International Baccalaureate®
Baccalauréat International
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MARKSCHEME

May 2012

MATHEMATICS

Higher Level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics HL: Guidance for e-marking May 2012**". It is essential that you read this document before you start marking. In particular, please note the following:

- Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.
- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.
- All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a misread. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

Candidates should ***NO LONGER*** *be penalized for an accuracy error* ***(AP)****.*

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for ***FT****.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

SECTION A

1. let $f(x) = 2x^3 + kx^2 + 6x + 32$

let $g(x) = x^4 - 6x^2 - k^2x + 9$

$f(-1) = -2 + k - 6 + 32 (= 24 + k)$ ***A1***

$g(-1) = 1 - 6 + k^2 + 9 (= 4 + k^2)$ ***A1***

$\Rightarrow 24 + k = 4 + k^2$ ***M1***

$\Rightarrow k^2 - k - 20 = 0$

$\Rightarrow (k-5)(k+4) = 0$ ***(M1)***

$\Rightarrow k = 5, -4$ ***A1A1***

[6 marks]

2. perpendicular when $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix} = 0$ ***(M1)***

$\Rightarrow -1 + 4\sin x \cos x = 0$ ***A1***

$\Rightarrow \sin 2x = \frac{1}{2}$ ***M1***

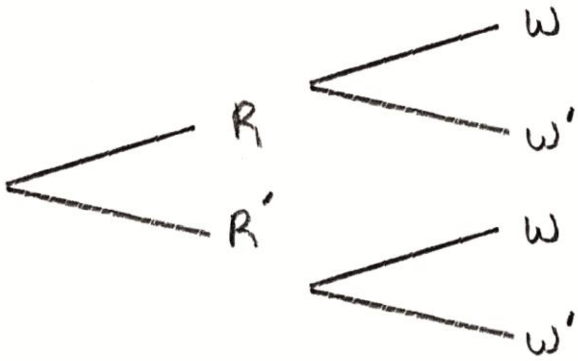
$\Rightarrow 2x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ ***A1A1***

Note: Accept answers in degrees.

[5 marks]

3. (a) let R be "it rains" and W be "the 'Tigers' soccer team win"



A1

[1 mark]

(b) $\mathrm{P}(W)=\frac{2}{5}\times\frac{2}{7}+\frac{3}{5}\times\frac{4}{7}$ ***(M1)***

$=\frac{16}{35}$ ***A1***

[2 marks]

(c) $\mathrm{P}(R|W)=\frac{\frac{2}{5}\times\frac{2}{7}}{\frac{16}{35}}$ ***(M1)***

$=\frac{1}{4}$ ***A1***

[2 marks]

Total [5 marks]

4. (a) $\left(x-\frac{2}{x}\right)^4=x^4+4x^3\left(-\frac{2}{x}\right)+6x^2\left(-\frac{2}{x}\right)^2+4x\left(-\frac{2}{x}\right)^3+\left(-\frac{2}{x}\right)^4$ ***(A2)***

Note: Award ***(A1)*** for 3 or 4 correct terms.

Note: Accept combinatorial expressions, *e.g.* $\binom{4}{2}$ for 6.

$=x^4-8x^2+24-\frac{32}{x^2}+\frac{16}{x^4}$ ***A1***

[3 marks]

(b) constant term from expansion of $(2x^2+1)\left(x-\frac{2}{x}\right)^4=-64+24=-40$ ***A2***

Note: Award ***A1*** for – 64 or 24 seen.

[2 marks]

Total [5 marks]

5. (a) $4\boldsymbol{A} - 5\boldsymbol{BX} = \boldsymbol{B}$

$\Rightarrow \boldsymbol{BX} = \frac{4}{5}\boldsymbol{A} - \frac{1}{5}\boldsymbol{B}$ ***M1***

$\Rightarrow \boldsymbol{X} = \frac{1}{5}\boldsymbol{B}^{-1}(4\boldsymbol{A} - \boldsymbol{B})\ \left(= \frac{4}{5}\boldsymbol{B}^{-1}\boldsymbol{A} - \frac{1}{5}\boldsymbol{I}\right)$ ***A1***

