International Baccalaureate®
Baccalauréat International
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MARKSCHEME

May 2011

MATHEMATICS

Higher Level

Paper 2

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then* ***FT*** *marks should be awarded if appropriate.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right) \qquad \textbf{\textit{A1}}$$

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

*Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**. Award the marks as usual then write **(AP)** against the answer. On the **front** cover write –1**(AP)**. Deduct 1 mark from the total for the paper, not the question.*

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation
The Mathematics HL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

SECTION A

1. (a) (i) median $= 104$ grams *A1*

Note: Accept 105.

(ii) 30^{th} percentile $= 90$ grams *A1*

(b) $80 - 49$ *(M1)*
$= 31$ *A1*

Note: Accept answers 30 to 32.

[4 marks]

2. (a) $f'(x) = 3x^2 - 6x - 9 \ (= 0)$ *(M1)*
$(x+1)(x-3) = 0$
$x = -1;\ x = 3$
$(\max)(-1,\ 15);(\min)(3,\ -17)$ *A1A1*

Note: The coordinates need not be explicitly stated but the values need to be seen.

$y = -8x + 7$ *A1* *N2*

(b) $f''(x) = 6x - 6 = 0 \Rightarrow$ inflexion $(1,\ -1)$ *A1*
which lies on $y = -8x + 7$ *R1AG*

[6 marks]

3. **METHOD 1**

$$\frac{\sin C}{7} = \frac{\sin 40}{5}$$ *M1(A1)*

$\mathrm{B\hat{C}D} = 64.14...^\circ$ *A1*
$\mathrm{CD} = 2 \times 5 \cos 64.14...$ *M1*

Note: Also allow use of sine or cosine rule.

$\mathrm{CD} = 4.36$ *A1*

[5 marks]

METHOD 2

let $\mathrm{AC} = x$
cosine rule
$5^2 = 7^2 + x^2 - 2 \times 7 \times x \cos 40$ *M1A1*
$x^2 - 10.7...x + 24 = 0$

$$x = \frac{10.7... \pm \sqrt{(10.7...)^2 - 4 \times 24}}{2}$$ *(M1)*

$x = 7.54;\ 3.18$ *(A1)*
CD is the difference in these two values $= 4.36$ *A1*

Note: Other methods may be seen.

[5 marks]

4. (a) $f(a) = 4a^3 + 2a^2 - 7a = -10$ ***M1***

$4a^3 + 2a^2 - 7a + 10 = 0$

$(a+2)(4a^2 - 6a + 5) = 0$ or sketch or GDC ***(M1)***

$a = -2$ ***A1***

(b) substituting $a = -2$ into $f(x)$

$f(x) = 4x^3 - 4x + 14 = 0$ ***A1***

EITHER

graph showing unique solution which is indicated (must include max and min) ***R1***

OR

convincing argument that only one of the solutions is real ***R1***

$(-1.74, 0.868 \pm 1.12\text{i})$

[5 marks]

5. (a) $2x^2 + x - 3 = (2x+3)(x-1)$ ***A1***

Note: Accept $2\left(x + \dfrac{3}{2}\right)(x-1)$.

Note: Either of these may be seen in (b) and if so ***A1*** should be awarded.

(b) **EITHER**

$(2x^2 + x - 3)^8 = (2x+3)^8(x-1)^8$ ***M1***

$= \left(3^8 + 8(3^7)(2x) + \ldots\right)\left((-1)^8 + 8(-1)^7(x) + \ldots\right)$ ***(A1)***

coefficient of $x = 3^8 \times 8 \times (-1)^7 + 3^7 \times 8 \times 2 \times (-1)^8$ ***M1***

