



**MATHEMATICS  
HIGHER LEVEL  
PAPER 2**

Friday 5 November 2010 (morning)

Candidate session number

2 hours

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**INSTRUCTIONS TO CANDIDATES**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Triangle ABC has  $AB = 5 \text{ cm}$ ,  $BC = 6 \text{ cm}$  and area  $10 \text{ cm}^2$ .

(a) Find  $\sin \hat{B}$ . [2 marks]

(b) Hence, find the two possible values of AC, giving your answers correct to two decimal places. [4 marks]

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2. [Maximum mark: 4]

The company *Fresh Water* produces one-litre bottles of mineral water. The company wants to determine the amount of magnesium, in milligrams, in these bottles.

A random sample of ten bottles is analysed and the results are as follows:

6.7, 7.2, 6.7, 6.8, 6.9, 7.0, 6.8, 6.6, 7.1, 7.3.

Find unbiased estimates of the mean and variance of the amount of magnesium in the one-litre bottles.

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3. [Maximum mark: 5]

The weight loss, in kilograms, of people using the slimming regime *SLIM3M* for a period of three months is modelled by a random variable  $X$ . Experimental data showed that 67 % of the individuals using *SLIM3M* lost up to five kilograms and 12.4 % lost at least seven kilograms. Assuming that  $X$  follows a normal distribution, find the expected weight loss of a person who follows the *SLIM3M* regime for three months.

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4. [Maximum mark: 7]

Find the equation of the normal to the curve  $x^3y^3 - xy = 0$  at the point (1, 1).

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5. [Maximum mark: 5]

Solve the equations

$$\ln \frac{x}{y} = 1$$

$$\ln x^3 + \ln y^2 = 5.$$

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6. [Maximum mark: 7]

Consider the polynomial  $p(x) = x^4 + ax^3 + bx^2 + cx + d$ , where  $a, b, c, d \in \mathbb{R}$ .

Given that  $1+i$  and  $1-2i$  are zeros of  $p(x)$ , find the values of  $a, b, c$  and  $d$ .

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7. [Maximum mark: 6]

The random variable  $X$  follows a Poisson distribution with mean  $m$  and satisfies

$$P(X = 1) + P(X = 3) = P(X = 0) + P(X = 2).$$

(a) Find the value of  $m$  correct to four decimal places. [4 marks]

(b) For this value of  $m$ , calculate  $P(1 \leq X \leq 2)$ . [2 marks]

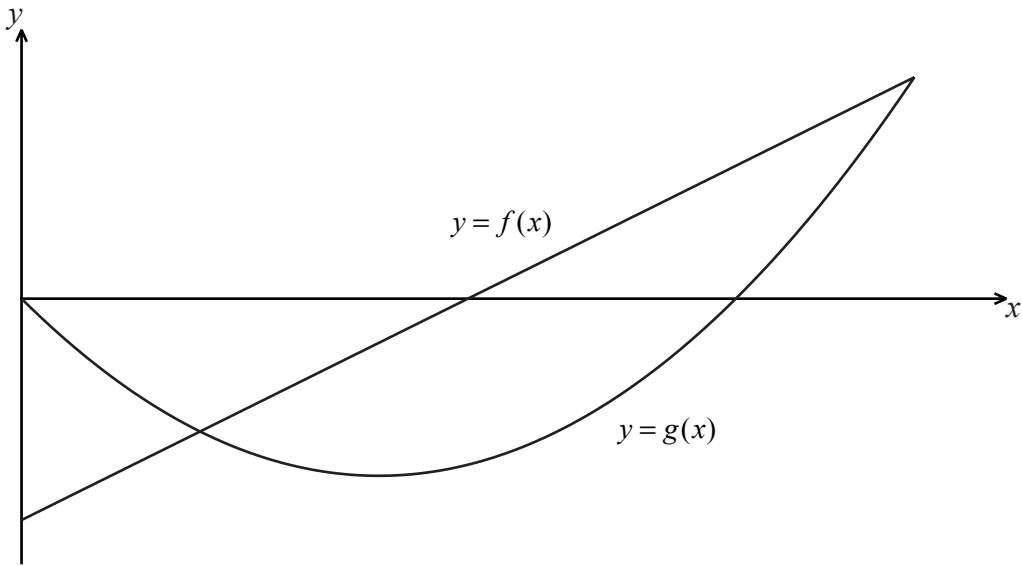
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8. [Maximum mark: 5]

The diagram shows the graphs of a linear function  $f$  and a quadratic function  $g$ .



On the same axes sketch the graph of  $\frac{f}{g}$ . Indicate clearly where the  $x$ -intercept and the asymptotes occur.

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9. [Maximum mark: 8]

Consider the vectors  $\mathbf{a} = \sin(2\alpha)\mathbf{i} - \cos(2\alpha)\mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = \cos\alpha\mathbf{i} - \sin\alpha\mathbf{j} - \mathbf{k}$ , where  $0 < \alpha < 2\pi$ .

Let  $\theta$  be the angle between the vectors  $\mathbf{a}$  and  $\mathbf{b}$ .