[2 marks]

(b) if $\boldsymbol{A} = 2\boldsymbol{B}$ then $\boldsymbol{B}^{-1}\boldsymbol{A} = 2\boldsymbol{I}$ ***M1***

$\Rightarrow \boldsymbol{X} = \frac{8}{5}\boldsymbol{I} - \frac{1}{5}\boldsymbol{I}$

$\Rightarrow \boldsymbol{X} = \frac{7}{5}\boldsymbol{I}\ \left(= \begin{pmatrix} \frac{7}{5} & 0 \\ 0 & \frac{7}{5} \end{pmatrix} \right)$ ***A2***

[3 marks]

Total [5 marks]

6. (a) attempt to equate real and imaginary parts ***M1***

equate real parts: $4m + 4n = 16$; equate imaginary parts: $-5m = 15$ ***A1***

$\Rightarrow m = -3,\ n = 7$ ***A1***

[3 marks]

(b) let $m = x + \mathrm{i}y$, $n = x - \mathrm{i}y$ ***M1***

$\Rightarrow (4 - 5\mathrm{i})(x + \mathrm{i}y) + 4(x - \mathrm{i}y) = 16 + 15\mathrm{i}$

$\Rightarrow 4x - 5\mathrm{i}x + 4\mathrm{i}y + 5y + 4x - 4\mathrm{i}y = 16 + 15\mathrm{i}$

attempt to equate real and imaginary parts ***M1***

$8x + 5y = 16$, $-5x = 15$ ***A1***

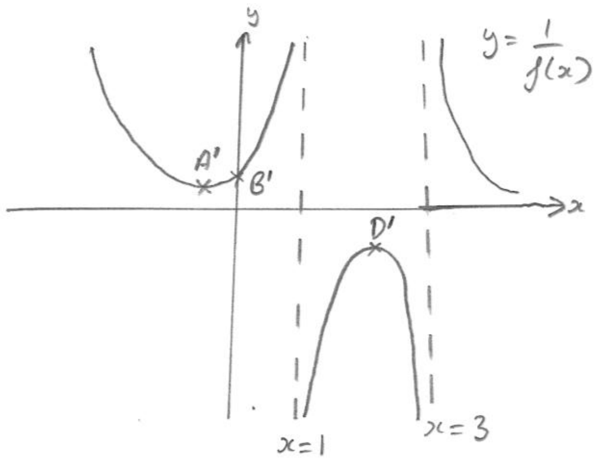
$\Rightarrow x = -3,\ y = 8$ ***A1***

$(\Rightarrow m = -3 + 8\mathrm{i},\ n = -3 - 8\mathrm{i})$

[4 marks]

Total [7 marks]

7. (a)

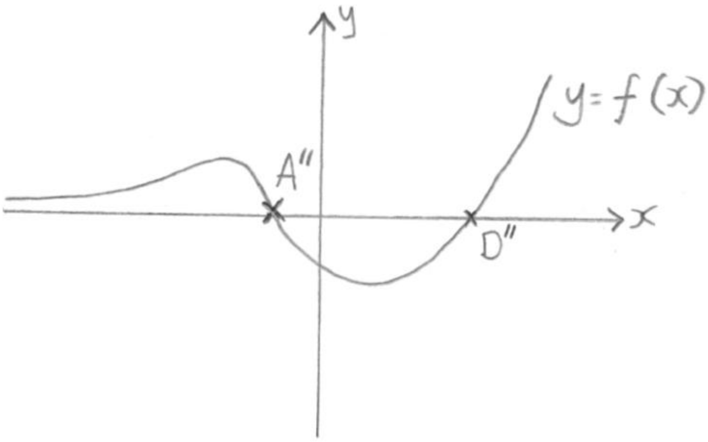


A1A1A1

Note:	Award ***A1*** for correct shape.
	Award ***A1*** for two correct asymptotes, $x=1$ and $x=3$.
	Award ***A1*** for correct coordinates, $\text{A}'\left(-1, \frac{1}{4}\right)$, $\text{B}'\left(0, \frac{1}{3}\right)$ and $\text{D}'\left(2, -\frac{1}{3}\right)$.