$= -17\,496$ ***A1***

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

OR

$(2x^2 + x - 3)^8 = \left(3 - (x - 2x^2)\right)^8$ ***M1***

$= 3^8 + 8\left(-(x - 2x^2)(3^7) + \ldots\right)$ ***(A1)***

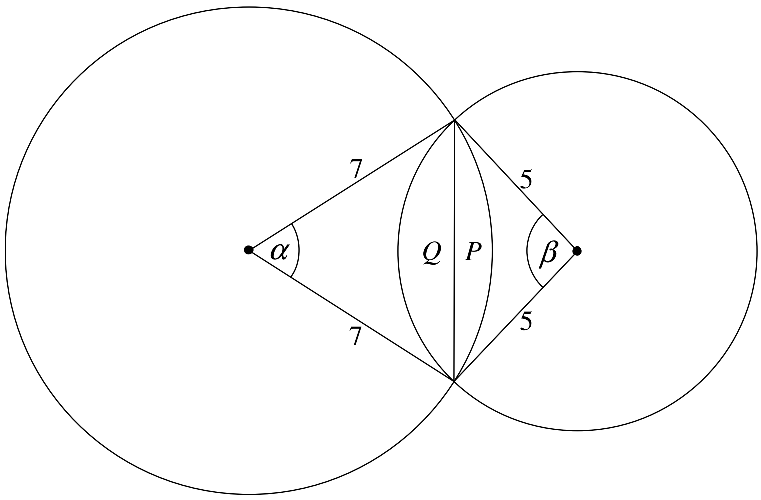
coefficient of $x = 8 \times (-1) \times 3^7$ ***M1***

$= -17\,496$ ***A1***

Note: Under ft, final ***A1*** can only be achieved for an integer answer.

[5 marks]

6.



$\alpha = 2\arcsin\left(\frac{4.5}{7}\right)$ $(\Rightarrow \alpha = 1.396\ldots = 80.010°\ldots)$ ***M1(A1)***

$\beta = 2\arcsin\left(\frac{4.5}{5}\right)$ $(\Rightarrow \beta = 2.239\ldots = 128.31°\ldots)$ ***(A1)***

Note:	Allow use of cosine rule.

area $P = \frac{1}{2}\times 7^2 \times (\alpha - \sin\alpha) = 10.08...$ ***M1(A1)***

area $Q = \frac{1}{2}\times 5^2 \times (\beta - \sin\beta) = 18.18...$ ***(A1)***

Note:	The ***M1*** is for an attempt at area of sector minus area of triangle.

Note:	The use of degrees correctly converted is acceptable.

area $= 28.3(\text{cm}^2)$ ***A1***

[7 marks]

7. (a) $k\int_{-2}^{0}(x+2)^2\,dx + \int_{0}^{\frac{4}{3}} k\,dx = 1$ ***M1***

$\frac{8k}{3} + \frac{4k}{3} = 1$

$k = \frac{1}{4}$ ***A1***

Note:	Only ft on positive values of k.

continued …

Question 7 continued

(b) (i) $\mathrm{E}(X) = \frac{1}{4}\int_{-2}^{0} x(x+2)^2\,\mathrm{d}x + \frac{1}{4}\int_{0}^{\frac{4}{3}} x\,\mathrm{d}x$ ***M1***

$= \frac{1}{4}\times\frac{-4}{3}+\frac{2}{9}$

$= -\frac{1}{9}$ (-0.111) ***A1***

(ii) median given by a such that $\mathrm{P}(X < a) = 0.5$

$\frac{1}{4}\int_{-2}^{a}(x+2)^2\,\mathrm{d}x = 0.5$ ***M1***

$\left[\frac{(x+2)^3}{3}\right]_{-2}^{a} = 2$ ***(A1)***

$(a+2)^3 - 0 = 6$

$a = \sqrt[3]{6} - 2$ $(= -0.183)$ ***A1***

[7 marks]

8. (a) equation of line in graph $a = -\frac{25}{60}t + 15$ ***A1***

$$\left(a = -\frac{5}{12}t + 15\right)$$

(b) $\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{5}{12}t + 15$ ***(M1)***

$v = -\frac{5}{24}t^2 + 15t + c$ ***(A1)***

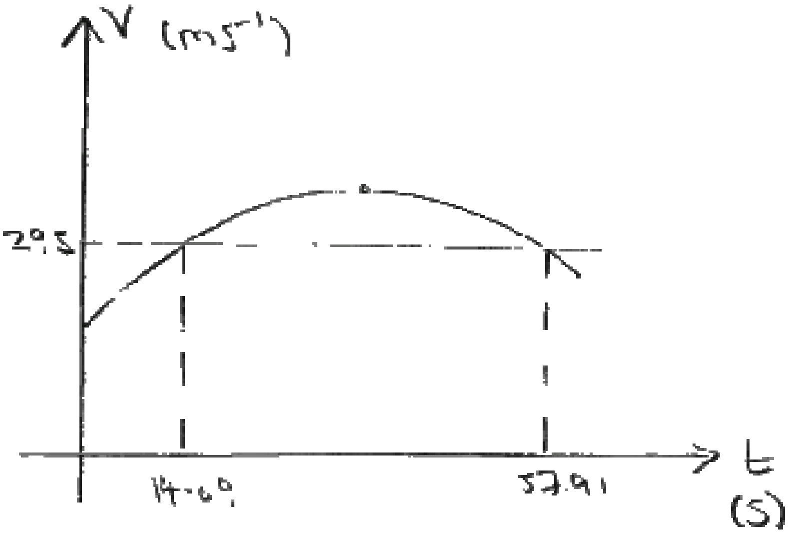
when $t = 0$, $v = 125 \text{ m s}^{-1}$

