Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – discrete mathematics

Wednesday 18 May 2016 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

2216 – 7207

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

(a) Use the Euclidean algorithm to show that 1463 and 389 are relatively prime. [4]

(b) Find positive integers a and b such that $1463a - 389b = 1$. [5]

2. [Maximum mark: 12]

The weights of the edges in the complete graph G are shown in the following table.

	A	B	C	D	E	F
A	–	14	10	8	12	9
B	14	–	9	12	10	13
C	10	9	–	7	8	13
D	8	12	7	–	9	11
E	12	10	8	9	–	11
F	9	13	13	11	11	–

(a) Starting at A, use the nearest neighbour algorithm to find an upper bound for the travelling salesman problem for G. [5]

(b) By first removing A, use the deleted vertex algorithm to find a lower bound for the travelling salesman problem for G. [7]

3. [Maximum mark: 10]

Throughout this question, $(abc\ldots)_n$ denotes the number $abc\ldots$ written with number base n. For example $(359)_n = 3n^2 + 5n + 9$.

(a) (i) Given that $(43)_n \times (56)_n = (3112)_n$, show that $3n^3 - 19n^2 - 38n - 16 = 0$.

(ii) Hence determine the value of n. [3]

(b) Determine the set of values of n satisfying $(13)_n \times (21)_n = (273)_n$. [3]

(c) Show that there are no possible values of n satisfying $(32)_n \times (61)_n = (1839)_n$. [4]

4. [Maximum mark: 17]

(a) Solve the recurrence relation $v_n + 4v_{n-1} + 4v_{n-2} = 0$ where $v_1 = 0$, $v_2 = 1$. [6]

(b) Use strong induction to prove that the solution to the recurrence relation $u_n - 4u_{n-1} + 4u_{n-2} = 0$ where $u_1 = 0$, $u_2 = 1$ is given by $u_n = 2^{n-2}(n-1)$. [8]

(c) Find a simplified expression for $u_n + v_n$ given that,

(i) n is even.

(ii) n is odd. [3]

5. [Maximum mark: 12]

The simple, connected graph G has e edges and v vertices, where $v \geq 3$.

(a) Show that the number of edges in G', the complement of G, is $\frac{1}{2}v^2 - \frac{1}{2}v - e$. [2]

Given that both G and G' are planar and connected,

(b) show that the sum of the number of faces in G and the number of faces in G' is independent of e; [3]

(c) show that $v^2 - 13v + 24 \leq 0$ and hence determine the maximum possible value of v. [7]