[3 marks]

(b)



A1A1A1

Note:	Award ***A1*** for correct general shape including the horizontal asymptote.
	Award ***A1*** for recognition of 1 maximum point and 1 minimum point.
	Award ***A1*** for correct coordinates, $\text{A}''(-1, 0)$ and $\text{D}''(2, 0)$.

[3 marks]

Total [6 marks]

8. $x^3 y = a\sin nx$

attempt to differentiate implicitly *M1*

$\Rightarrow 3x^2 y + x^3 \frac{\mathrm{d}y}{\mathrm{d}x} = an\cos nx$ *A2*

Note: Award ***A1*** for two out of three correct, ***A0*** otherwise.

$\Rightarrow 6xy + 3x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + 3x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + x^3 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -an^2 \sin nx$ *A2*

Note: Award ***A1*** for three or four out of five correct, ***A0*** otherwise.

$\Rightarrow 6xy + 6x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + x^3 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -an^2 \sin nx$

$\Rightarrow x^3 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 6x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + 6xy + n^2 x^3 y = 0$ *A1*

$\Rightarrow x^3 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 6x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + (n^2 x^2 + 6)xy = 0$ *AG*

[6 marks]

9. **METHOD 1**

$\frac{\cos A + \sin A}{\cos A - \sin A} = \sec 2A + \tan 2A$

consider right hand side

$\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$ *M1A1*

$= \frac{\cos^2 A + 2\sin A\cos A + \sin^2 A}{\cos^2 A - \sin^2 A}$ *A1A1*

Note: Award ***A1*** for recognizing the need for single angles and ***A1*** for recognizing $\cos^2 A + \sin^2 A = 1$.

$= \frac{(\cos A + \sin A)^2}{(\cos A + \sin A)(\cos A - \sin A)}$ *M1A1*

$= \frac{\cos A + \sin A}{\cos A - \sin A}$ *AG*

METHOD 2

$\frac{\cos A + \sin A}{\cos A - \sin A} = \frac{(\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$ *M1A1*

$= \frac{\cos^2 A + 2\sin A\cos A + \sin^2 A}{\cos^2 A - \sin^2 A}$ *A1A1*

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.

$= \frac{1 + \sin 2A}{\cos 2A}$ *M1A1*

$= \sec 2A + \tan 2A$ *AG*

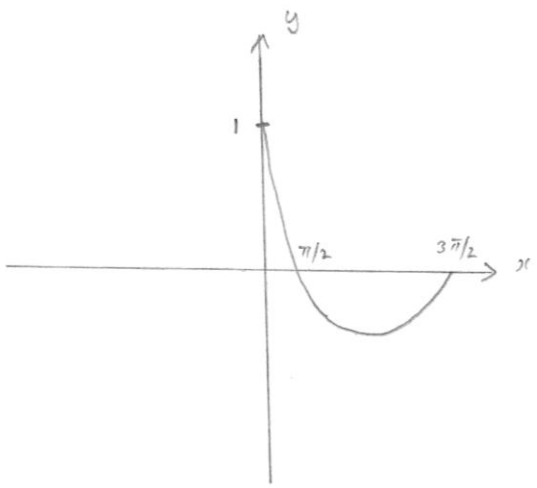
[6 marks]

10. (a) $e^{-x}\cos x = 0$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ ***A1***

[1 mark]

(b)



A1

Note:	Accept any form of concavity for $x \in [0, \frac{\pi}{2}]$.

Note:	Do not penalize unmarked zeros if given in part (a).

Note:	Zeros written on diagram can be used to allow the mark in part (a) to be awarded retrospectively.

[1 mark]

continued …

Question 10 continued

(c) attempt at integration by parts ***M1***

EITHER

$I = \int e^{-x}\cos x\mathrm{d}x = -e^{-x}\cos x\mathrm{d}x - \int e^{-x}\sin x\mathrm{d}x$ ***A1***

$\Rightarrow I = -e^{-x}\cos x\mathrm{d}x - \left[-e^{-x}\sin x + \int e^{-x}\cos x\mathrm{d}x\right]$ ***A1***

$\Rightarrow I = \frac{e^{-x}}{2}(\sin x - \cos x) + C$ ***A1***

Note: Do not penalize absence of *C*.

