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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Monday 7 May 2012 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 14]*

A baker produces loaves of bread that he claims weigh on average 800 g each. Many customers believe the average weight of his loaves is less than this. A food inspector visits the bakery and weighs a random sample of 10 loaves, with the following results, in grams:

783, 802, 804, 785, 810, 805, 789, 781, 800, 791.

Assume that these results are taken from a normal distribution.

(a) Determine unbiased estimates for the mean and variance of the distribution. *[3 marks]*

In spite of these results the baker insists that his claim is correct.

(b) Stating appropriate hypotheses, test the baker's claim at the 10 % level of significance. *[7 marks]*

The inspector informs the baker that he must improve his quality control and reject any loaf that weighs less than 790 g. The baker changes his production methods and asserts that he has reduced the number of low weight loaves. On a subsequent visit to the bakery the inspector tests a random sample of loaves for sale. Of the 40 loaves tested, 5 should have been rejected.

(c) Calculate a 95 % confidence interval for the proportion of loaves for sale that should be rejected. *[4 marks]*

2. *[Maximum mark: 6]*

The random variable X has a geometric distribution with parameter p.

(a) Show that $\mathrm{P}(X \le n) = 1-(1-p)^n$, $n \in \mathbb{Z}^+$. *[3 marks]*

(b) Deduce an expression for $\mathrm{P}(m < X \le n)$, $m, n \in \mathbb{Z}^+$ and $m < n$. *[1 mark]*

(c) Given that $p = 0.2$, find the least value of n for which $\mathrm{P}(1 < X \le n) > 0.5$, $n \in \mathbb{Z}^+$. *[2 marks]*

3. *[Maximum mark: 13]*

Each week the management of a football club recorded the number of injuries suffered by their playing staff in that week. The results for a 52-week period were as follows:

Number of injuries per week	0	1	2	3	4	5	6
Number of weeks	6	14	15	9	5	2	1

(a) Calculate the mean and variance of the number of injuries per week. *[2 marks]*

(b) Explain why these values provide supporting evidence for using a Poisson distribution model. *[1 mark]*

(c) Stating your hypotheses, test whether a Poisson distribution is a suitable model for the number of injuries per week at the 5 % level of significance using a χ^2 test. *[10 marks]*

4. *[Maximum mark: 19]*

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 2x, & 0 \le x \le 0.5, \\ \frac{4}{3} - \frac{2}{3}x, & 0.5 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch the function f and show that the lower quartile is 0.5. *[3 marks]*

(b) (i) Determine $\mathrm{E}(X)$.

(ii) Determine $\mathrm{E}(X^2)$. *[4 marks]*

Two independent observations are made from X and the values are added. The resulting random variable is denoted Y.

(c) (i) Determine $\mathrm{E}(Y - 2X)$.

(ii) Determine $\mathrm{Var}(Y - 2X)$. *[5 marks]*

(d) (i) Find the cumulative distribution function for X.

(ii) Hence, or otherwise, find the median of the distribution. *[7 marks]*

5. *[Maximum mark: 8]*

The random variable $X \sim \mathrm{Po}(m)$. Given that $\mathrm{P}(X = k-1) = \mathrm{P}(X = k+1)$, where k is a positive integer,

(a) show that $m^2 = k(k+1)$; *[2 marks]*

(b) hence show that the mode of X is k. *[6 marks]*