(a) Express  $\cos\theta$  in terms of  $\alpha$ . [2 marks]

(b) Find the acute angle  $\alpha$  for which the two vectors are perpendicular. [2 marks]

(c) For  $\alpha = \frac{7\pi}{6}$ , determine the vector product of  $\mathbf{a}$  and  $\mathbf{b}$  and comment on the geometrical significance of this result. [4 marks]

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10. [Maximum mark: 7]

The line  $y = m(x - m)$  is a tangent to the curve  $(1 - x)y = 1$ .

Determine  $m$  and the coordinates of the point where the tangent meets the curve.

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Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

**SECTION B**

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

11. [Maximum mark: 21]

Tim throws two identical fair dice simultaneously. Each die has six faces: two faces numbered 1, two faces numbered 2 and two faces numbered 3. His score is the sum of the two numbers shown on the dice.

- (a) (i) Calculate the probability that Tim obtains a score of 6.
- (ii) Calculate the probability that Tim obtains a score of at least 3. [3 marks]

Tim plays a game with his friend Bill, who also has two dice numbered in the same way. Bill’s score is the sum of the two numbers shown on his dice.

- (b) (i) Calculate the probability that Tim and Bill **both** obtain a score of 6.
- (ii) Calculate the probability that Tim and Bill obtain the same score. [4 marks]

(c) Let  $X$  denote the largest number shown on the four dice.

(i) Show that  $P(X \leq 2) = \frac{16}{81}$ .

(ii) Copy and complete the following probability distribution table.

$x$	1	2	3
$P(X = x)$	$\frac{1}{81}$		

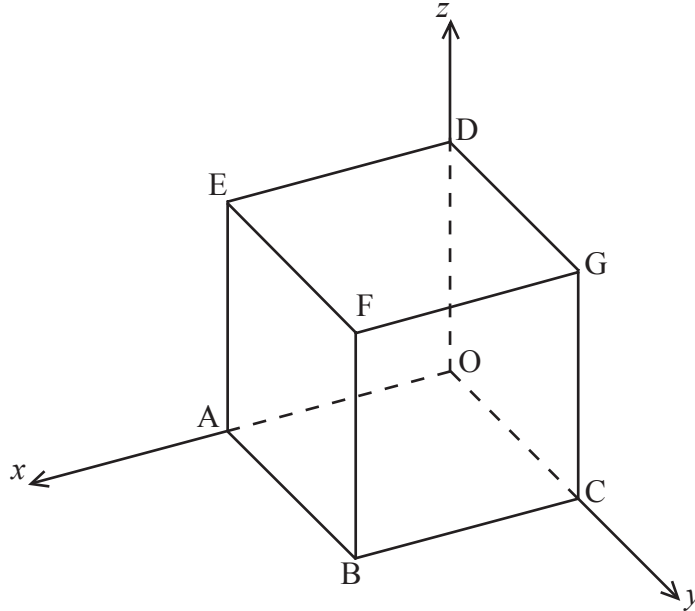
- (iii) Calculate  $E(X)$  and  $E(X^2)$  and hence find  $\text{Var}(X)$ . [10 marks]
- (d) Given that  $X = 3$ , find the probability that the sum of the numbers shown on the four dice is 8. [4 marks]



Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

12. [Maximum mark: 20]

The diagram shows a cube OABCDEFG.



Let O be the origin, (OA) the  $x$ -axis, (OC) the  $y$ -axis and (OD) the  $z$ -axis. Let M, N and P be the midpoints of [FG], [DG] and [CG], respectively. The coordinates of F are (2, 2, 2).

- (a) Find the position vectors  $\vec{OM}$ ,  $\vec{ON}$  and  $\vec{OP}$  in component form. [3 marks]
- (b) Find  $\vec{MP} \times \vec{MN}$ . [4 marks]
- (c) **Hence,**
  - (i) calculate the area of the triangle MNP;
  - (ii) show that the line (AG) is perpendicular to the plane MNP;
  - (iii) find the equation of the plane MNP. [7 marks]
- (d) Determine the coordinates of the point where the line (AG) meets the plane MNP. [6 marks]



Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

13. [Maximum mark: 19]

Let  $f(x) = \frac{a + be^x}{ae^x + b}$ , where  $0 < b < a$ .

(a) Show that  $f'(x) = \frac{(b^2 - a^2)e^x}{(ae^x + b)^2}$ . [3 marks]

(b) Hence justify that the graph of  $f$  has no local maxima or minima. [2 marks]

(c) Given that the graph of  $f$  has a point of inflexion, find its coordinates. [6 marks]

(d) Show that the graph of  $f$  has exactly two asymptotes. [3 marks]

(e) Let  $a = 4$  and  $b = 1$ . Consider the region  $R$  enclosed by the graph of  $y = f(x)$ , the  $y$ -axis and the line with equation  $y = \frac{1}{2}$ .

Find the volume  $V$  of the solid obtained when  $R$  is rotated through  $2\pi$  about the  $x$ -axis. [5 marks]