$v = -\frac{5}{24}t^2 + 15t + 125$ ***A1***

from graph or by finding time when $a = 0$

maximum $= 395 \text{ m s}^{-1}$ ***A1***

(c) **EITHER**



graph drawn and intersection with $v = 295 \text{ m s}^{-1}$ ***(M1)(A1)***

$t = 57.91 - 14.09 = 43.8$ ***A1***

OR

$295 = -\frac{5}{24}t^2 + 15t + 125 \Rightarrow t = 57.91...; 14.09...$ ***(M1)(A1)***

$t = 57.91\ldots - 14.09\ldots = 43.8 \ \left(8\sqrt{30}\right)$ ***A1***

[8 marks]

9. $\log_{x+1} y = 2$

$\log_{y+1} x = \frac{1}{4}$

so $(x+1)^2 = y$ ***A1***

$(y+1)^{\frac{1}{4}} = x$ ***A1***

EITHER

$x^4 - 1 = (x+1)^2$ ***M1***

$x = -1$, not possible ***R1***

$x = 1.70,\ y = 7.27$ ***A1A1***

OR

$(x^2 + 2x + 2)^{\frac{1}{4}} - x = 0$ ***M1***

attempt to solve or graph of LHS ***M1***

$x = 1.70,\ y = 7.27$ ***A1A1***

[6 marks]

10. METHOD 1

equation of journey of ship S_1

$$\boldsymbol{r}_1 = t\begin{pmatrix}10\\20\end{pmatrix}$$

equation of journey of speedboat S_2 ,setting off k minutes later

$$\boldsymbol{r}_2 = \begin{pmatrix}70\\30\end{pmatrix} + (t-k)\begin{pmatrix}-60\\30\end{pmatrix}$$ ***M1A1A1***

Note: Award ***M1*** for perpendicular direction, ***A1*** for speed, ***A1*** for change in parameter (*e.g.* by using $t-k$ or T, k being the time difference between the departure of the ships).

solve $t\begin{pmatrix}10\\20\end{pmatrix} = \begin{pmatrix}70\\30\end{pmatrix} + (t-k)\begin{pmatrix}-60\\30\end{pmatrix}$ ***(M1)***

Note: ***M*** mark is for equating their two expressions.

$10t = 70 - 60t + 60k$

$20t = 30 + 30t - 30k$ ***M1***

Note*:* ***M*** mark is for obtaining two equations involving two different parameters.

$7t - 6k = 7$

$-t + 3k = 3$

$k = \frac{28}{15}$ ***A1***

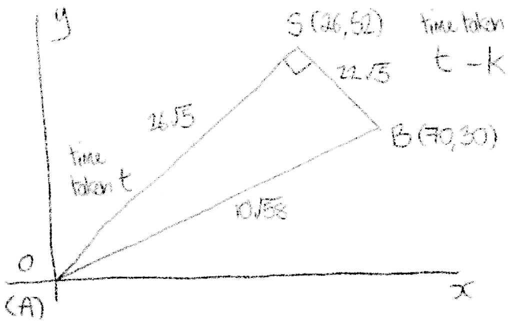
latest time is 11:52 ***A1***

[7 marks]

continued …

Question 10 continued

METHOD 2



SB $= 22\sqrt{5}$ ***M1A1***

(by perpendicular distance)

SA $= 26\sqrt{5}$ ***M1A1***

(by Pythagoras or coordinates)

$t = \frac{26\sqrt{5}}{10\sqrt{5}}$ ***A1***

$t - k = \frac{22\sqrt{5}}{30\sqrt{5}}$ ***A1***

$k = \frac{28}{15}$ leading to latest time 11:52 ***A1***

[7 marks]

SECTION B

11. (a) $\begin{pmatrix} 0 & 2 & 1 \\ -1 & 1 & 3 \\ -2 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ k \end{pmatrix}$

$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1 & 3 \\ -2 & 1 & 2 \end{vmatrix} = 0 - 2(-2+6) + (-1+2) = -7$ ***M1A1***

since determinant $\neq 0 \Rightarrow$ unique solution to the system ***R1***

planes intersect in a point ***AG***

> **Note:** For any method, including row reduction, leading to the explicit solution $\left(\frac{6-5k}{7}, \frac{10+k}{7}, \frac{1-2k}{7}\right)$, award ***M1*** for an attempt at a correct method ***A1*** for two correct coordinates and ***A1*** for a third correct coordinate.

