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MARKSCHEME

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MATHEMATICS
STATISTICS AND PROBABILITY

Higher Level

Paper 3

12 pages

Instructions to Examiners

Abbreviations

M   Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A   Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R   Marks awarded for clear **Reasoning**.

N   Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1   General

Mark according to Scoris instructions and the document "**Mathematics HL: Guidance for e-marking November 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.

- If a part is completely wrong, stamp **A0** by the final answer.

- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by Scoris.

2   Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, for example, **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (for example, substitution into a formula) and **A1** for using the **correct** values.
- Where the markscheme specifies **(M2)**, **N3**, *etc*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3   *N* **marks**

*Award N marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of *N* and other marks.
- There may be fewer *N* marks available than the total of *M*, *A* and *R* marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4   **Implied marks**

*Implied marks appear in **brackets, for example, (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5   **Follow through marks**

*Follow through **(FT)** marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent** *A* marks can be awarded, but *M* marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6   **Mis-read**

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a mis-read. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (for example, $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7   **Discretionary marks *(d)***

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8   Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9   Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

   Example: for differentiating $f(x) = 2\sin(5x - 3)$, the markscheme gives:

   $$f'(x) = (2\cos(5x - 3))\,5 \quad (= 10\cos(5x - 3)) \qquad\qquad\qquad \textit{A1}$$

   Award *A1* for $(2\cos(5x - 3))\,5$, even if $10\cos(5x - 3)$ is not seen.

10   Accuracy of Answers

*Candidates should **NO LONGER** be penalized for an accuracy error **(AP)**.*

*If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.*

11   Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12   Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (for example, TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:
        Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13   More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1.   (a)   (i) $\bar{v} = \dfrac{1}{1000}(55 \times 5 + 65 \times 13 + \ldots + 145 \times 31)$ ***A1M1***

> **Note:** *A1* for mid-points, *M1* for use of the formula.

$= \dfrac{113210}{1000} = 113.21$ ***AG***

(ii) $s^2 = \dfrac{(55-113.21)^2 \times 5 + (65-113.21)^2 \times 13 + \ldots + (145-113.21)^2 \times 31}{999}$ ***(M1)***

$= \dfrac{362295.9}{999} = 362.6585\ldots = 363$ ***A1***

> **Note:** Award *A1* if answer rounds to 362 or 363.
>
> **Note:** Condone division by 1000.

[4 marks]

(b) $\bar{v} \pm \dfrac{t_{0.025} \times s}{\sqrt{n}}$ ***(M1)***

hence the confidence interval $I = [112.028, 114.392]$ ***A1***

> **Note:** Accept answers which round to 112 and 114.
>
> **Note:** Condone the use of $z_{0.025}$ for $t_{0.025}$ and σ for s.

[2 marks]

(c)   less confidence implies narrower interval   ***R2***

> **Note:** Accept equivalent statements or arguments having a meaningful diagram and/or relevant percentiles.

hence the confidence interval I at the 95 % level contains the confidence interval J at the 90 % level   ***AG***

[2 marks]

Total [8 marks]

2.   (a)   let $W = X - 1.5Y$ **(M1)**
           $E(W) = 180 - 1.5 \times 150 (= -45)$ **A1**
           $Var(W) = 14^2 + 2.25 \times 12^2 \; (= 520 \text{ or } 22.8^2)$ **(M1)A1**
           $W \sim N(-45, 520)$ **(M1)**
           $P(W > 0) = 0.0242$ **(M1)A1**   **N4**

Note: The penultimate **(M1)** is for recognising normality.

[7 marks]

(b)   let $T = X_1 + X_2 + X_3 + X_4 + Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6$ (grams) denote
        the total weight   **(M1)**
        $E(T) = 4 \times 180 + 6 \times 150 \; (= 1620)$ **A1**

Note: Condone correct expected value from $T = 4X + 6Y$.

        $Var(T) = 4 \times 14^2 + 6 \times 12^2 \; (= 1648 \text{ or } 40.6^2)$ **(M1)A1**
        then $T \sim N(1620, 1648)$ **(M1)**
        $P(T > 1500) = 0.998$ **(M1)A1**   **N4**

Note: Accept answers which round to 0.998.

Note: The penultimate **(M1)** is for recognising normality.

[7 marks]

Total [14 marks]

3. (a) (i) H_0: all coins are fair (or the data are represented by $B(7, 0.5)$)   *A1*
         H_1: not all coins are fair (or the data are not represented by $B(7, 0.5)$) *A1*

   (ii) $\chi^2_{cal} > \chi^2_{critical}$ (14.067) or p-value $< 0.05 \Rightarrow$ reject null hypothesis   *A1*

   (iii) let T be the number of tails obtained, T is binomially distributed   *(M1)*
         $T \sim B(7, 0.5)$ *(A1)*

T	0	1	2	3	4	5	6	7
f_0	6	19	141	218	203	117	38	8
f_e	**5.859…**	**41.01…**	**123.0…**	**205.0…**	**205.0…**	**123.0…**	**41.01…**	**5.859…**

A2

> **Note:** Allow tabular values which are correct to 3 significant figures.
>
> **Note:** Award *A1* for 6 or 7 correct values.