OR

$I = \int e^{-x}\cos x\mathrm{d}x = e^{-x}\sin x + \int e^{-x}\sin x\mathrm{d}x$ ***A1***

$\Rightarrow I = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\mathrm{d}x$ ***A1***

$\Rightarrow I = \frac{e^{-x}}{2}(\sin x - \cos x) + C$ ***A1***

Note: Do not penalize absence of *C*.

THEN

$\int_0^{\frac{\pi}{2}} e^{-x}\cos x\mathrm{d}x = \left[\frac{e^{-x}}{2}(\sin x - \cos x)\right]_0^{\frac{\pi}{2}} = \frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}$ ***A1***

$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} e^{-x}\cos x\mathrm{d}x = \left[\frac{e^{-x}}{2}(\sin x - \cos x)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -\frac{e^{-\frac{3\pi}{2}}}{2} - \frac{e^{-\frac{\pi}{2}}}{2}$ ***A1***

ratio of $A:B$ is $\dfrac{\frac{e^{-\frac{\pi}{2}}}{2} + \frac{1}{2}}{\frac{e^{-\frac{3\pi}{2}}}{2} + \frac{e^{-\frac{\pi}{2}}}{2}}$

$= \dfrac{e^{\frac{3\pi}{2}}\left(e^{-\frac{\pi}{2}} + 1\right)}{e^{\frac{3\pi}{2}}\left(e^{-\frac{3\pi}{2}} + e^{-\frac{\pi}{2}}\right)}$ ***M1***

$= \dfrac{e^{\pi}\left(e^{\frac{\pi}{2}} + 1\right)}{e^{\pi} + 1}$ ***AG***

[7 marks]

Total [9 marks]

SECTION B

11. (a) $f(x) \geq \frac{1}{25}$ ***A1***

$g(x) \in \mathbb{R}, g(x) \geq 0$ ***A1***

[2 marks]

(b) $f \circ g(x) = \dfrac{2\left(\frac{3x-4}{10}\right)^2 + 3}{75}$ ***M1A1***

$= \dfrac{\frac{2(9x^2-24x+16)}{100} + 3}{75}$ ***(A1)***

$= \dfrac{9x^2-24x+166}{3750}$ ***A1***

[4 marks]

(c) (i) **METHOD 1**

$y = \dfrac{2x^2+3}{75}$

$x^2 = \dfrac{75y-3}{2}$ ***M1***

$x = \sqrt{\dfrac{75y-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\dfrac{75x-3}{2}}$ ***A1***

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form $f^{-1}(x) =$.

METHOD 2

$y = \dfrac{2x^2+3}{75}$

$x = \dfrac{2y^2+3}{75}$ ***M1***

$y = \sqrt{\dfrac{75x-3}{2}}$ ***(A1)***

$\Rightarrow f^{-1}(x) = \sqrt{\dfrac{75x-3}{2}}$ ***A1***

Note: Accept ± in line 3 for the ***(A1)*** but not in line 4 for the ***A1***.
Award the ***A1*** only if written in the form $f^{-1}(x) =$.

(ii) domain: $x \geq \frac{1}{25}$; range: $f^{-1}(x) \geq 0$ ***A1***

[4 marks]

continued …

Question 11 continued

(d) probabilities from $f(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

probabilities from $g(x)$:

X	0	1	2	3	4
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

A2

Note: Award ***A1*** for one error, ***A0*** otherwise.

only in the case of $f(x)$ does $\sum P(X=x)=1$, hence only $f(x)$ can be used as a probability mass function ***A2***

[6 marks]

(e)

$$E(x)=\sum x\cdot \mathrm{P}(X=x)$$ ***M1***

$$=\frac{5}{75}+\frac{22}{75}+\frac{63}{75}+\frac{140}{75}=\frac{230}{75}\left(=\frac{46}{15}\right)$$ ***A1***

[2 marks]

Total [18 marks]

