M09/5/MATHL/HP2/ENG/TZ2/XX+

22097206

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 8 May 2009 (morning)

2 hours

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 4]*

In a class of 20 students, 12 study Biology, 15 study History and 2 students study neither Biology nor History.

(a) Illustrate this information on a Venn diagram. *[2 marks]*

(b) Find the probability that a randomly selected student from this class is studying both Biology and History. *[1 mark]*

(c) Given that a randomly selected student studies Biology, find the probability that this student also studies History. *[1 mark]*

2. *[Maximum mark: 5]*

Given that $\boldsymbol{a} = 2\sin\theta\,\boldsymbol{i} + (1 - \sin\theta)\,\boldsymbol{j}$, find the value of the acute angle θ, so that $\boldsymbol{a}$ is perpendicular to the line $x + y = 1$.

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3. *[Maximum mark: 6]*

(a) Differentiate $f(x) = \arcsin x + 2\sqrt{1-x^2}$, $x \in [-1, 1]$. *[3 marks]*

(b) Find the coordinates of the point on the graph of $y = f(x)$ in $[-1, 1]$, where the gradient of the tangent to the curve is zero. *[3 marks]*

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4. *[Maximum mark: 6]*

(a) The graph of $y = \ln(x)$ is transformed into the graph of $y = \ln(2x+1)$. Describe two transformations that are required to do this. *[2 marks]*

(b) Solve $\ln(2x+1) > 3\cos(x)$, $x \in [0, 10]$. *[4 marks]*

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5. *[Maximum mark: 5]*

An influenza virus is spreading through a city. A vaccination is available to protect against the virus. If a person has had the vaccination, the probability of catching the virus is 0.1; without the vaccination, the probability is 0.3. The probability of a randomly selected person catching the virus is 0.22.

(a) Find the percentage of the population that has been vaccinated. *[3 marks]*

(b) A randomly chosen person catches the virus. Find the probability that this person has been vaccinated. *[2 marks]*

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6. *[Maximum mark: 6]*

The acceleration in m s^{-2} of a particle moving in a straight line at time t seconds, $t \geq 0$, is given by the formula $a = -\frac{1}{2}v$. When $t = 0$, the velocity is 40 m s^{-1}.

Find an expression for v in terms of t.

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7. *[Maximum mark: 8]*

The cubic curve $y = 8x^3 + bx^2 + cx + d$ has two distinct points P and Q, where the gradient is zero.

(a) Show that $b^2 > 24c$. *[4 marks]*

(b) Given that the coordinates of P and Q are $\left(\frac{1}{2}, -12\right)$ and $\left(-\frac{3}{2}, 20\right)$ respectively, find the values of b, c and d. *[4 marks]*

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8. *[Maximum mark: 5]*

Six people are to sit at a circular table. Two of the people are not to sit immediately beside each other. Find the number of ways that the six people can be seated.

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9. *[Maximum mark: 8]*

Using the substitution $x = 2\sin\theta$, show that

$$\int \sqrt{4-x^2}\,\mathrm{d}x = Ax\sqrt{4-x^2} + B\arcsin\left(\frac{x}{2}\right) + \text{constant},$$

where A and B are constants whose values you are required to find.

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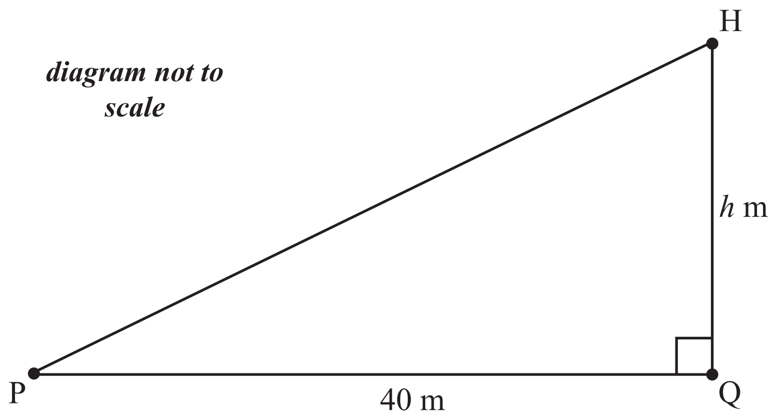
10. *[Maximum mark: 7]*

A helicopter H is moving vertically upwards with a speed of 10 m s^{-1}. The helicopter is h m directly above the point Q which is situated on level ground. The helicopter is observed from the point P which is also at ground level and $\text{PQ} = 40$ m. This information is represented in the diagram below.