[3 marks]

(b) $\begin{vmatrix} a & 2 & 1 \\ -1 & a+1 & 3 \\ -2 & 1 & a+2 \end{vmatrix} = a\big((a+1)(a+2)-3\big) - 2\big(-1(a+2)+6\big) + \big(-1+2(a+1)\big)$ ***M1(A1)***

planes not meeting in a point $\Rightarrow$ no unique solution *i.e.* determinant $= 0$ ***(M1)***

$a(a^2+3a-1)+(2a-8)+(2a+1)=0$

$a^3+3a^2+3a-7=0$ ***A1***

$a=1$ ***A1***

[5 marks]

continued …

Question 11 continued

(c) $\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ -2 & 1 & 3 & k \end{pmatrix}$ $r_1 + r_2$ ***M1***

$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ 0 & 5 & 5 & 6+k \end{pmatrix}$ $2r_1 + r_3$ ***(A1)***

$\begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 0 & 4+4k \end{pmatrix}$ $4r_3 - 5r_2$ ***(A1)***

for an infinite number of solutions to exist, $4 + 4k = 0 \Rightarrow k = -1$ ***A1***

$x + 2y + z = 3$
$y + z = 1$ ***M1***

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ***A1***

Note: Accept methods involving elimination.

Note: Accept any equivalent form *e.g.* $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

Award ***A0*** if $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ or $\boldsymbol{r} =$ is absent.

[6 marks]

Total [14 marks]

12. (a) $\mathrm{P}(X < 30) = 0.4$
$\mathrm{P}(X < 55) = 0.9$
or relevant sketch ***(M1)***
given $Z = \dfrac{X - \mu}{\sigma}$
$\mathrm{P}(Z < z) = 0.4 \Rightarrow \dfrac{30-\mu}{\sigma} = -0.253...$ ***(A1)***
$\mathrm{P}(Z < z) = 0.9 \Rightarrow \dfrac{55-\mu}{\sigma} = 1.28...$ ***(A1)***
$\mu = 30 + (0.253...) \times \sigma = 55 - (1.28...) \times \sigma$ ***M1***
$\sigma = 16.3$, $\mu = 34.1$ ***A1***

Note: Accept 16 and 34.

Note: Working with 830 and 855 will only gain the two ***M*** marks.

[5 marks]

(b) $X \sim \mathrm{N}(34.12\ldots, 16.28\ldots^2)$
late to school $\Rightarrow X > 60$
$\mathrm{P}(X > 60) = 0.056...$ ***(A1)***
number of students late $= 0.0560\ldots \times 1200$ ***(M1)***
$= 67$ (to nearest integer) ***A1***

Note: Accept 62 for use of 34 and 16.

[3 marks]

(c) $\mathrm{P}(X > 60 \mid X > 30) = \dfrac{\mathrm{P}(X > 60)}{\mathrm{P}(X > 30)}$ ***M1***
$= 0.0935$ (accept anything between 0.093 and 0.094) ***A1***

Note: If 34 and 16 are used 0.0870 is obtained. This should be accepted.

[2 marks]

(d) let L be the random variable of the number of students who leave school in a 30 minute interval
since $24 \times 30 = 720$ ***A1***
$L \sim \mathrm{Po}(720)$
$\mathrm{P}(L \geq 700) = 1 - \mathrm{P}(L \leq 699)$ ***(M1)***
$= 0.777$ ***A1***

Note: Award ***M1A0*** for $\mathrm{P}(L > 700) = 1 - \mathrm{P}(L \leq 700)$ (this leads to 0.765).

[3 marks]

continued …

Question 12 continued

(e) (i) $Y \sim \mathrm{B}(200,\ 0.7767\ldots)$ ***(M1)***

$\mathrm{E}(Y) = 200 \times 0.7767\ldots = 155$ ***A1***

Note: On ft, use of 0.765 will lead to 153.

(ii) $\mathrm{P}(Y > 150) = 1 - \mathrm{P}(Y \leq 150)$ ***(M1)***

$= 0.797$ ***A1***

Note: Accept 0.799 from using rounded answer.

Note: On ft, use of 0.765 will lead to 0.666.