12. Part A

(a) (i) $(x+\mathrm{i}y)^2 = -5+12\mathrm{i}$

$x^2 + 2\mathrm{i}xy + \mathrm{i}^2 y^2 = -5+12\mathrm{i}$ ***A1***

(ii) equating real and imaginary parts ***M1***

$x^2 - y^2 = -5$ ***AG***

$xy = 6$ ***AG***

[2 marks]

(b) substituting ***M1***

EITHER

$x^2 - \frac{36}{x^2} = -5$

$x^4 + 5x^2 - 36 = 0$ ***A1***

$x^2 = 4, -9$ ***A1***

$x = \pm 2$ and $y = \pm 3$ ***(A1)***

OR

$\frac{36}{y^2} - y^2 = -5$

$y^4 - 5y^2 - 36 = 0$ ***A1***

$y^2 = 9, -4$ ***A1***

$y = \pm 3$ **and** $x = \pm 2$ ***(A1)***

Note: Accept solution by inspection if completely correct.

THEN

the square roots are $(2+3\mathrm{i})$ and $(-2-3\mathrm{i})$ ***A1***

[5 marks]

(c) **EITHER**

consider $z = x+\mathrm{i}y$

$z^* = x - \mathrm{i}y$

$(z^*)^2 = x^2 - y^2 - 2\mathrm{i}xy$ ***A1***

$(z^2) = x^2 - y^2 + 2\mathrm{i}xy$ ***A1***

$(z^2)^* = x^2 - y^2 - 2\mathrm{i}xy$ ***A1***

$(z^*)^2 = (z^2)^*$ ***AG***

OR

$z^* = r\mathrm{e}^{-\mathrm{i}\theta}$

$(z^*)^2 = r^2\mathrm{e}^{-2\mathrm{i}\theta}$ ***A1***

$z^2 = r^2\mathrm{e}^{2\mathrm{i}\theta}$ ***A1***

continued …

Question 12 continued

$(z^2)^* = r^2 e^{-2i\theta}$ **A1**

$(z^*)^2 = (z^2)^*$ **AG**

[3 marks]

(d) $(2-3i)$ and $(-2+3i)$ **A1A1**

[2 marks]

Part B

(a) the graph crosses the x-axis twice, indicating two real roots **R1**

since the quartic equation has four roots and only two are real, the other two roots must be complex **R1**

[2 marks]

(b) $f(x) = (x+4)(x-2)(x^2 + cx + d)$ **A1A1**

$f(0) = -32 \Rightarrow d = 4$ **A1**

Since the curve passes through $(-1, -18)$,

$-18 = 3 \times (-3)(5-c)$ **M1**

$c = 3$ **A1**

Hence $f(x) = (x+4)(x-2)(x^2 + 3x + 4)$

[5 marks]

(c) $x = \dfrac{-3 \pm \sqrt{9-16}}{2}$ ***(M1)***

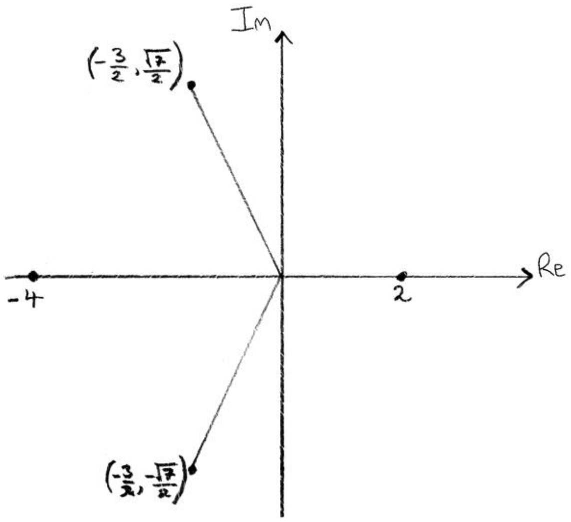
$\Rightarrow x = -\dfrac{3}{2} \pm i\dfrac{\sqrt{7}}{2}$ **A1**

[2 marks]

continued …

Question 12 continued

(d)



A1A1

Note:	Accept points or vectors on complex plane. Award ***A1*** for two real roots and ***A1*** for two complex roots.

