88097207

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Wednesday 18 November 2009 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

* Do not open this examination paper until instructed to do so.
* Answer all the questions.
* Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 9]*

    An arithmetic sequence has first term 2 and common difference 4. Another arithmetic sequence has first term 7 and common difference 5. Find the set of all numbers which are members of both sequences.

2. *[Maximum mark: 16]*

   The diagram below shows the weighted graph G.
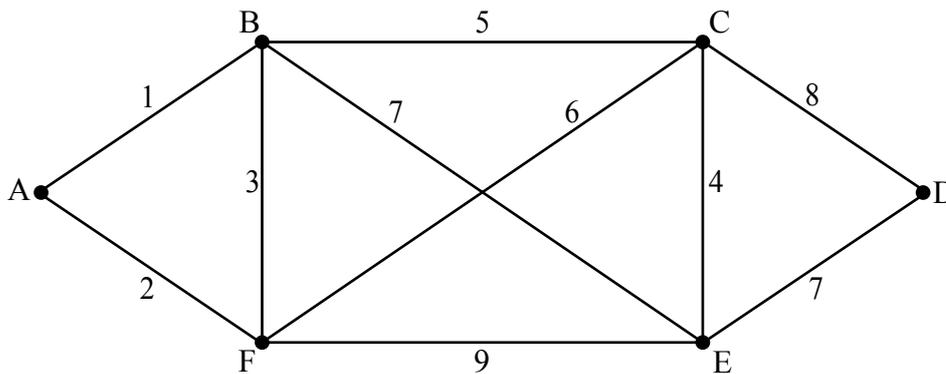
    (a) (i)   What feature of the graph enables you to deduce that G contains an Eulerian circuit?

         (ii) Find an Eulerian circuit.   *[3 marks]*

    (b) (i)   Find the number of distinct walks of length 4 beginning and ending at F.

         (ii) Determine which of these walks has the minimum weight.

         (iii) Determine which of these walks has the maximum weight.   *[7 marks]*

    (c) Use Kruskal's Algorithm to find the minimum spanning tree for G, showing the order in which the edges are added.   *[6 marks]*

3.   *[Maximum mark: 11]*

(a)   The planar graph G and its complement G' are both simple and connected. Given that G has 6 vertices and 10 edges, show that G' is a tree.   *[5 marks]*

(b)   Consider the following graphs.

Graph 1   **Graph 2**
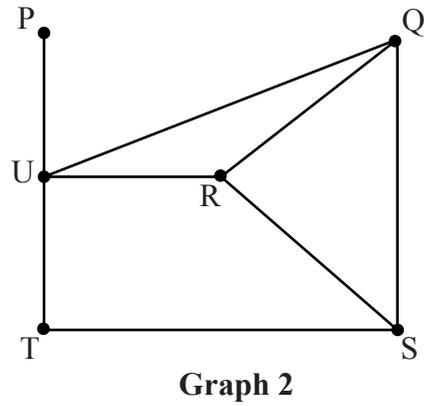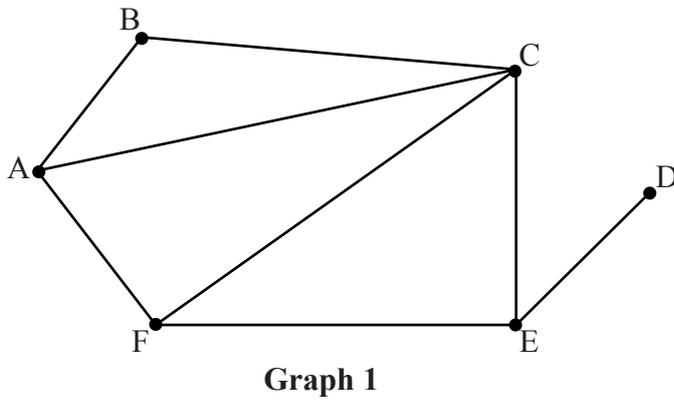
(i)   Give a reason why these two graphs are not isomorphic.

(ii)   Show, however, that an isomorphism can be produced by removing the edge UQ in Graph 2 and replacing it by a different edge. In addition to stating the new edge, your solution should indicate the bijection between the two graphs.   *[6 marks]*

4.   *[Maximum mark: 16]*

(a)   Show that a positive integer, written in base 10, is divisible by 9 if the sum of its digits is divisible by 9.   *[7 marks]*

(b)   The representation of the positive integer N in base p is denoted by $(N)_p$. If $(5^{(126)_7})_7 = (a_n a_{n-1} \ldots a_1 a_0)_7$, find a_0.   *[9 marks]*

5.   *[Maximum mark: 8]*

Show that a graph is bipartite if and only if it contains only cycles of even length.