[4 marks]

Total [17 marks]

13. (a) $\boldsymbol{A}^2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$= \begin{pmatrix} \cos^2\theta - \sin^2\theta & \cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta - \cos\theta\sin\theta & -\sin^2\theta + \cos^2\theta \end{pmatrix}$ ***M1(A1)***

$= \begin{pmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix}$ ***A1***

$= \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$ ***AG***

[3 marks]

continued …

Question 13 continued

(b) let $\mathrm{P}(n)$ be the proposition that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix}$

for all $n \in \mathbb{Z}^+$

$\mathrm{P}(1)$ is true ***A1***

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^1 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

assume $\mathrm{P}(k)$ to be true ***A1***

Note:	Must see the word 'true' or equivalent, that makes clear an assumption is being made that $\mathrm{P}(k)$ is true.

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^k = \begin{pmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{pmatrix}$$

consider $\mathrm{P}(k+1)$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^{k+1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^k \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$ ***(M1)***

$$= \begin{pmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$ ***A1***

$$= \begin{pmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & \cos k\theta\sin\theta + \sin k\theta\cos\theta \\ -\sin k\theta\cos\theta - \cos k\theta\sin\theta & -\sin k\theta\sin\theta + \cos k\theta\cos\theta \end{pmatrix}$$ ***A1***

$$= \begin{pmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{pmatrix}$$ ***A1***

if $\mathrm{P}(k)$ is true then $\mathrm{P}(k+1)$ is true and since $\mathrm{P}(1)$ is true then $\mathrm{P}(n)$ is true for all $n \in \mathbb{Z}^+$ ***R1***

Note:	The final ***R1*** can only be gained if the ***M1*** has been gained.

[7 marks]

continued …

Question 13 continued

(c) **EITHER**

$$\boldsymbol{A}^{-1} = \begin{pmatrix} \cos(-\theta) & \sin(-\theta) \\ -\sin(-\theta) & \cos(-\theta) \end{pmatrix} \text{ from formula}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ ***A1***

$$\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{A}\boldsymbol{A}^{-1} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ ***M1***

Note: Accept either just $\boldsymbol{A}^{-1}\boldsymbol{A}$ or just $\boldsymbol{A}\boldsymbol{A}^{-1}$.

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ ***A1***

$\therefore \boldsymbol{A}^{-1}$ is inverse of $\boldsymbol{A}$

OR

$$\boldsymbol{A}^{-1} = \frac{1}{\cos^2\theta + \sin^2\theta}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ ***M1***

$$\boldsymbol{A}^{-1} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ ***A1***

putting $n = -1$ in formula gives inverse ***A1***

[3 marks]

Total [13 marks]

14. (a) volume $= \pi\int_0^h x^2\,\mathrm{d}y$ ***(M1)***

$\pi\int_0^h y\,\mathrm{d}y$ ***M1***

$= \pi\left[\frac{y^2}{2}\right]_0^h = \frac{\pi h^2}{2}$ ***A1***

[3 marks]

(b) $\frac{\mathrm{d}V}{\mathrm{d}t} = -3\times\text{surface area}$ ***A1***

surface area $= \pi x^2$ ***(M1)***

$= \pi h$ ***A1***

since $V = \frac{\pi h^2}{2} \Rightarrow h = \sqrt{\frac{2V}{\pi}}$ ***M1A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\pi\sqrt{\frac{2V}{\pi}}$ ***A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\sqrt{2\pi V}$ ***AG***

Note: Assuming that $\frac{\mathrm{d}h}{\mathrm{d}t} = -3$ without justification gains no marks.

[6 marks]

continued …

Question 14 continued

(c) $V_0 = 5000\pi \ \ (= 15700 \text{ cm}^3)$ ***A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = -3\sqrt{2\pi V}$

attempting to separate variables ***M1***

EITHER

$\int \frac{\mathrm{d}V}{\sqrt{V}} = -3\sqrt{2\pi}\int \mathrm{d}t$ ***A1***

$2\sqrt{V} = -3\sqrt{2\pi}t + c$ ***A1***

$c = 2\sqrt{5000\pi}$ ***A1***

$V = 0$ ***M1***

$\Rightarrow t = \frac{2}{3}\sqrt{\frac{5000\pi}{2\pi}} = 33\tfrac{1}{3}$ hours ***A1***

OR

$\int_{5000\pi}^{0} \frac{\mathrm{d}V}{\sqrt{V}} = -3\sqrt{2\pi}\int_0^T \mathrm{d}t$ ***M1A1A1***

Note:	Award ***M1*** for attempt to use definite integrals, ***A1*** for correct limits and ***A1*** for correct integrands.

$\left[2\sqrt{V}\right]_{5000\pi}^{0} = -3\sqrt{2\pi}T$ ***A1***

$T = \frac{2}{3}\sqrt{\frac{5000\pi}{2\pi}} = 33\tfrac{1}{3}$ hours ***A1***

[7 marks]

Total [16 marks]