[2 marks]

(e) real roots are $4e^{i\pi}$ and $2e^{i0}$ ***A1A1***

considering $-\frac{3}{2} \pm i\frac{\sqrt{7}}{2}$

$r = \sqrt{\frac{9}{4} + \frac{7}{4}} = 2$ ***A1***

finding θ using $\arctan\left(\frac{\sqrt{7}}{3}\right)$ ***M1***

$\theta = \arctan\left(\frac{\sqrt{7}}{3}\right) + \pi$ or $\theta = \arctan\left(-\frac{\sqrt{7}}{3}\right) + \pi$ ***A1***

$\Rightarrow z = 2e^{i\left(\arctan\left(\frac{\sqrt{7}}{3}\right)+\pi\right)}$ or $\Rightarrow z = 2e^{i\left(\arctan\left(\frac{-\sqrt{7}}{3}\right)+\pi\right)}$ ***A1***

Note:	Accept arguments in the range $-\pi$ to π or 0 to 2π. Accept answers in degrees.

[6 marks]

Total [29 marks]

13. (a) let $f(x)=\dfrac{1}{2x+1}$ and using the result $f'(x)=\lim_{h\to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right)$

$$f'(x)=\lim_{h\to 0}\left(\frac{\frac{1}{2(x+h)+1}-\frac{1}{2x+1}}{h}\right) \qquad \textbf{\textit{M1A1}}$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\left(\frac{[2x+1]-[2(x+h)+1]}{h[2(x+h)+1][2x+1]}\right) \qquad \textbf{\textit{A1}}$$

$$\Rightarrow f'(x)=\lim_{h\to 0}\left(\frac{-2}{[2(x+h)+1][2x+1]}\right) \qquad \textbf{\textit{A1}}$$

$$\Rightarrow f'(x)=\frac{-2}{(2x+1)^2} \qquad \textbf{\textit{AG}}$$

[4 marks]

(b) let $y=\dfrac{1}{2x+1}$

we want to prove that $\dfrac{d^n y}{dx^n}=(-1)^n\dfrac{2^n n!}{(2x+1)^{n+1}}$

let $n=1 \Rightarrow \dfrac{dy}{dx}=(-1)^1\dfrac{2^1 1!}{(2x+1)^{1+1}}$ ***M1***

$\Rightarrow \dfrac{dy}{dx}=\dfrac{-2}{(2x+1)^2}$ which is the same result as part (a)

hence the result is true for $n=1$ ***R1***

assume the result is true for $n=k$: $\dfrac{d^k y}{dx^k}=(-1)^k\dfrac{2^k k!}{(2x+1)^{k+1}}$ ***M1***

$$\frac{d^{k+1}y}{dx^{k+1}}=\frac{d}{dx}\left[(-1)^k\frac{2^k k!}{(2x+1)^{k+1}}\right] \qquad \textbf{\textit{M1}}$$

$$\Rightarrow \frac{d^{k+1}y}{dx^{k+1}}=\frac{d}{dx}\left[(-1)^k 2^k k!(2x+1)^{-k-1}\right] \qquad \textbf{\textit{(A1)}}$$

$$\Rightarrow \frac{d^{k+1}y}{dx^{k+1}}=(-1)^k 2^k k!(-k-1)(2x+1)^{-k-2}\times 2 \qquad \textbf{\textit{A1}}$$

$$\Rightarrow \frac{d^{k+1}y}{dx^{k+1}}=(-1)^{k+1}2^{k+1}(k+1)!(2x+1)^{-k-2} \qquad \textbf{\textit{(A1)}}$$

$$\Rightarrow \frac{d^{k+1}y}{dx^{k+1}}=(-1)^{k+1}\frac{2^{k+1}(k+1)!}{(2x+1)^{k+2}} \qquad \textbf{\textit{A1}}$$

hence if the result is true for $n=k$, it is true for $n=k+1$
since the result is true for $n=1$, the result is proved by mathematical induction ***R1***

Note: Only award final ***R1*** if all the ***M*** marks have been gained.

[9 marks]

Total [13 marks]