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MATHEMATICS
HIGHER LEVEL
PAPER 3 – SERIES AND DIFFERENTIAL EQUATIONS

Monday 15 November 2010 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 7]*

Find $\lim\limits_{x \to 0} \left(\dfrac{1 - \cos x^6}{x^{12}} \right)$.

2. *[Maximum mark: 16]*

Determine whether or not the following series converge.

(a) $\displaystyle\sum_{n=0}^{\infty} \left(\sin \frac{n\pi}{2} - \sin \frac{(n+1)\pi}{2} \right)$ *[3 marks]*

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n - 1}{\pi^n}$ *[7 marks]*

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n+1}}{n(n-1)}$ *[6 marks]*

3. *[Maximum mark: 9]*

(a) Using the Maclaurin series for the function e^x, write down the first four terms of the Maclaurin series for $e^{-\frac{x^2}{2}}$.   *[3 marks]*

(b) Hence find the first four terms of the series for $\displaystyle\int_0^x e^{-\frac{u^2}{2}} \, du$.   *[3 marks]*

(c) Use the result from part (b) to find an approximate value for $\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^1 e^{-\frac{x^2}{2}} \, dx$.   *[3 marks]*

4. *[Maximum mark: 13]*

Solve the differential equation

$$(x-1)\frac{dy}{dx} + xy = (x-1)e^{-x}$$

given that $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.

5. *[Maximum mark: 15]*

Consider the infinite series

$$\frac{1}{2\ln 2} - \frac{1}{3\ln 3} + \frac{1}{4\ln 4} - \frac{1}{5\ln 5} + \dots .$$

(a) Show that the series converges. *[4 marks]*

(b) Determine if the series converges absolutely or conditionally. *[11 marks]*