N09/5/MATHL/HP3/ENG/TZ0/DM/M+

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MARKSCHEME

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MATHEMATICS
DISCRETE MATHEMATICS

Higher Level

Paper 3

8 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**: often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g*. ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = 2\cos(5x-3)\ \ 5 \quad = 10\cos(5x-3)$ ***A1***

Award ***A1*** for $2\cos(5x-3)\ \ 5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

*Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**. Award the marks as usual then write **(AP)** against the answer. On the **front** cover write –1**(AP)**. Deduct 1 mark from the total for the paper, not the question*.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct answers not given to three significant figures.

If there is no working shown, and answers are given to the correct two significant figures, apply the ***AP***. However, do **not** accept answers to one significant figure without working.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

1. the mth term of the first sequence $= 2 + 4(m-1)$ ***(M1)(A1)***
the nth term of the second sequence $= 7 + 5(n-1)$ ***(A1)***

EITHER

equating these, ***M1***

$5n = 4m - 4$

$5n = 4(m-1)$ ***(A1)***

4 and 5 are coprime ***(M1)***

$\Rightarrow 4 \mid n$ so $n = 4s$ or $5 \mid (m-1)$ so $m = 5s+1,\ s \in \mathbb{Z}^+$ ***(A1)A1***

thus the common terms are of the form $\{2 + 20s;\ s \in \mathbb{Z}^+\}$ ***A1***

OR

the numbers of both sequences are
$2, 6, 10, 14, 18, 22$
$7, 12, 17, 22$ ***A1***

so 22 is common ***A1***

identify the next common number as 42 ***(M1)A1***

the general solution is $\{2 + 20s;\ s \in \mathbb{Z}^+\}$ ***(M1)A1***

Total [9 marks]

2. (a) (i) all the vertices have even degree ***A1***

(ii) for example ABCDECFBEFA ***A2***

[3 marks]

(b) (i) the adjacency matrix is

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$ ***(M1)(A1)***

attempting to find $\boldsymbol{A}^4$ ***(M1)***

$$\left(\boldsymbol{A}^4 = \begin{pmatrix} 14 & 17 & 22 & 8 & 22 & 17 \\ 17 & 38 & 32 & 22 & 32 & 37 \\ 22 & 32 & 38 & 17 & 37 & 32 \\ 8 & 22 & 17 & 14 & 17 & 22 \\ 22 & 32 & 37 & 17 & 38 & 32 \\ 17 & 37 & 32 & 22 & 32 & 38 \end{pmatrix} \right)$$

the required number of walks is the (F, F) element in $\boldsymbol{A}^4$ which is 38 ***A2***

(ii) FABAF (6) ***A1***

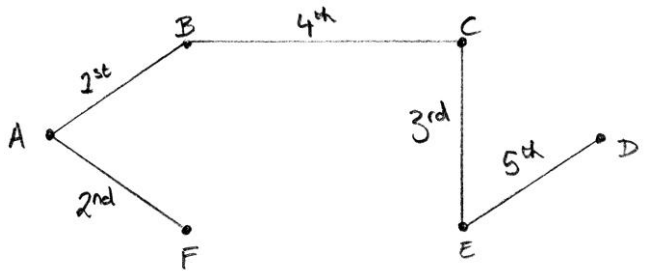
(iii) FEFEF (36) ***A1***

[7 marks]

continued ...

Question 2 continued

(c) the edges are included in the order shown



M1A1A1A1A1A1

Note:	Award each ***A1*** for the edge added in the correct order. Award no further marks after the first error.

[6 marks]

Total [16 marks]

3. (a) the complete graph with 6 vertices has 15 edges so G' has 6 vertices and 5 edges ***M1A1***

the number of faces in G', $f = 2 + e - v = 1$ ***M1A1***

it is therefore a tree because $f = 1$ ***R1***

Note:	Accept it is a tree because $v = e + 1$.

[5 marks]

(b) (i) They are not isomorphic because, for example, in Graph 1 the vertex of degree 1 is adjacent to a vertex of degree 3; in Graph 2 the vertex of degree 1 is adjacent to a vertex of degree 4. ***R2***

(ii) remove UQ and insert RT ***A2***

the bijection is

A, B, C, D, E, F $\rightarrow$ S, Q, R, P, U, T ***A2***

[6 marks]

Total [11 marks]

4. (a) consider the decimal number $A = a_n a_{n-1} \ldots a_0$ ***M1***

$A = a_n \times 10^n + a_{n-1} \times 10^{n-1} + \ldots + a_1 \times 10 + a_0$ ***M1***

$= a_n \times (10^n - 1) + a_{n-1} \times (10^{n-1} - 1) + \ldots + a_1 \times (10 - 1) + a_n + a_{n-1} + \ldots + a_0$ ***M1A1***

$= a_n \times 99\ldots9(n \text{ digits}) + a_{n-1} \times 99\ldots9(n-1 \text{ digits}) + \ldots + 9a_1 + a_n + a_{n-1} + \ldots + a_0$ ***A1***

all the numbers of the form 99…9 are divisible by 9 (to give 11…1), ***R1***

hence A is divisible by 9 if $\sum_{i=0}^{n} a_i$ is divisible by 9 ***R1***

Note: A method that uses the fact that $10^t \equiv 1 (\text{mod } 9)$ is equally valid.

[7 marks]

(b) by Fermat's Little Theorem $5^6 \equiv 1 (\text{mod } 7)$ ***M1A1***

$(126)_7 = (49 + 14 + 6)_{10} = (69)_{10}$ ***M1A1***

$5^{(126)_7} \equiv 5^{(11 \times 6 + 3)_{10}} \equiv 5^{(3)_{10}} (\text{mod } 7)$ ***M1A1***

$5^{(3)_{10}} = (125)_{10} = (17 \times 7 + 6)_{10} \equiv 6 (\text{mod } 7)$ ***M1A1***

hence $a_0 = 6$ ***A1***

[9 marks]

Total [16 marks]

5. Suppose the graph is bipartite so that the vertices belong to one of two disjoint sets M, N. ***M1***

Then consider any vertex V in M. To generate a cycle returning to V, we must go to a vertex in N, then to a vertex in M, then to a vertex in N, then to a vertex in M, *etc.* ***R1***

To return to V, therefore, which belongs to M, an even number of steps will be required. ***R1***

Now suppose the graph contains only cycles of even length. ***M1***

Starting at any vertex V, define the set M as containing those vertices accessible from V in an even number of steps and the set N as containing those vertices accessible from V in an odd number of steps. ***R1***

Suppose that the vertex X belongs to both M and N. Then consider the closed walk from V to X one way and back to V the other way. This closed walk will be of odd length. This closed walk can be contracted to a cycle which will also be of odd length, giving a contradiction to the initial assumption. ***R1***

There can therefore be no vertices common to M and N which shows that the vertices can be divided into two disjoint sets and the graph is bipartite. ***R1***

Consider any edge joining P to vertex Q. Then either $P \in M$ in which case $Q \in N$ or vice versa. In either case an edge always joins a vertex in M to a vertex in N so the graph is bipartite. ***R1***

Total [8 marks]