When $h = 30$,

(a) show that the rate of change of $\text{H}\hat{\text{P}}\text{Q}$ is 0.16 radians per second; *[3 marks]*

(b) find the rate of change of PH. *[4 marks]*

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SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 20]*

Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212. *[1 mark]*

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water. *[3 marks]*

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water, is greater than 0.99. *[4 marks]*

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B. *[6 marks]*

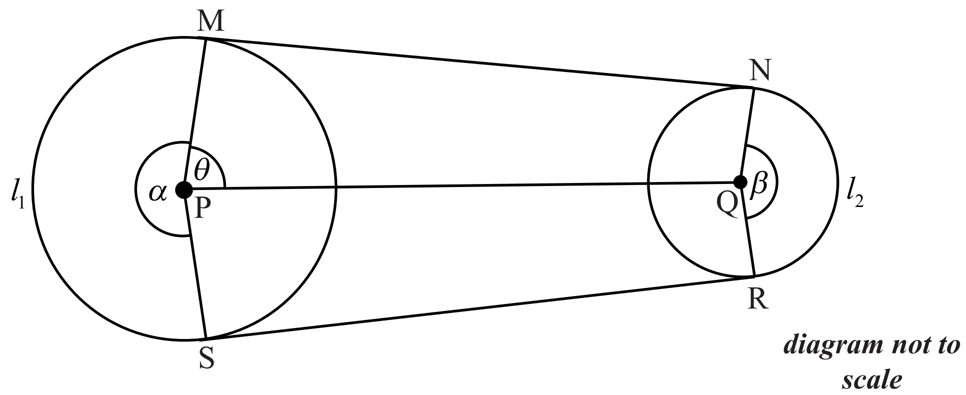
(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X, follows a Poisson distribution such that $\mathrm{P}(X=2)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$.

(i) Find the value of m.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period. *[6 marks]*

12. *[Maximum mark: 18]*

Two non-intersecting circles C_1, containing points M and S, and C_2, containing points N and R, have centres P and Q where $PQ = 50$. The line segments [MN] and [SR] are common tangents to the circles. The size of the reflex angle MPS is α, the size of the obtuse angle NQR is β, and the size of the angle MPQ is θ. The arc length MS is l_1 and the arc length NR is l_2. This information is represented in the diagram below.



The radius of C_1 is x, where $x \geq 10$ and the radius of C_2 is 10.

(a) Explain why $x < 40$. *[1 mark]*

(b) Show that $\cos\theta = \dfrac{x-10}{50}$. *[2 marks]*

(c) (i) Find an expression for MN in terms of x.

(ii) Find the value of x that maximises MN. *[2 marks]*

(d) Find an expression in terms of x for

(i) α;

(ii) β. *[4 marks]*

(e) The length of the perimeter is given by $l_1 + l_2 + \text{MN} + \text{SR}$.

(i) Find an expression, $b(x)$, for the length of the perimeter in terms of x.

(ii) Find the maximum value of the length of the perimeter.

(iii) Find the value of x that gives a perimeter of length 200. *[9 marks]*

13. *[Maximum mark: 22]*

Consider the graphs $y = e^{-x}$ and $y = e^{-x}\sin 4x$, for $0 \le x \le \frac{5\pi}{4}$.

(a) On the same set of axes draw, on graph paper, the graphs, for $0 \le x \le \frac{5\pi}{4}$. Use a scale of 1 cm to $\frac{\pi}{8}$ on your x-axis and 5 cm to 1 unit on your y-axis. *[3 marks]*

(b) Show that the x-intercepts of the graph $y = e^{-x}\sin 4x$ are $\frac{n\pi}{4}$, $n = 0, 1, 2, 3, 4, 5$. *[3 marks]*

(c) Find the x-coordinates of the points at which the graph of $y = e^{-x}\sin 4x$ meets the graph of $y = e^{-x}$. Give your answers in terms of π. *[3 marks]*

(d) (i) Show that when the graph of $y = e^{-x}\sin 4x$ meets the graph of $y = e^{-x}$, their gradients are equal.

(ii) Hence explain why these three meeting points are not local maxima of the graph $y = e^{-x}\sin 4x$. *[6 marks]*

(e) (i) Determine the y-coordinates, y_1, y_2 and y_3, where $y_1 > y_2 > y_3$, of the local maxima of $y = e^{-x}\sin 4x$ for $0 \le x \le \frac{5\pi}{4}$. You do not need to show that they are maximum values, but the values should be simplified.

(ii) Show that y_1, y_2 and y_3 form a geometric sequence and determine the common ratio r. *[7 marks]*