   $\chi^2_{calc} = 16.576…$ *A1*

> **Note:** Accept answer which round to 16.6.

   (iv) $v = 7$ *(A1)*
         since $16.576… > 14.067$ or $p = 0.02(034…) < 0.05$, H_0 is rejected   *R1*

> **Note:** Follow through their χ^2_{calc} or *p*-value for the *R1*.

[10 marks]

(b) reduce the significance level (or equivalent statement)   *R2*

[2 marks]

(c) (i) accepting H_0 (or failing to reject H_0) when it is false (or equivalent)   *A1*

   (ii) increase the number of trials   *A1*

[2 marks]

Total [14 marks]

4. H_0: the training schedule does not help improve times (or $\mu = 0$)   *A1*
     H_1: the training schedule does help improve times (or $\mu > 0$)   *A1*

> **Note:** Subsequent marks can be awarded even if the hypotheses are not stated. (Assuming difference of times is normally distributed.)

let d = time before training – time after training   *(M1)*

Competitor	A	B	C	D	E
Time before training (in seconds)	75	74	60	69	69
Time after training (in seconds)	73	69	55	72	65
Difference d	2	5	5	–3	4

A1

EITHER

$n = 5, \sum d = 13, \sum d^2 = 79 \Rightarrow s_{n-1}^2 = \frac{1}{4}\left(79 - \frac{169}{5}\right) = 11.3$ *(M1)*

(small sample) so use a one-sided t-test   *(M1)*

> **Note:** The "one-sided" t-test may have been seen above when stating H_1.

$t = \dfrac{2.6}{\sqrt{\dfrac{11.3}{5}}} = 1.7\ldots$ *A1*

$v = 4$,   *A1*
at the 1 % level the critical value is 3.7   *A1*
since $3.7 > 1.7\ldots$
H_0 is accepted (insufficient evidence to reject H_0)   *R1*

> **Note:** Follow through their t-value.

OR

(small sample) so use a one-sided t-test   *(M1)*
 $p = 0.079\ldots$ *A4*
since $0.079\ldots > 0.01$
H_0 is accepted (insufficient evidence to reject H_0)   *R1*

> **Note:** Follow through their p-value.
>
> **Note:** Accept d = time after training – time before training throughout.

[10 marks]

5.   (a) $E(S) = 2E(X) + 3E(Y) = 6 + 6 = 12$ *A1*
$Var(S) = 4Var(X) + 9Var(Y) = 12 + 18 = 30$ *A1*

[2 marks]

(b)   *S* does not have a Poisson distribution   *A1*
because $Var(S) \neq E(S)$ *R1*

> **Note:** Follow through their E(*S*) and Var(*S*) if different.

[2 marks]

(c)   **EITHER**

$P(T = 3) = P\big((X, Y) = (3, 0)\big) + P\big((X, Y) = (2, 1)\big) +$
$\qquad\qquad + P\big((X, Y) = (1, 2)\big) + P\big((X, Y) = (0, 3)\big)$ *(M1)*
$\qquad = P(X = 3)P(Y = 0) + P(X = 2)P(Y = 1) +$
$\qquad\qquad + P(X = 1)P(Y = 2) + P(X = 0)P(Y = 3)$ *(M1)*
$\qquad = \dfrac{125e^{-5}}{6}(= 0.140)$ *A2*

> **Note:** Accept answers which round to 0.14.

OR

T is $P_o(2 + 3) = P_o(5)$ *(M1)(A1)*

$P(T = 3) = \dfrac{125e^{-5}}{6}(= 0.140)$ *A2*

> **Note:** Accept answers which round to 0.14.

[4 marks]

(d) $P(T = t) = P\big((X, Y) = (0, t)\big) + P\big((X, Y) = (1, t - 1)\big) + \ldots P\big((X, Y) = (t, 0)\big)$ *(M1)*
$\qquad = P(X = 0)P(Y = t) + P(X = 1)P\big(Y = t - 1\big) + \ldots + P(X = t)P(Y = 0)$ *A1*
$\qquad = \displaystyle\sum_{r=0}^{t} P(X = r)P(Y = t - r)$ *AG*

[2 marks]

(e) $P(T = t)$ $= \sum_{r=0}^{t} P(X = r) P(Y = t - r)$

$= \sum_{r=0}^{t} \dfrac{e^{-3} 3^r}{r!} \times \dfrac{e^{-2} 2^{t-r}}{(t-r)!}$ *M1A1*

$= \dfrac{e^{-5}}{t!} \sum_{r=0}^{t} \dfrac{t!}{r!(t-r)!} \times 3^r 2^{t-r}$ *M1*

$= \dfrac{e^{-5}}{t!} (3+2)^t$ *A1*

$\left(= \dfrac{e^{-5} 5^t}{t!} \right)$

hence T follows a Poisson distribution with mean 5   *AG*

[4 marks]

Total [14 marks]