle PQR, $\text{PR} = 12\,\text{cm}$, $\text{QR} = p\,\text{cm}$, $\text{PQ} = r\,\text{cm}$ and $\text{Q}\hat{\text{P}}\text{R} = 30^\circ$.

(a) Use the cosine rule to show that $r^2 - 12\sqrt{3}\,r + 144 - p^2 = 0$. [2]

Consider the possible triangles with $\text{QR} = 8\,\text{cm}$.

(b) Calculate the two corresponding values of PQ. [3]

(c) Hence, find the area of the smaller triangle. [3]

Consider the case where p, the length of QR is not fixed at $8\,\text{cm}$.

(d) Determine the range of values of p for which it is possible to form two triangles. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 9]

Xavier, the parachutist, jumps out of a plane at a height of h metres above the ground. After free falling for 10 seconds his parachute opens.

His velocity, $v\,\text{ms}^{-1}$, t seconds after jumping from the plane, can be modelled by the function

$$v(t) = \begin{cases} 9.8t, & 0 \le t \le 10 \\ \dfrac{98}{\sqrt{1 + (t-10)^2}}, & t > 10 \end{cases}.$$

(a) Find his velocity when $t = 15$. [2]

(b) Calculate the vertical distance Xavier travelled in the first 10 seconds. [2]

His velocity when he reaches the ground is $2.8\,\text{ms}^{-1}$.

(c) Determine the value of h. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Consider $f(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right)$.

(a) Find the largest possible domain D for f to be a function. [2]

The function f is defined by $f(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right),\ x \in D$.

(b) Sketch the graph of $y = f(x)$ showing clearly the equations of asymptotes and the coordinates of any intercepts with the axes. [3]

(c) Explain why f is an even function. [1]

(d) Explain why the inverse function f^{-1} does not exist. [1]

The function g is defined by $g(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right),\ x \in]1, \infty[$.

(e) Find the inverse function g^{-1} and state its domain. [4]

(f) Find $g'(x)$. [3]

(g) Hence, show that there are no solutions to

(i) $g'(x) = 0$;

(ii) $\left(g^{-1}\right)'(x) = 0$. [4]